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Research on Risk Identification and Decision Support Models for PPP Project Management Based on Fuzzy Mathematics

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SUMMARY: *With China's vigorous socioeconomic development, risk management and decision support for social capital partners in PPP projects have garnered increased attention. This paper adopts fuzzy mathematics as a method for identifying risks in PPP project management, providing an overall assessment of project risk management. Building upon this, it integrates the Analytic Hierarchy Process (AHP) to jointly evaluate project management risks under the PPP model. A risk hierarchy structure is established, a set of risk factor evaluations is determined, the probability distribution of secondary indicator risks is calculated, and the overall risk management evaluation results for the project are generated. The full lifecycle management process of the R County infrastructure PPP project received an overall risk score of 3.2, indicating a relatively high level of risk significance. Net Present Value (NPV) and Internal Rate of Return (IRR) were selected as evaluation indicators for the decision support model to assess the financial sensitivity of R County's PPP project. Under an adverse scenario where construction costs increased by 10% while benefits remained unchanged, the net present value reached 165,325 million yuan, demonstrating the project's continued profitability. This indicates that the project possesses a certain degree of risk resistance during its construction phase.*

KEYWORDS: *Fuzzy Mathematics; Analytic Hierarchy Process; Decision Support Model; PPP Project Management Risk; Financial Sensitivity*

1 Introduction

The Public-Private Partnership (PPP) model establishes a long-term cooperative relationship in infrastructure and public service sectors. Currently promoted and implemented across multiple regions, PPP alleviates fiscal pressures on governments at all levels by introducing private capital. This approach accelerates project implementation and completion, enhances the efficiency and quality of public service delivery, and elevates overall service standards. It has become a key policy instrument complementing effective market forces and proactive government action [1-4]. Under this model, governments and private investors jointly participate in project design, financing, construction, operation, and maintenance, sharing project risks while allocating responsibilities based on their respective strengths and resources. This significantly boosts private investors' confidence and enthusiasm, strengthens the cooperative relationship between governments and private capital, and facilitates mutually beneficial development [5-9].

Compared to conventional projects, PPPs are capital-intensive, requiring substantial

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funding with extended capital recovery periods. Involving multiple stakeholders with diverse objectives and typically bearing significant social benefits, their complexity, diversity, dynamism, and phased nature create a unique risk environment [10-12]. Globally, 15% of PPP projects fail due to mismanagement. Currently, risk management knowledge systems lack clear guidance on the specific management responsibilities of key stakeholders, particularly among participants in large-scale projects. Existing frameworks primarily focus on outlining risk management steps, processes, and methodologies [13-15]. Traditional risk management models not only struggle to ensure efficient project execution but may also harbor latent risks of project failure. Ambiguity and uncertainty in PPP projects remain persistent challenges, largely because conventional approaches tend to address each risk point in isolation. Without systematic strategies, these methods fail to effectively integrate and allocate resources [16-18]. Given that project risks evolve over time and with project progress, leading to dynamic changes in risk states, risk identification is crucial [19, 20]. Initiating risk identification in PPP project management to achieve comprehensive risk management is a key factor influencing the economic and social benefits of PPP projects.

Fuzzy mathematics, a branch of mathematics addressing ambiguity, provides support for tackling uncertainty issues. It has been preliminarily applied in various fields including fuzzy control, fuzzy recognition, fuzzy cluster analysis, fuzzy decision-making, fuzzy evaluation, and information retrieval [21-23].

Conducting project risk identification and classification is a prerequisite for risk management. Common risks in PPP projects include political risks, legal risks, economic risks, and force majeure risks. Liu and Wei [24] employed a questionnaire survey to explore factors in electric vehicle charging infrastructure PPP projects, identifying seven risk factors encompassing political, legal, economic, and environmental aspects. They evaluated project risk levels using the Fuzzy Ranking Similarity Ideal Solution method. Mao and Zhang [25] employed expert interviews to identify risks in public tunnel PPP projects, categorizing them by severity. They then allocated risks based on three principles and proposed corresponding risk management measures. Liu and Fang [26] employed a stakeholder-perspective expert weighting method to randomly select expert interest coefficients. They determined expert comprehensive evaluations and weights, then performed weighted calculations using expert weights and the Analytic Hierarchy Process (AHP) to identify and evaluate PPP project risks for underground utility tunnels through comprehensive weighting. Xu et al. [27] designed a fiscal risk matrix model and a fuzzy evaluation model for PPP fiscal risks based on the Analytic Hierarchy Process (AHP), identifying and evaluating PPP project fiscal risks from multiple dimensions. Hai-Min et al. [28] employed hierarchical holographic modeling, explanatory structural modeling, and OWA-fuzzy clustering evaluation to respectively identify risks, analyze factor relationships, and conduct comprehensive evaluations for transportation corridor PPP projects, thereby supporting scientific decision-making for project stakeholders. Zhang et al. [29] employed the Decision Experiment and Evaluation Laboratory (DEMATEL) method to rank risks in sponge city PPP projects while analyzing correlations among risk indicators to identify critical project risks. Zhang et al. [30] developed a risk identification and analysis model for electric vehicle charging infrastructure PPP projects using the DEMATEL method and a two-component linguistic representation model. This model assessed risks across linguistic, political, legal, temporal, economic, social, environmental, technological, and project-specific dimensions. Qian and Li [31] employed Holzer's three-dimensional model to identify port risks in the Taiping Port PPP project, providing risk management references for project administrators. Han et al. [32] combined an explanatory structural model with an influence matrix cross-reference multiplication applied to classification analysis to examine correlations and degrees of association among risks in PPP projects, thereby identifying key project factors. Among these identification and evaluation studies, most models were developed for specific industries or project phases, lacking universality. Additionally, some research omitted post-identification decision recommendations, making it difficult to close the loop between identification results and decision support. The application characteristics of fuzzy mathematics offer new insights for identifying and evaluating the fuzzy and uncertain factors inherent in PPP project risks.

This paper employs a structured analytical approach to elucidate investment risks in infrastructure projects under the PPP model. Building upon fuzzy mathematical evaluation methods, the Analytic Hierarchy Process (AHP) is introduced to perform fuzzy computations on PPP projects, yielding evaluation outcomes. A set of risk factor indicators for PPP project management is constructed. Through practical case analysis, the applicability of the PPP project risk evaluation model is assessed. Net Present Value (NPV) and Internal Rate of Return (IRR) are selected as the two key indicators for PPP project investment decision evaluation. Monte Carlo simulation is applied to determine the probability distributions of each variable, with the simulated data incorporated into the investment decision model. Risk management countermeasures are designed, and the decision optimization effects of the selected schemes are evaluated. By comparing the basic scheme with the analyzed scheme, decision-makers can select appropriate risk response strategies.

2 Related Concepts and Theoretical Approaches

2.1 PPP-Related Concepts

2.1.1 Definition of PPP Projects

The PPP project model originated in the United Kingdom during the 1980s. In 1989, the British government, building upon the removal of restrictions from the original model, pioneered a pathway for private capital to integrate into public service sectors. This approach was primarily applied to infrastructure projects such as transportation and piped water supply, as well as public service domains, and it was the first to propose the Private Finance Initiative (PFI) model [33]. Since then, the PPP project model has gradually taken center stage in history. With the development of project financing, the PPP model has also become increasingly popular.

The definition of the PPP model—a cooperative framework established between governments and social capital—can generally be understood from both broad and narrow perspectives. Broadly speaking, the PPP model refers to a long-term cooperative relationship established between the public sector and the private sector to provide public goods or services. In a narrow sense, PPP refers to the government employing commercial rather than administrative actions to participate in projects by holding shares in newly established project companies, thereby achieving project control. This model also involves complementary strengths, risk sharing, and maximizing benefits during corporate collaboration. According to data from China's National Development and Reform Commission, PPP is typically applied to transportation infrastructure or public service projects characterized by large investment scales, stable long-term demand, and high marketization levels. These projects often follow a government-initiated model for establishing public-private partnerships.

The international academic community now increasingly endorses the broader definition of PPP as an umbrella term for various long-term government-enterprise collaboration approaches. This includes methods such as BOT, TOT, and PFI, emphasizing objectives like risk sharing, profit distribution, and maintaining public sector control over projects during public-private partnerships.

2.1.2 Characteristics of PPP Projects

Although the PPP model has only been in existence for a relatively short time since its inception and global adoption, it is evident that PPP represents more than just a financing approach. Its management philosophy represents significant breakthroughs and changes compared to conventional engineering projects. Whether in terms of participating entities or collaboration methods, PPP can be described as a fusion of management and financing. This integration transforms the delivery of public services, enabling both private capital and the public sector to leverage their respective strengths while enhancing service provision capabilities. In summary, PPP projects exhibit the following three characteristics:

(1) Long-Term Partnership

PPP implementation constitutes a long-term collaborative process between governments and private investors. Typically grounded in shared objectives, this model seeks to maximize mutual benefits through optimal resource allocation over an extended partnership. Unlike conventional construction projects, PPP initiatives feature prolonged implementation phases and long-term contracts. This necessitates mutual agreement between parties to leverage complementary strengths through partnership, thereby minimizing costs and risks while maximizing public benefits and societal value.

(2) Shared Benefits

Benefit sharing in PPP means all project participants achieve predetermined objectives within the cooperative framework. Each party leverages its strengths: governments utilize policy-making and legal expertise, while private investors contribute capital mobilization and operational efficiency. The government achieves efficient delivery of public services, provides public goods, shares social benefits, facilitates public convenience, and gains political achievements. The private sector secures long-term, stable investment returns.

(3) Risk Sharing

Given the inherent high risks in PPP projects, designing risk-sharing mechanisms to clearly allocate risk factors among partners is essential for smooth project execution. Risk allocation schemes are influenced by factors such as each party's risk tolerance, investment return expectations, and management capabilities. A reasonable allocation leverages each party's strengths, mitigates their weaknesses, and balances the benefits of risk-bearing. Only by clearly assigning all potential risk factors to the most suitable bearers in a rational manner can systematic risk management be achieved in PPP projects, thereby fulfilling the objective of shared risk.

2.2 Risk Management Theory

2.2.1 Risk Identification

Risk identification is the first and crucial step in project risk management. It requires a comprehensive assessment conducted prior to project implementation, utilizing relevant methodologies to identify risks based on their origins or the various project phases, and classifying them accordingly. Failure to thoroughly identify all potential risk points at this stage renders subsequent risk management efforts meaningless. Therefore, scientifically and effectively identifying each risk factor lays a solid foundation for subsequent work.

2.2.2 Risk Assessment

Risk assessment involves applying appropriate methodologies to evaluate the probability of occurrence and corresponding impact levels of each risk factor within the final risk identification list. This typically requires a comprehensive evaluation combining qualitative and quantitative approaches to ensure the final determination of critical risks is free from logical bias, thereby providing reliable mathematical support for subsequent risk response strategies.

Numerous risk assessment methods exist. Through extensive literature review and consideration of the study's focus on PPP projects and method applicability, this paper selects fuzzy mathematics as the risk identification approach for PPP project management [34].

Fuzzy Comprehensive Evaluation Method: Originally proposed by American automation control experts to express uncertainty in phenomena [35]. This method employs fuzzy mathematics as its fundamental approach. Based on the membership theory of fuzzy mathematics, it converts qualitative evaluation problems into quantitative assessments. Subsequently, following membership principles, it transforms qualitative evaluations into quantitative outcomes. It serves as a comprehensive evaluation method suitable for assessing problems constrained by multiple factors.

3 Risk Assessment of Project Management under the PPP Model

3.1 Risk Evaluation Process

Investment in infrastructure projects can be categorized as sustainable development risk management. Its risk assessment process can be articulated using structured analysis methods (IDEFO method), which fundamentally involves expressing all functions hierarchically according to their distinct characteristic structures. A concrete representation is shown in Figure 1. As illustrated in Figure 1, the outcomes of risk assessment are influenced by multiple factors.

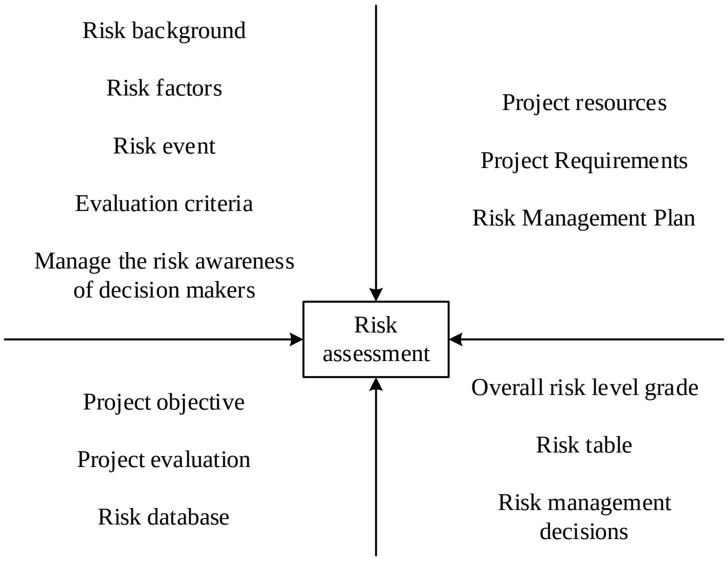

Figure 1: Structured Analysis Diagram of Risk assessment

3.2 Risk Evaluation Methods for PPP Projects

3.2.1 Analytic Hierarchy Process

The fundamental steps in applying the Analytic Hierarchy Process (AHP) begin with establishing a hierarchical model. Based on in-depth analysis of the actual problem, relevant factors are decomposed top-down into multiple levels according to different attributes. Factors at the same level are subordinate to factors at the level above or influence factors at the level above, while simultaneously governing factors at the level below or being influenced by factors at the level below [36]. The top layer represents the objective layer, typically containing only one factor. The bottom layer usually constitutes the solution or object layer. Intermediate layers may include one or more levels, typically designated as criterion or indicator layers. When criteria exceed a certain threshold (e.g., more than nine), they should be further decomposed into sub-criterion layers.

Step 2: Construct a pairwise comparison matrix. Starting from the second level of the hierarchical model, form pairwise comparison matrices using the pairwise comparison method and a 1–9 rating scale for all factors at the same level that are subordinate to (or influence) each factor at the preceding level, continuing down to the lowest level. For each pairwise comparison matrix, calculate the maximum eigenvalue and corresponding eigenvector, then perform consistency tests using the consistency index, random consistency index, and consistency ratio. If the test passes, the eigenvectors (after normalization) become the weight vectors. If it fails, the pairwise comparison matrix must be reconstructed. Step 3: Construct a judgment matrix. A key feature of the Analytic Hierarchy Process is expressing the relative importance levels of two alternatives as pairwise importance ratios. For example, for a given criterion, pairwise comparisons are made between all alternatives under it, and their importance levels are rated accordingly.

3.2.2 Fuzzy Mathematical Evaluation Method

When faced with problems where uncertainty plays a significant role, there has been a tradition in science and engineering of turning to probability theory. This shift was reasonable when no alternative tools existed to handle uncertainty. Today, this is no longer the case. Fuzzy logic offers a rich and meaningful complement to standard logic. This theory is based on membership functions operating within the real number range [0,1]. New operations for logical calculus are also introduced, and in principle, they demonstrate at least a generalization of classical logic [1,2]. Although both operate within the same numerical range [0,1], fuzzy set theory differs from probability theory. Thus, it also analyzes problems of non-determinism. In this paper, it can quantify the overall risk of a project. As a comprehensive evaluation method, it can integrate various indicator factors in risk assessment. Therefore, this paper also adopts this method for venture capital analysis. In fuzzy mathematics, determining fuzzy subsets and membership functions forms the foundation for establishing mathematical models. Let U be the domain. The mapping:

$$A(x):U \to [0.1] \tag{1}$$

A fuzzy subset A is defined on U. The mapping $A(x)$ is called the membership function of A, representing the degree of membership of x in A. The point x that satisfies $A(x)=0.5$ is called a transition point of A, where this point exhibits the highest degree of fuzziness. When the mapping $A(x)$ takes only 0 or 1, the fuzzy subset A becomes a classical subset, and $A(x)$ is its characteristic function. Thus, classical subsets are a special case of fuzzy subsets. Three common methods for determining membership functions are fuzzy statistical methods, assignment methods, and borrowing existing “objective” scales. After determining the fuzzy set and membership functions, fuzzy comprehensive evaluation can proceed as follows:

(1) Establish the set of factors influencing the evaluation object:

$$U = \{u_1, u_2, \cdots u_n\} \tag{2}$$

Factors refer to the various metric elements that require evaluation.

(2) Establish an evaluation set:

$$Y = \left\{Y_1, Y_2, \cdots, Y_m\right\} \tag{3}$$

A set of evaluation criteria typically constitutes a collection of grading levels, with the number of criteria determined based on each evaluator's understanding of the nature of the indicator factors.

(3) Establish a single-factor evaluation

This involves constructing a fuzzy mapping from U to $F(Y)$:

$$f = U \to F(Y) \tag{4}$$

From f, the fuzzy relation R can be derived, leading to the fuzzy relation matrix R:

$$(R_{ij})_{n\times m} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & & \\ R_{n1} & R_{n2} & \cdots & R_{nm} \end{bmatrix} \tag{5}$$

R_{ij} denotes the membership degree of the i th factor for the j th judgment.

(4) Establishing the Weight Set

Generally, the influence of each evaluation factor on the evaluation result varies. Therefore, a weight coefficient must be assigned to each factor, denoted as:

$$A = \{a_1, a_2 \cdots, a_n\} \tag{6}$$

In the formula, a_i represents the weight assigned to the i th factor, typically defined as:

$$\sum_{i=1}^{0} a_i = 1 \tag{7}$$

(5) Comprehensive Evaluation

Let B denote the fuzzy comprehensive evaluation of the subject being evaluated, then:

$$B = A \times R = (a_1, a_2, \cdots, a_n) \times \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & & \\ R_{n1} & R_{n2} & \cdots & R_{nm} \end{bmatrix} = (b_1, \quad b_2, \cdots, \quad b_m) \tag{8}$$

3.2.3 Fuzzy Hierarchical Evaluation

The Fuzzy Hierarchical Evaluation Method is an analytical approach that integrates fuzzy mathematics evaluation techniques with the Analytic Hierarchy Process (AHP). Its operational procedure involves first categorizing the elements under analysis into hierarchical levels to establish a corresponding hierarchical structure. Subsequently, pairwise comparisons are conducted to assess the relative importance of elements at each level, constructing a corresponding indicator-element judgment matrix. The weights for each indicator are then calculated. Finally, the membership degrees of each factor set—derived from experts' theoretical and practical insights—are applied to perform fuzzy calculations, ultimately yielding the evaluation results.

3.3 Establish a Hierarchy

The risk factor indicators are stratified into target-level and indicator-level categories to form the project management risk factor indicator set under the PPP model, as shown in Table 1.

Table 1: The management risk factors of the project are composed in the PPP model

Target layer	Risk factors	
	First-level indicator	Secondary indicators
PPP project management risks	Political and legal risks	Policy change risk
		Administrative intervention risk
		Economic risk
		Bidding risk
	Financial risk	Financing risk
		Liquidity risk
		Market risk
		Budgetary risk
	Design and construction risks	Construction risk
		The design party case changes the risk
		Cost risk of construction
		Quality risk of construction
		Construction safety risk
	Operational management risk	Project change risk
		Material supply risk
		Organized risk
		Market risk

3.4 Calculation of Element Weights at Each Level

3.4.1 Establishing a Pairwise Comparison Judgment Burden of Proof

When constructing a risk evaluation model, the first step is to compare the importance of factors within the same indicator layer. After determining the relative importance of each factor, establish the corresponding judgment matrix. Risk factors at the criterion level are denoted as Q_{ij} and belong to the objective level Q_i. In the Analytic Hierarchy Process (AHP), the importance of elements in the judgment matrix is quantified. The importance of each risk factor at the criterion level relative to the objective level is expressed numerically in a quantitative matrix, as shown in Equation (9):

$$(Q_{ij})_{n\times m}=\begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1m} \\ Q_{21} & Q_{22} & \cdots & Q_{2m} \\ \vdots & \vdots & & \\ Q_{n1} & Q_{n2} & \cdots & Q_{nm} \end{bmatrix} \tag{9}$$

This paper employs the 1-9 scale method primarily because it aligns more closely with people's psychological habits of judgment and comparison.

3.4.2 Calculate the Weight Values for Each Layer Factor

The primary methods for calculating the weight vector values of a comparison judgment matrix include:

(1) Summation method

$$W_i=\frac{1}{n}\sum_{j=1}^{n}\frac{Q_{ij}}{\sum_{k=1}^{n}Q_{kj}}(i=1,2,3,\cdots,n) \tag{10}$$

The calculation steps are as follows:

1) Normalize each column vector of Q

2) Sum the normalized vectors over j:

3) Multiply by $1/n$ to obtain the weight vector

(2) Root Method

Perform geometric averaging on Q followed by normalization. Compute the product of Q and raise it to the power of $1/n$ to obtain:

$$\bar{W}_t=n\sqrt{\prod_{j=1}^{n}Q_{ij}} \tag{11}$$

To calculate the weights, refer to the formula:

$$W_i=\frac{\bar{W}_i}{\sum_{i=1}^{n}\bar{W}_i} \tag{12}$$

The calculation steps are as follows:

1) Multiply Q by j

2) Raise each component to the power of $1/n$

3) Normalize to obtain the weight vector

(3) Optimal Transfer Matrix Method

To calculate the factor weights of the decision matrix, follow these steps:

1) From $\overline{Wi}=(T_i)^{1/n}(i=1,2,\cdots,5)$ we obtain:

$$\bar{W}_1,\bar{W}_2,\cdots,\bar{W}_5 \tag{13}$$

2) Normalize $\bar{W}_i$ using formula (14) to obtain W, i.e., the indicator element weight:

$$W = \frac{\bar{W}_i}{\sum_{i=1}^{5} \bar{W}_i} \quad (i = 1, 2, \cdots, 5) \tag{14}$$

3.4.3 Judgment Matrix Consistency Test

To ensure the applicability of the method and guarantee that all obtained weights are logically consistent and reasonable, a consistency check is performed on the judgment matrix. When the judgment matrix satisfies:

$$CR = \frac{CI}{RI} < 0.1 \tag{15}$$

Indicates that the judgment matrix satisfies the consistency test. Here, CI denotes the consistency index:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{16}$$

In the equation, $\lambda_{\max}$ is the maximum eigenvalue of the matrix, calculated according to Equation (17):

$$\lambda_{\max} = \sum_{i=1}^{n} \frac{(UW)_i}{nW_i} \tag{17}$$

In the formula, $(UW)_i$ denotes the i th element of the vector UW, and RI represents the average random uniform indicator.

3.4.4 Factor Evaluation Scale Set

(1) Level-1 Fuzzy Comprehensive Evaluation

After determining the risk weights for each factor, an evaluation metric is required to assess the relative impact of each risk factor on the project. This necessitates establishing a set of evaluation grades.

$$Y = \{Y_1, Y_2, \cdots, Y_m\} \tag{18}$$

Among these, $Y_i (j = 1, 2, \cdots, m)$ represents possible evaluation outcomes.

Y={very small, small, moderate, large, larger, very large}={0.1, 0.3, 0.5, 0.7, 0.9}.

In fuzzy mathematics, factor quantification analysis typically assigns values within the fuzzy linguistic range [0, 1]. This yields the standard membership degrees for the evaluation grade set: $V = (0.1, 0.3, 0.5, 0.7, 0.9)$. Using this evaluation grade set to assess risk factor indicators across each indicator layer produces the first-level fuzzy comprehensive evaluation matrix:

$$(R_{ij})_{n\times m}=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & & \\ R_{n1} & R_{n2} & \cdots & R_{nm} \end{bmatrix} \tag{19}$$

In the formula, $R_i j=\dfrac{\text{Number of persons registered for level } i \text{ evaluation}}{\text{All evaluators}}$.

(2) Second-level fuzzy comprehensive evaluation

According to the formula $B_i = U_i \times R_i$ the second-level fuzzy comprehensive evaluation assesses all indicators within each indicator subset to determine the evaluation grades for each indicator at the criterion level.

(3) Third-Level Fuzzy Comprehensive Evaluation

Based on the formula $B = W \times P$, a third-level fuzzy comprehensive evaluation is conducted between criterion layers. The comprehensive membership degree of the objective layer U is then calculated as: $G = B \times V^T$. This is compared against the evaluation set Y to determine the risk level of the objective layer U. The above outlines the risk evaluation method combining the Analytic Hierarchy Process (AHP) with fuzzy mathematics. This approach quantifies the importance of risk factors across levels and enables quantitative risk analysis. It allows investors to clearly identify potential risks at each project stage and pinpoint critical risks, facilitating targeted risk management and enhancing risk management efficiency.

3.5 Project Management Risk Identification Based on Fuzzy Mathematics Comprehensive Evaluation

3.5.1 Case Study

(1) Project Background and Operation Model

R County adopted the Public-Private Partnership (PPP) model to complete the project in 1988. Following completion, the R County People's Government entrusted the social capital party to undertake project construction, operation, and maintenance. The R County Housing and Urban-Rural Development Bureau serves as the project implementation agency, while the R County government investment platform acts as the government's capital contribution representative. Together with the social capital party selected through procurement, they legally established a project company in R County. The R County Government Investment Platform holds a 30% equity stake in the project company, while the social capital partner holds 70%. The total investment for the PPP project amounts to 211.45 million yuan, with a construction period of 2 years—from the formal signing of this contract until the project passes final acceptance inspection. The operational period spans 25 years, commencing from the date of final acceptance for the newly constructed project.

(2) Project Return Mechanism This project is classified as a quasi-operational project, employing a “Feasibility Gap Subsidy” model for its return mechanism. The project company will receive operational income and concurrently obtain a feasibility gap subsidy from the R County People's Government when actual operational water volume falls short of the designed operational volume. The maximum total feasibility gap subsidy is 583.64 million yuan.

3.5.2 Application of PPP Project Risk Evaluation Models

(1) Determination of Risk Factor Evaluation Set

Based on the actual results of expert scoring, this paper generated a risk factor evaluation set for the infrastructure PPP project in County R, as shown in Table 2. Taking policy change risk as an example, the evaluation set for this indicator is (3,4,2,2,0), with other indicators following the same pattern.

Table 2: R infrastructure PPP project risk factor evaluation set

Target layer	Risk factors		Risk Comment Collection				
	First-level indicator	Secondary indicators	High	Higher	Medium	Lower	Low
/	/	/	5	4	3	2	1
PPP project management risks	Political and legal risks	Policy change risk	3	4	2	2	0
		Administrative intervention risk	4	3	2	2	0
		Economic risk	2	2	4	2	1
		Bidding risk	1	2	4	1	3
	Financial risk	Financing risk	3	1	4	3	0
		Liquidity risk	1	3	3	4	0
		Market risk	1	2	4	1	3
		Budgetary risk	1	5	3	2	0
	Design and construction risks	Construction risk	2	2	4	3	0
		The design party case changes the risk	1	2	4	4	0
		Cost risk of construction	1	1	3	3	3
		Quality risk of construction	1	2	4	4	0
		Construction safety risk	2	1	3	5	0
	Operational management risk	Project change risk	1	1	4	2	3
		Material supply risk	1	1	3	5	1
		Organized risk	1	2	4	4	0
		Market risk	1	1	4	3	2

(2) Determination of Probability Distribution for Secondary Risk Indicators

When calculating the probability distribution of secondary indicators, this paper primarily references Table 3 and employs the formula: $P_{ij} = n_{ij} / n$. In this formula, P_{ij} denotes the probability weight, n_{ij} represents the number of scores assigned to indicator factor i relative to indicator factor j in the importance evaluation, and n indicates the number of experts participating in the risk indicator importance scoring. Similarly, taking policy change risk as an example, the corresponding risk probability distribution is (0.285, 0.368, 0.185, 0.185, 0).

Table 3: R Infrastructure PPP Project Risk Factor Score

Target layer	Risk factors		Risk Comment Collection				
	First-level indicator	Secondary indicators	High	Higher	Medium	Lower	Low
			5	4	3	2	1
PPP project management risks	Political and legal risks	Policy change risk	0.285	0.368	0.185	0.185	0
		Administrative intervention risk	0.368	0.285	0.185	0.185	0
		Economic risk	0.185	0.185	0.368	0.185	0.07
		Bidding risk	0.07	0.185	0.368	0.07	0.285
	Financial risk	Financing risk	0.285	0.07	0.368	0.285	0
		Liquidity risk	0.07	0.285	0.285	0.368	0
		Market risk	0.07	0.185	0.368	0.07	0.285
		Budgetary risk	0.07	0.455	0.285	0.185	0
	Design and construction risks	Construction risk	0.185	0.185	0.368	0.285	0
		The design party case changes the risk	0.07	0.185	0.368	0.368	0
		Cost risk of construction	0.07	0.07	0.285	0.285	0.285
		Quality risk of construction	0.07	0.185	0.368	0.368	0
		Construction safety risk	0.185	0.07	0.285	0.455	0
	Operational management risk	Project change risk	0.07	0.07	0.368	0.185	0.285
		Material supply risk	0.07	0.07	0.285	0.455	0.07
		Organized risk	0.07	0.185	0.368	0.368	0
		Market risk	0.07	0.07	0.368	0.285	0.185

(3) Handling Evaluation Criteria

The weighted sets for each layer of factors are summarized as follows:

$$\begin{aligned} W &= (0.293, 0.418, 0.175, 0.114)^T \\ W_1 &= (0.433, 0.342, 0.134, 0.091)^T \\ W_2 &= (0.504, 0.249, 0.179, 0.068)^T \\ W_3 &= (0.408, 0.162, 0.145, 0.248, 0.037)^T \\ W_4 &= (0.452, 0.264, 0.14, 0.144)^T \end{aligned}$$

C1, C2, C3, and C4 represent the lowest-level scheme indicators in the indicator hierarchy. Establishing a judgment matrix for C1 yields:

$$R_1 = \begin{Bmatrix} 0.285\ 0.368\ 0.185\ 0.185\ 0 \\ 0.368\ 0.285\ 0.185\ 0.185\ 0 \\ 0.185\ 0.185\ 0.368\ 0.185\ 0.07 \\ 0.07\ 0.185\ 0.368\ 0.07\ 0.285 \end{Bmatrix}$$

Subsequently, the result for the matrix R_i can be computed using the formula $B_i = W_i R_i$, namely:

$$B_1 = W_1 R_1 = (0.433, 0.342, 0.134, 0.091) *$$
$$\begin{Bmatrix} 0.285\ 0.368\ 0.185\ 0.185\ 0 \\ 0.368\ 0.285\ 0.185\ 0.185\ 0 \\ 0.185\ 0.185\ 0.368\ 0.185\ 0.07 \\ 0.07\ 0.185\ 0.368\ 0.07\ 0.285 \end{Bmatrix} = (0.28, 0.3, 0.23, 0.17, 0.03)$$

Repeat the above process to establish decision matrices R2, R3, and R4 for C2, C3, and C4 respectively, and calculate their results B2, B3, and B4:

$$\begin{aligned} B_2 &= (0.178, 0.17, 0.342, 0.26, 0.05) \\ B_3 &= (0.12, 0.16, 0.35, 0.03, 0.04) \\ B_4 &= (0.07, 0.09, 0.4, 0.3, 0.17) \end{aligned}$$

U = (U1, U2, U3, U4) = (Political and Legal Risk, Financial Risk, Design and Construction Risk, Operational Management Risk), with weights $W = (0.293, 0.418, 0.175, 0.114)$.

By reintegrating the results obtained from the calculated matrices R_1, R_2, R_3, and R_4, namely B_1, B_2, B_3, and B_4, the final outcome for the overall risk assessment can be generated:

$B = W * R = (0.2, 0.2, 0.3, 0.2, 0.1)$

To quantify the importance of each indicator factor in the overall risk assessment, this paper assigns specific scores of 5, 4, 3, 2, and 1 to the comment set V based on the different risk assessment levels. Building upon this foundation, this paper conducted integrated calculations to derive the importance scores for the top-level objective indicator and the four intermediate criterion-level indicators within the risk hierarchy of the R infrastructure PPP project during the overall risk assessment. The specific results for each indicator are presented in Table 4.

The R Infrastructure PPP project's full lifecycle management process received an overall risk score of 3.2. This score exceeds 3 (corresponding to importance level V3, moderate) but falls below 4 (corresponding to importance level V2, high), exhibiting characteristics leaning toward the V2 level. Given the project's nature as infrastructure construction, whose outcomes hold significant value and importance for both contractors and society, this study concludes that its overall risk level warrants a V2 rating.

Additionally, the importance scores of the four intermediate-level indicators in the overall risk assessment, ranked from highest to lowest, are 3.66, 3.166, 2.68, and 2.39. The highest score of 3.66 corresponds to the political-legal indicator, classified as High (V2) importance. This indicates the highest probability of risk occurrence for this indicator, demanding the highest level of attention. The second-highest score of 3.16 corresponds to the financial indicator, also classified as High (V2) importance. The operational management risk indicator follows with a score of 2.68, classified as Moderate importance, slightly lower than the preceding two. Finally, the design and construction risk indicator scored 2.39 points, corresponding to the lowest importance level, which can be considered moderately low. The risk domains (criterion level) in the evaluation results cover the entire project implementation cycle, and the final risk indicators, ranked from highest to lowest, correspond to the project phases. Political and legal risks scored the highest, often requiring the earliest consideration during the project preparation phase. The lowest score for design and construction risks indicates that as the project enters its final stages, risk sources become relatively concentrated. However, this does not imply that risk management becomes simpler.

Considering the specific stages where problems arise and their underlying causes, the resulting consequences can be broadly categorized into two types. The first involves the inability to complete the project construction phase, preventing a smooth transition into the operational phase. For example, during the project planning stage, inadequate coordination in the implementation plan—failing to holistically consider project scope, policy requirements, and revenue logic—may result in the project being rejected for inclusion in the project pipeline, fundamentally lacking the conditions for implementation. Another example involves failure to engage financial institutions early in the financing process, resulting in unsecured funding. This not only halts the project but also tarnishes the local government's reputation. The second category involves projects that successfully complete construction and enter the operational phase. However, due to insufficient professional expertise in the initial planning stage, operational sub-projects fail to achieve expected returns or become inoperable. This subsequently prevents repayment of principal and interest to financial institutions, creating hidden government debt. This impacts local credit ratings and severely constrains future regional development.

Supported by scientific analytical methods—the Analytic Hierarchy Process (AHP) and Fuzzy Evaluation Method—this chapter completed a graded assessment of the risk level throughout the entire lifecycle management process of R infrastructure PPP projects. The results indicate a relatively high risk level, establishing a reliable practical foundation for subsequent development of corresponding risk prevention and response measures.

Table 4: R infrastructure PPP project risk factor assessment set

Evaluation grade	High(V1)	Higher(V2)	Medium(V3)	Lower(V4)	Low(V5)
Evaluation index	5	4	3	2	1
R infrastructure PPP project risk U	0.2*5+0.2*4+0.3*3+0.2*2+0.1*1=3.2				
Political legal risk U1	0.28*5+0.3*4+0.23*3+0.17*2+0.03*1=3.66				
Financial risk U2	0.178*5+0.17*4+0.342*3+0.26*2+0.05*1=3.166				
Design construction risk U3	0.12*5+0.16*4+0.35*3+0.03*2+0.04*1=2.39				
Operational management risk U4	0.07*5+0.09*4+0.4*3+0.3*2+0.17*1=2.68				

4 Development of a Decision Support Model for Projects Under the PPP Model

4.1 Investment Evaluation Indicators

Project investment refers to activities that generate specific economic or social benefits by injecting a defined amount of capital into a project. Therefore, the investment objective of social capital in R County's PPP projects is also to achieve certain economic returns. Consequently, economic benefit indicators serve as the most critical investment decision criteria for social capital participating in R County's PPP projects. The R County PPP project must employ dynamic investment return calculation methods, as its concession period typically spans 40 to 50 years. Consequently, this paper selects Net Present Value (NPV) and Internal Rate of Return (IRR) as investment evaluation metrics.

4.1.1 Net Present Value

Net Present Value (NPV) enables dynamic evaluation of projects. It represents the difference between the present value of a project's expected cash inflows and the present value of its planned cash outflows. When $NPV \geq 0$, the expected return on investment for the project is greater than or equal to the benchmark rate of return, indicating the project is financially viable. If $NPV < 0$, the project is financially unfeasible. An increase in NPV indicates greater net cash inflows generated by the investment project, making it more advantageous for investors.

The formula is as follows:

$$NPV = \sum_{t=0}^{n} \frac{A_t}{(1+i)^t} \tag{20}$$

In equation (20), n represents the investment period, A_t denotes the cash flow in year t, and i is the discount rate.

4.1.2 Internal Rate of Return

Internal Rate of Return (IRR) is the discount rate at which the net present value of a project's annual cash inflows equals the net present value of its annual cash outflows over the entire investment period. It serves as a key indicator for measuring a project's profitability, representing the rate at which funds invested in the project generate returns.

The formula is as follows:

$$\sum_{t=0}^{n}(CI-CO)_t(1+IRR)^{-t}=0 \tag{21}$$

In Equation (21):

CI: Economic benefit inflow, CO: Economic benefit outflow.

If IRR is less than R_e, it indicates the investment project is not viable, meaning the PPP project's return level falls significantly short of investor requirements. If IRR exceeds R_e, it signifies the project is feasible, indicating the PPP project's return level aligns with investor expectations. Here, R_e is the benchmark rate of return for the PPP project.

4.2 Determination of Decision Models

4.2.1 Determination of Decision Model Variables

Based on the actual operational conditions in County R and the characteristics of the Monte Carlo simulation method (a quantitative measurement approach), this paper selects quantifiable risk indicators among the key risks faced by the PPP project in County R:

(1) Labor Allocation Policy

The labor allocation policy refers to a series of measures adopted by the government to regulate the labor market, protect workers' rights, and promote economic development.

(2) Resistance to Schedule Adjustments

Schedule adjustment policies aim to reasonably determine and effectively control construction timelines, ensure project quality and safety, and protect the legitimate rights and interests of all participating parties.

(3) Risk Ripple Effect

In economics, the risk ripple effect describes the process where the diffusion of technology, information, experience, and new ideas across economic regions gradually diminishes in energy, slows in speed, and reduces in impact—much like ripples spreading across a pond.

Therefore, the variables selected for the decision-making model are labor rationing, schedule adjustment, and the risk ripple effect.

4.2.2 Building Decision Models

This paper identifies risk factors influencing investment decisions for PPP projects in County R. By employing fuzzy mathematical models, it determines the primary risk factors. Under the condition that enterprises share these key risks, random variables are defined. Simultaneously, Monte Carlo simulations are applied to determine the probability distributions of each variable. Finally, the simulated data is incorporated into the investment decision model for calculation and evaluation. As shown in the formula:

$$NPV/T_n = NPV/T_a + \sum_{t=T_a}^{n}\frac{CI_t-CO_t}{(1+i)^t}(T_a \le n \le T) \tag{22}$$

$$\sum_{t=1}^{n}\frac{CI_t-CO_t}{(1+IRR)^t}=0 \tag{23}$$

$$NPV_t=\frac{CI_t-CO_t}{(1+i)^t} \tag{24}$$

In the formula, T denotes the project's concession period, T_c represents the construction period, O_t is the investment amount in year t, NPV / Ta indicates the cumulative discounted value of investments during the construction period, reflecting the accumulated discounted value at the end of year n—which is the net present value in year t. IRR stands for internal rate of return, and i represents the discount rate.

4.3 Empirical Research

4.3.1 Model Parameter Settings

Due to numerous unpredictable risk factors during the project's construction phase, cost escalation and schedule delays caused by construction modifications, inflation, changes in scope of work, and price fluctuations represent the most sensitive variables. Sensitivity analysis and financial sensitivity analysis were conducted separately for these factors. The results of the sensitivity analysis and financial sensitivity analysis are presented in Tables 5 and 6, respectively.

Analysis of Table 5 indicates that the project possesses strong risk resilience. Even under the most unfavorable scenario—a 10% increase in construction costs and a 10% reduction in benefits—the project remains economically viable, achieving an internal rate of return (IRR) of 10.478%. Analysis of Table 6 indicates the project possesses sound financial profitability and is financially viable. Even under an unfavorable scenario where construction costs increase by 10% due to construction period risks while benefits remain unchanged, the project remains profitable, achieving a net present value of 165,325 million yuan. The comprehensive sensitivity analysis results demonstrate that the project's construction period exhibits a certain degree of risk resistance.

Table 5: Sensitivity analysis result

Benefit variation	Construction fee	-10%	0	10%
-10%	ENPV(Ten thousand yuan)	1330525	1123048	910636
	EBCR	1.748	1.536	1.436
	EIRR(%)	12.599	11.548	**10.478**
0	ENPV(Ten thousand yuan)	1676068	1469536	1236154
	EBCR	1.948	1.798	1.549
	EIRR(%)	13.536	12.536	11.636
10%	ENPV(Ten thousand yuan)	2023578	1863124	1569315
	EBCR	2.164	1.948	1.748
	EIRR(%)	14.469	13.498	12.596

Table 6: Financial sensitivity analysis results

Benefit variation	Construction fee	-10%	0	10%
-10%	ENPV(Ten thousand yuan)	314048	80321	-154663
	EBCR	1.165	1.069	0.945
	EIRR(%)	5.498	4.448	3.636
0	ENPV(Ten thousand yuan)	631263	378696	**165325**
	EBCR	1.299	1.148	1.045
	EIRR(%)	6.512	5.569	4.745
10%	ENPV(Ten thousand yuan)	945362	715365	498365
	EBCR	1.348	1.254	1.169
	EIRR(%)	7.698	6.488	5.745

4.3.2 Model Testing

The behavioral reproduction capability test evaluates whether the model can accurately replicate project behavior by integrating specific engineering examples. As shown in Figure 2, the simulation results of this model demonstrate a comparison between the actual completion progress of the R County PPP project and the model's projected completion progress.

Calculations reveal a correlation coefficient R = 0.979 and a coefficient of determination $R^2 = 0.967$ between the actual data and simulated data. These figures indicate the model exhibits a high degree of alignment with the real-world engineering project.

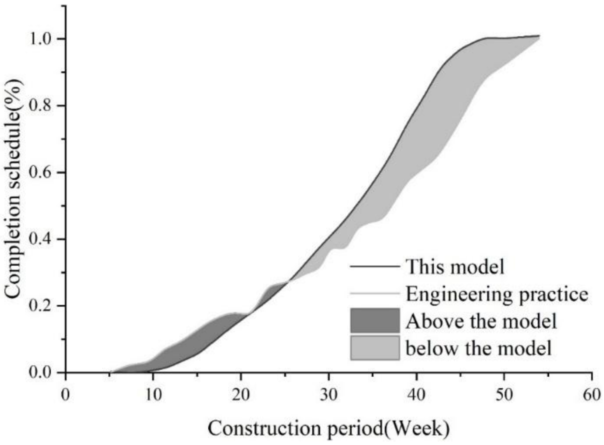

Figure 2: Progress of project actual completion and model completion

4.4 Risk Management Strategy Design and Evaluation

4.4.1 Single-Factor Influence Analysis

In the analysis scenarios, each factor value is benchmarked against the baseline scenario. Within a range of [-50%, +50%], 11 values are selected at 10% intervals. Consequently, each influencing factor generates 11 scenarios. The simulation results for scenario design are shown in Figure 3, with completion time as the observed metric. The straight line in the figure represents the baseline—the baseline scenario without any policy adjustments—where completion time is 46 weeks.

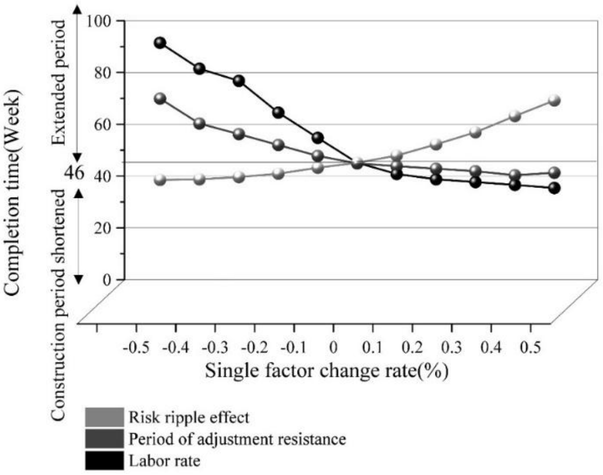

Figure 3: Solution design simulation results

4.4.2 Risk Response Strategy Design

Based on single-factor sensitivity analysis, the following risk response strategies were designed: labor rationing policy, schedule adjustment policy, and ripple effect each assigned high (H), medium (M), and low (L) values, representing three scenarios for each policy. The M value serves as the baseline scenario, the H value is 130% of the baseline, and the L value is 70% of the baseline. This policy combination approach generates 27 combined strategy scenarios. By substituting the values of each combined strategy into the model for simulation, the completion time indicator is extracted, and the simulation results of the 27 scenarios are compared and analyzed. Table 7 presents the design and simulation results of the risk response strategy scenarios, while Figure 4 compares the effectiveness of the baseline scenario with the analyzed scenarios.

Model simulations indicate that all risk response schemes effectively shorten the duration of Item H, with Scheme 1 demonstrating the most pronounced effect. It reduces the completion time to 30 periods, achieving an optimization rate of 34.78%. Of course, these optimization effects were achieved by substantially increasing labor supply, reducing ripple effects, and increasing resistance to schedule adjustments. Decision-makers should select risk response strategies based on comprehensive considerations of factors such as cost, resource availability, and communication convenience.

Table 7: Risk coping strategy design and simulation results

Scheme	Solution content			Progress optimization		
	Labor rate	Period of adjustment resistance	Ripple effect	Completion time	Optimized value	Optimization rate
1	H	H	L	30	16	34.78%
2	H	H	L	32	14	30.43%
3	H	M	M	34	12	26.09%
4	M	M	L	35	11	23.91%
5	M	M	M	37	9	19.57%

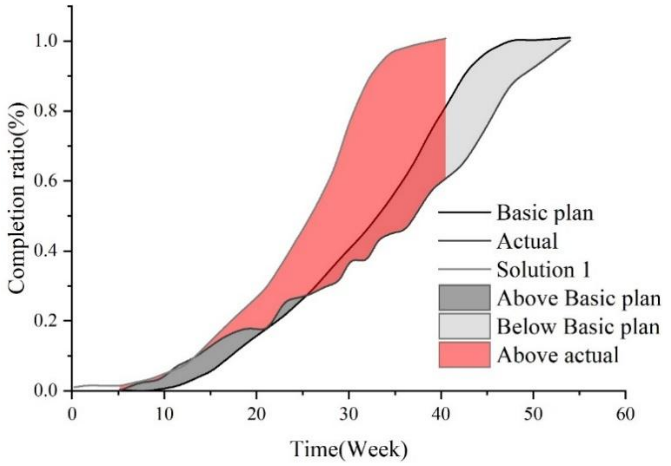

Figure 4: The basic solution is compared with the analysis scheme

5 Conclusion

This paper reviews key concepts linking PPP projects and risk management theory. It evaluates risk management under the PPP model by integrating the Analytic Hierarchy Process (AHP) with fuzzy mathematical evaluation methods. A hierarchical structure is established to calculate weight values for elements at each level, forming a set of factor evaluation scales. Applying the PPP project risk evaluation model to a case study, the subject infrastructure PPP project received a full lifecycle management risk score of 3.2, indicating a relatively high level of risk significance. Scores for political and legal risks, financial indicator risks, operational management risks, and design and construction risks were 3.66, 3.16, 2.68, and 2.39, respectively, with political and legal risks and financial indicator risks being of higher importance.

NPV and IRR were selected as decision-support evaluation indicators for the PPP project. The project demonstrated strong overall risk resilience. Even under the worst-case scenario of a 10% increase in construction costs and a 10% decrease in benefits, the internal rate of return remained at 10.478%, preserving the project's economic viability. The model generated 27 portfolio strategy options, with Option 1 demonstrating the most significant effect. It reduced the completion time from 46 to 30 construction periods, achieving an optimization rate of 34.78% and effectively shortening the project cycle.

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