



The integration path of mathematical thinking training and artificial intelligence-assisted instruction

Na Li^{1,*} and Zhiming Liu²

¹ School of Mathematics and Statistics, Hubei University of Education, Wuhan, Hubei, 430025, China

² School of Computer and Artificial Intelligence, Hubei University of Education, Wuhan, Hubei, 430025, China

SUMMARY: *In order to play the role of artificial intelligence assistance to improve students' mathematical thinking level, the study proposes to use artificial intelligence technology to construct educational knowledge map, collect learning data to generate learner profiles, use clustering algorithms to generate learning communities, and use ant colony optimization algorithms to recommend learning paths and other teaching assistance methods. Through the intelligent education cloud platform, multiple teaching assistance methods can be used in collaboration, and students can realize the cultivation of mathematical thinking based on the learning paths and teaching resources recommended by the platform. Taking the teaching of Preliminary Geometry as an example, after students used the Smart Education Cloud Platform for unit review, 82.5% and 81.3% of students were satisfied with the learning paths and learning resources recommended by the platform respectively. Students' math thinking test scores improved by 10.26 points, especially on students' creativity, algorithmic thinking and problem solving ability. Through the role of artificial intelligence technology empowerment, students' mathematical thinking was developed and the quality of teaching was improved.*

KEYWORDS: *Artificial Intelligence; Knowledge Graph; Learner Portrait; Clustering Algorithm; Ant Colony Optimization Algorithm; Mathematical Thinking*

1 Introduction

Advancements in science and technology have led to increased expectations in terms of talent nurturing, and this will always depend on education. In the current era, the education industry faces new challenges. The Ministry of Education issued guidelines in the year 2014 with the objective of reinforcing the reform in the curriculum and realizing the goal of nurturing morality. In this respect, thinking skills have been identified as key aspects in education, thus requiring consideration [1]. The study of pedagogy in mathematics highlights the importance of developing the mathematical thinking ability of students [2]. However, in reality, classroom instruction demonstrates the inadequacy of the development of this ability on the part of many students, limiting their ability to conduct proper analysis of mathematical problems [3, 4]. Under the old paradigm of mathematics education, students usually solve mathematical problems and develop a problem reflex, which not only adversely affects the improvement of their mathematical literacy but also hinders the improvement of thinking skills [5-8]. Digitalization and information technology are two significant characteristics in today's society.

*nana2000_01@126.com

<https://doi.org/10.65102/is2026160>

Whether the students have developed the skill of mathematical thinking will determine whether they can quickly respond to problems in the future [9]. Thus, for that reason, the reform of mathematical education needs to include the development of mathematical thinking as part of its main curriculum. At the same time, the implementation of a structured training system must also be carried out. This way, the abilities of students in terms of analytical and creative thinking skills will be improved and used in other fields like innovation in science and technology, management, and social governance, which together form the picture of abilities called core literacy [10].

When addressing the definition and connotation of mathematical thinking, literature [11] provides a detailed investigation into the education of mathematical thinking in mathematics instruction, suggesting that teaching students their creativity and developing mathematical thinking ability would involve a recognition of what kind of subject the mathematics actually is, and that involves the education of thinking methods through the mathematics instruction. Literature [12] argues that the ability of mathematical thinking of students refers to their ability of solving problems using scientific approaches such that problem solving would be realized simultaneously with gaining the related knowledge. The description of the students' ability of mathematical thinking by means of mathematical problem-learning model includes three steps: first, observe the process of learning in the classroom, then, examine the students, and at last conduct qualitative data analysis. The results suggest that the mathematical problem-learning model has played an important part in the mathematical thinking activities of the students, which have been realized step by step. On another occasion, literature [13] designed scales of mathematical thinking, critical thinking, creativity, and problem solving based on the concept of existing mathematical thinking, and proved the validity and reliability of the above scales by conducting factor analysis and reliability testing in a scientific way. Literature [14] points out that examples in the classroom help identify and determine the key factors of students' mathematical thinking, offering an insight and creating a common language; the aspect of the issue reflects the significance of mathematical thinking skills from the viewpoint of teachers' efficient utilization of the students' mathematical thinking skills. Without any doubt, mathematical thinking is mainly manifested in the aspects of arithmetic skills and reasoning awareness or reasoning ability [15]. Students should gain the necessary mathematical thinking skills including arithmetic and reasoning skills through mathematics instruction. It can also be seen that the mathematical activities constitute the formation of mathematical knowledge and the developing of students' mathematical thinking skills [16].

As to the mathematical thinking ability training strategy, there have been different approaches highlighted in the literature. For instance, as stated by Reference [17], a learner-centered approach has been proposed to facilitate the improvement of students' mathematical thinking. In this case, the initial stages involved using advanced level tests of mathematical thinking along with questionnaires and observation methods to collect relevant data; based on the findings, the researchers suggested a mathematical learning model based on the constructivist theoretical frameworks regarding mathematical learning. At last, a model promoting the enhancement of students' mathematical thinking abilities was proposed. Reference [18] analyzed the interaction between learners' creative thinking and mathematics teaching practices, showing that both aspects are linked and that the ability to think mathematically can be encouraged through problem solving in mathematics. Reference [19] used a computer-assisted learning software called GeoGebra to facilitate mathematics learning. GeoGebra is a web-based platform for the simulation of mathematical concepts including graphing, geometry, algebra, 3D modeling, statistics, and probability. It facilitates problem-based learning models to improve students' mathematical thinking. Reference [20] investigated the connection between math anxiety and mathematical thinking of African learners from 2005

to 2021, and the results show the presence of a negative correlation between the variables. Moreover, the study suggests that there is a need for developing a curriculum encouraging reforms and development in the mathematics curriculum. According to literature [21], higher-order thinking skills can be improved using the situational thinking learning model in mathematics classroom practices. This approach utilizes the provision of situational problems, asking critical and analytical questions, investigation of individual questions through collaboration, demonstrations, discussions, reflection and higher-order thinking assessment. Literature suggests that through constant engagement with this process, higher-order thinking skills will be reinforced. According to literature [22], instructional practices can represent cognitive action problems of learners and encourage reflective abstraction. Problem based learning, according to literature, can help in improving higher-order thinking skills among students. Moreover, reflective abstraction of students' mental actions, processes, objects, and plans can be achieved through problem based learning.

Currently, the accelerated growth of artificial intelligence (AI) technology is presenting more opportunities than ever before in the teaching model of the mathematics classroom, and AI technology, which has excellent natural language processing capability, data analysis capability and adaptive learning technology is introducing new viewpoints and opportunities to the study and application of the mathematics classroom [23]. The AI technology contributes positively to the personalization and dynamism of the materials used in mathematical thinking training leading to better learning experiences among students. In terms of how AI technology affects mathematical thinking ability, literature [24], indicates that the connotation and role of mathematical thinking have been analyzed and that mathematical thinking can be regarded as the foundation of AI development and that it can be developed within the context of AI and the two are mutually beneficial. In the same line, according to the findings, the literature [25] has suggested that the implementation of AI technologies facilitates the formation of a dynamic monitoring system for fostering students' mathematical thinking capabilities. It gathers detailed information about learners' instructional pathways and develops a personal profile of the students' progress in developing mathematical thinking. According to the literature [26], which examines the application of computational thinking (CT) and AI in mathematics education, it is crucial to utilize AI technology for instilling mathematical thinking capabilities. literature [27] which assessed the efficacy of AI technology and its software in improving secondary school students' mathematical thinking, analyzed the mean score differences between those who were exposed to AI technology and found that their performance improved remarkably in mathematical critical thinking in aspects like deduction, explanation, inference, and evaluation. On the other hand, the synthesis implies that although AI applications could help in fostering critical thinking in mathematics and promote interaction in learning, depending excessively on AI may have adverse effects and result in algorithmic problem-solving without conceptual comprehension [28]. Literature [29] investigated the use of AI technology into primary school mathematics instruction using teaching experiments and revealed that the AI technology-based math teaching and training can significantly enhance the level of creativity in the thinking of students.

Mathematical thinking can be effectively developed through multidimensional integration of student data, consideration of students individual requirements and respect to their cognitive features. In this context, this paper implements artificial intelligence technology to build educational knowledge maps, produce learner profiles through gathered learning information, accurately depict the learning scenario, create learning community through clustering algorithms that would deliver effective assistance to students, recommend suitable learning paths based on deep learning algorithms, and achieve path integration with the assistance of the intelligent education cloud platform. The paper eventually validates the usefulness of the smart

education cloud platform and its training value in mathematical thinking upon teaching practice.

2 Method

2.1 Mathematical Thinking Training Strategies Assisted by Artificial Intelligence

2.1.1 Integrating Student Data to Capture Instructional Priorities and Difficulties

Artificial intelligence technology applied to teaching can integrate multi-dimensional data such as students' homework, test scores, classroom performance, learning resources utilization, etc., to build students' learning portraits and diagnose students' mastery of mathematical knowledge, thinking cultivation, learning methods, learning participation and other aspects. On the basis of the results obtained through diagnostic analysis using artificial intelligence, the teacher can find out the strengths and weaknesses of the students in their mathematical understanding, analyze the deficiencies in the students' knowledge base, and probe into the psychological state of mind and study habits of the learners. Through this process, the teacher will be able to understand the essential points in mathematical articulation.

2.1.2 Focus on students' individualized needs and on the development of teaching resources

The deviation of teaching resources from the basic needs of students will affect their motivation to utilize them. The gradual increase of online resources and offline educational resources under the concept of open education increases the difficulty of students' choice. The development of artificial intelligence technology provides information support for carrying out accurate teaching resource development. It is necessary to respect the cognitive characteristics and thinking characteristics of students to develop articulated teaching resources, and promote students to realize a smooth transition of mathematics learning in different school years. Currently, the large-unit theme teaching mode has been widely used in teaching, which can build a knowledge structure mapping system around the large-unit theme, set up a mathematical learning framework, guide students to intuitively perceive the intrinsic connection between mathematical themes, and cultivate students' structured thinking.

2.1.3 Respect students' cognitive characteristics and guide them to experience inquiry

The new curriculum concept emphasizes the need to build an inquiry-based teaching mode, guiding students to carry out investigations, enhancing students' independent learning ability, and expanding students' thinking. The teaching mode of mathematics under artificial intelligence changes from a two-dimensional subject of teacher-student to a four-dimensional subject of teacher-peer-artificial intelligence technology-student, so it is necessary to pay attention to the innovation of teaching methods in mathematical thinking training, and to enhance the initiative and inquiry of students in learning. . Through artificial intelligence technology to build a realization and immersive scene to inspire students to think. Teachers should play a guiding role, focusing on the deficiencies in students' articulation, putting forward problematic hypotheses or introducing real-life cases to promote the development of students' thinking from superficial thinking to creative thinking.

2.2 Technological Approaches to Artificial Intelligence-Assisted Teaching and Learning

2.2.1 Educational Knowledge Graph Construction Based on Artificial Intelligence Technology

The construction of an educational knowledge graph mostly involves the application of artificial intelligence technology to identify certain entities in the educational domain knowledge base, and create inter-entity association relationships. Semi-structured data and structured data are acquired through wrapper learning and entity recognition respectively, and knowledge extraction through entity extraction of knowledge system (conditional random field model), entity extraction of problem system (KNN algorithm), entity extraction of competence system (manually organized to be extracted by subject experts in the education field), and inter-entity relationship extraction (Markov Logic network). Knowledge fusion is achieved through the methods of entity alignment based on machine learning, and knowledge reasoning is achieved by the Path Ranking algorithm in the graph-based reasoning.

2.2.2 Learner Profile Construction Based on Learning Data

By using artificial intelligence technology, learner portrait construction involves obtaining the learner learning data, which is analyzed and calculated to offer a many-faceted description of the learner. Learner portrait construction is essentially an overall analysis of the data of learning process and results, and the data on the computational modeling of learning achievements, autonomous learning activities, learning reports, acquisition of knowledge, problem solving, cooperation and communication, summarizing, etc., to create a portrait of a learner in three dimensions, i.e. knowledge, problem and ability, and the construction process is depicted in Figure 1.

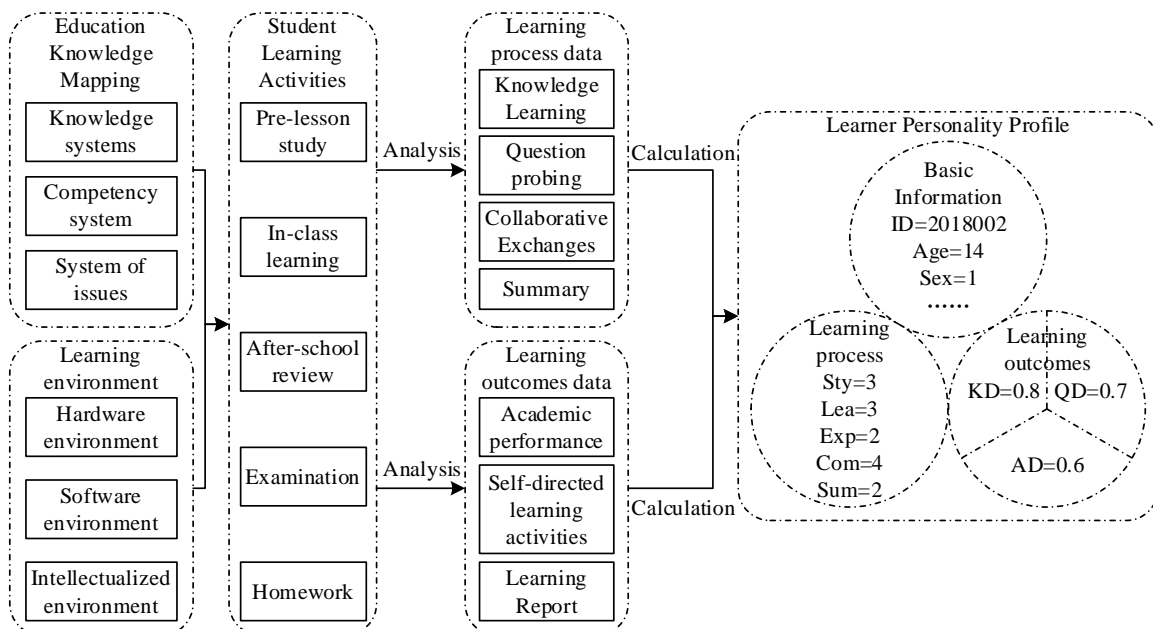


Figure 1: The learner's portrait construction process

2.2.3 Learning community construction based on clustering algorithm

The establishment of a learning community refers to organizing students into homogeneous or heterogeneous groups according to specific characteristics, so that the community shows both within-group diversity and between-group consistency. It is widely recognized that learning styles may directly influence learners' emotions and behaviors, thereby affecting autonomous study, inquiry-based learning, and collaborative interaction. For this reason, constructing such communities through clustering according to learning styles has become an important trend in this field. The present study applies a clustering method to generate learning communities automatically, mainly including divisive grouping, initial community formation, and hierarchical grouping.

(1) Division clustering

The mean shift method first partitions learners into different groups. This offset-based approach, derived from mean-value clustering, uses a sliding-window mechanism to locate the center point of a learner group. The window moves towards areas with higher density as a hill climbing algorithm. The next step involves updating the cluster center with respect to the average value of all samples contained in the window. The iteration goes on until convergence, and we have clusters and cluster centers, completing the clustering process, and its computational measures are shown in (1):

$$M_h(x) = \frac{\sum_{i=1}^n G\left(\left\|\frac{x_i - x}{h}\right\|^2\right) w(x_i)(x_i - x)}{\sum_{i=1}^n G\left(\left\|\frac{x_i - x}{h}\right\|^2\right) w(x_i)} \quad (1)$$

where $G\left(\left\|\frac{x_i - x}{h}\right\|^2\right)$ is the kernel function, h denotes the bandwidth, and $w(x_i)$ is the sample weight.

(2) Initial construction of learning community

Once the first step of clustering decomposition is done, students who have similar learning styles are then grouped into one cluster. The total number of communities that will be created is represented by N . In the first step, the cluster D_{max} having the largest density of nodes must be identified. The N nodes furthest away from their centers in D_{max} will be selected. The N selected nodes are the first communities.

(3) Hierarchical clustering

The present paper exploits the similarity in learning styles among learners using the AGNES clustering technique. The procedure begins by assigning each instance to a separate cluster, followed by the determination of two closest clusters and their merging in each step until a predetermined number of clusters is achieved. Following that, one more learner is added to each learning cluster in succession till all learners are clustered together in one single cluster. The complexity analysis of this algorithm is given by equation (2):

$$d_{avg}(C_i, C_j) = \frac{1}{|C_i||C_j|} \sum_{x \in C_i} \sum_{z \in C_j} dist(x, z) \quad (2)$$

where C_i and C_j denote the two clusters that need to be merged for computation.

2.2.4 Learning path recommendation based on artificial intelligence technology

Learning path acts as an operational guideline for performing intelligent learning. Learning paths recommendation includes four major steps that are, first, the annotation of the existing cognitive state; second, the annotation of the desired learning state; third, group learning path planning; and fourth, personal learning path recommendation.

(1) Group Learner Learning Path Planning

According to the educational knowledge graph and the learning information, the path transition probability and pheromone concentration for the purpose of learning pathway planning of group learners are determined using ant colony optimization algorithm. The path planning process, the transfer probability formula for ants moving from node i to node j is shown in (3):

$$P_{ij}^k(t) = \begin{cases} \frac{\tau_{ij}^\alpha(t)\eta_{ij}^\beta(t)}{\sum \tau_{ij}^\alpha(t)\eta_{ij}^\beta(t)}, & i, j \in \text{feasible domain} \\ 0, & \text{other} \end{cases} \quad (3)$$

where $\tau_{ij}(t)$ represents the pheromone concentration between nodes i and j on the search path, and $\eta_{ij}(t)$ is the heuristic information, which denotes the degree of expectation of transferring from node i to node j .

After each ant search, the pheromone concentration needs to be updated, and the equations are shown in (4) (5) (6):

$$\tau_{ij}(t+1) = \rho\tau_{ij}(t) + \Delta\tau_{ij}(t, t+1) \quad (4)$$

$$\Delta\tau_{ij}(t, t+1) = \sum_{k=1}^n \Delta\tau_{ij}^k(t, t+1) \quad (5)$$

$$\Delta\tau_{ij}^k(t) = \begin{cases} F / S_k, & \text{Ants } k \text{ after } ij \\ 0, & \text{Ant } k \text{ without passing } ij \end{cases} \quad (6)$$

where F denotes the pheromone intensity factor, which represents the total amount of pheromone released by an ant searching once. S_k denotes the total length of the path traveled by the k th ant during the search.

(2) Personalized learning path recommendation

Based on the aforementioned framework of group learning paths, a CNN architecture can be designed to help in generating personalized learning paths. In general, the CNN model contains three major components, which are input layer, hidden layer, and output layer. To be more specific, the input layer is responsible for inputting the group learning path, whereas the hidden layer includes three modules such as convolutional, pooling, and fully connected layers.

1) Input layer. The input layer of this study is the group learning path.

2) Convolutional layer. This layer is mainly responsible for feature extraction from the input data, and its kernel operation can be expressed by Equation (7):

$$\begin{aligned}
Z^{l+1}(i, j) &= [Z^l \times w^{l+1}](i, j) + b \\
&= \sum_{k=1}^{K_i} \sum_{x=1}^f \sum_{y=1}^f [Z_k^l(s_0 i + x, s_0 j + y) w_k^{l+1}(x, y)] + b
\end{aligned} \tag{7}$$

3) Pooling layer. The pooling stage summarizes local responses in the feature map by aggregating values from neighboring regions. In this study, the L_p pooling strategy is adopted, and its basic form is given in Equation (8):

$$A_k^l(i, j) = \left[\sum_{x=1}^f \sum_{y=1}^f A_k^l(s_0 i + x, s_0 j + y)^p \right]^{\frac{1}{p}} \tag{8}$$

4) Fully connected layer. Positioned at the final stage of the hidden part of the convolutional neural network, this layer transmits the learned representations to subsequent fully connected units.

5) Output Layer. At the final stage, a standard logistic regression model is employed to generate personalized learning paths.

2.3 Integration Path of Mathematical Thinking Training and Artificial Intelligence Technology

The previously mentioned technology methods based on AI-assisted teaching are integrated through the construction of a smart education cloud platform, and the cultivation of mathematical thinking is realized with the help of the platform.

The intelligent education cloud platform needs to be constructed based on multi-level architecture, a combination of public and private clouds, and differentiated distribution of education types, modes, themes and features from top to bottom, so that the intelligent education cloud is closer to the actual learning needs of learners. From the technical architecture level, the smart education cloud should be able to realize the inclusion and integration of existing education platforms, education systems, education resources and education services, and build a unified identity authentication, data sharing, interface specification and access portal. The hardware resources of the cloud platform should have elastic scalability and distributed storage capability, and the platform should be connected to a large-bandwidth high-speed network to provide highly available various educational services.

3 Results and Discussion

To evaluate the effectiveness of integration, two groups of students from Secondary School A were chosen to be subjects in this study; the experimental group and control group each having 45 students. The time point chosen for this case implementation was when the two classes had just finished learning the basics of each part of Preliminary Geometry and were about to conduct a unit review.

Before the experiment, the experimental class student accounts were set up and distributed to the students, so that the experimental class students logged into the platform for online learning: firstly, they completed the unit quiz homework assigned by the teacher online (using the homework module of the platform), and then carried out personalized online learning. During the experimental process, teachers monitored the learning information and provided targeted guidance based on the real-time feedback from the learning platform, such as explaining high-frequency errors in class and assigning post-class homework, etc. The whole

process lasted for one week. The control class conducted unit review according to traditional teaching methods.

At the end of the experiment, the actual application effect of the smart education cloud platform and its effect on the cultivation of mathematical thinking level in teaching practice were analyzed.

3.1 Evaluation of the Application Effect of Intelligent Education Cloud Platform

In order to analyze the application effect of the wisdom education cloud platform, a questionnaire survey was conducted on all students in the experimental class after the experiment, which involved platform acceptance, learning resource adaptability and other aspects.

(1) User acceptance

The results of the user acceptance survey are shown in Figure 2, with “1~5” indicating “very dissatisfied, dissatisfied, average, satisfied, very satisfied”. The data showed that the vast majority of students were satisfied with the learning support effect of the Smart Education Cloud Platform, of which 85% of the total number of students surveyed had a good impression of the Smart Education Cloud Platform as a whole, saying that they liked this way of learning. On the other hand, 4% of the students had a poor impression of the Smart Education Cloud Platform, mainly due to the following reasons: individual resources could not be used on the iPad, such as some flash animations were loaded with external resources; it was not convenient to do the questions, such as constantly switching between the screen and the paper when calculating.

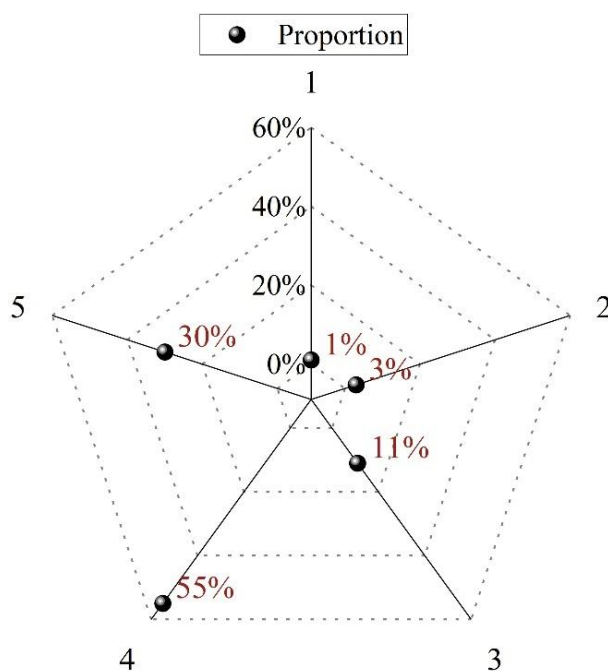


Figure 2: User acceptance survey results

(2) Satisfaction with Learning Paths and Learning Resources

The results of the survey on the satisfaction of learning paths and learning resources are shown in Figure 3, with “1~5” indicating “very unsuitable/very unsatisfactory, unsuitable/unsatisfactory, no feeling, basically suitable/comparatively satisfactory, suitable/very

satisfactory” respectively. According to the survey, most students think that the difficulty of the learning resources recommended by the platform is more suitable for their actual situation, and the students' views on the suitability of the difficulty of the recommended learning resources are in the following order: 82.5% of the students think that the difficulty is suitable or basically suitable, 6.3% think that the difficulty is not too suitable or very unsuitable for them, and 11.2% of the students have no feeling. Regarding students' satisfaction with the types of learning resources recommended by the platform, 81.3% of the students said they were very satisfied or quite satisfied, 12.6% said they had no feeling about it, and only 6.1% felt dissatisfied or very dissatisfied.

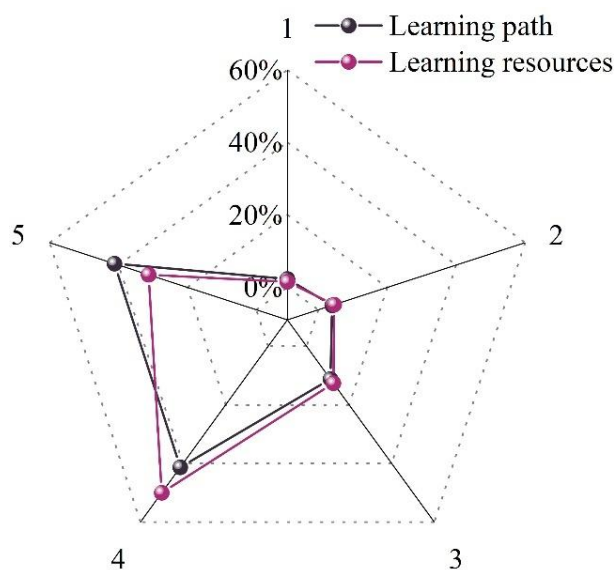


Figure 3: Learning path and learning resource satisfaction survey results

(3) Validity of learning resources

The validity of five types of learning resources, namely, text text, graphic images, audio and video, animation and inquiry tools, is verified and analyzed. Before the experiment, teachers have been asked to estimate the validity of the learning resources used in this unit, and after the experiment, the platform calculates the actual validity according to the students' learning process and results. For the average value of the effectiveness of five types of learning resources, the estimated value is compared with the actual value, the estimated value and the actual value exceeds 0.1 that there is a deviation, and the comparison results are shown in Table 1. The effectiveness calculated by the platform is generally similar to the teachers' estimated value, and the effectiveness of the actual application is basically in line with expectations. The validity of the text-based learning resources analyzed by the platform is 0.17 lower than the teachers' estimated value, while the validity of the tool-based learning resources exceeds the expected 0.11.

Table 1: Study resource effectiveness analysis

Type	Teacher prevaluation	Platform analysis results	Deviation
Text	0.79	0.62	↓
Graphic image	0.76	0.71	-
Audio video	0.73	0.78	-
Animation	0.79	0.82	-
Inquiry tool	0.83	0.94	↑

3.2 Analysis of the effect of cultivating the level of mathematical thinking

For testing the efficiency of the Smart Education Cloud Platform on the development of mathematical thinking among students, this research adopts the application of SPSS data analysis program for quantitative data analysis.

(1) Statistics and analysis of mathematical thinking pre-test questionnaire

The preliminary assessment of mathematical thinking was conducted during the preparation phase of the teaching practice. The instruments were designed to evaluate several dimensions, including creativity, algorithmic reasoning, cooperation, critical thinking, and problem-solving ability. Figure 4 presents the pre-intervention results for mathematical thinking in the two groups. The average baseline score was 72.96 for the experimental group and 73.30 for the control group.

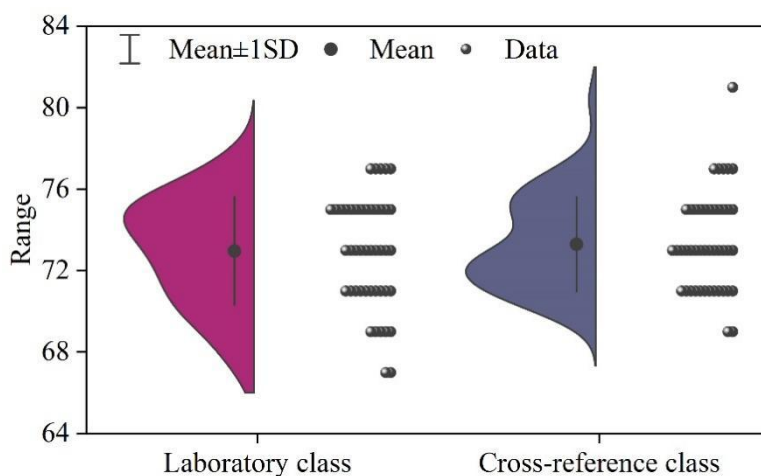


Figure 4: Test scores of experimental classes and comparison classes

A two-sample independent t-test was conducted on the pre-test scores of both groups to test whether there is a significant difference in their developmental level of mathematical thinking skills before implementing the teaching strategy. See Table 2 below for the test output. Test of Equality of Variances based on Levene’s test gave $F = 0.493$, $P = 0.412 > 0.05$, indicating equality of variance assumption, hence pooled (equal) variance was used. The t-statistic in equal variance group was $t = 0.203$, $P = 0.822$, implying that the difference is insignificant. Therefore, no difference existed in developmental level of mathematical thinking between the two classes before conducting the teaching practice.

Table 2: Test results of the independent sample t test for the previous data

	Levene variance equivalence test		Average equivalent t test						
	F	Sig.	t	Freedom	Sig(2-tail)	MD	SED	The difference is 95% confidence interval	
								Lower limit	Upper limit
Assumed equal variance	0.493	0.412	0.203	66	0.822	0.801	3.216	-6.523	7.963
Unassuming equal variance	-	-	0.203	65.232	0.822	0.801	3.216	-6.519	7.942

An independent-samples t-test was conducted to examine whether differences existed across the dimensions of mathematical thinking between the experimental and control groups prior to the teaching intervention. The statistical results are presented in Table 3. The analysis showed that no statistically significant variation was found between the two groups in any dimension ($P > 0.05$). Therefore, the experimental group was considered suitable for subsequent teaching practice research.

Table 3: Test the independent sample t test before each dimension

Dimension	Class	N	Mean	SD	t	P
Creativity	Laboratory class	45	14.24	3.016	-0.523	0.523
	Cross-reference class	45	14.81	3.017		
Algorithm thinking	Laboratory class	45	13.31	3.095	1.003	0.263
	Cross-reference class	45	13.41	3.283		
Cooperative communication	Laboratory class	45	16.72	3.403	1.423	0.256
	Cross-reference class	45	16.54	3.439		
Critical thinking	Laboratory class	45	13.41	3.44	-1.536	0.231
	Cross-reference class	45	13.22	3.464		
Problem solving	Laboratory class	45	15.28	3.487	0.523	0.242
	Cross-reference class	45	15.32	3.541		

(2) Statistics and Analysis of Mathematical Thinking Post-test Questionnaire

The post-test measure of mathematical thinking took place during the teaching summary stage. The efficiency of the process of the development of mathematical thinking through the practice of instruction was tested via post-test statistics and the analyses of mathematical thinking, using both cross-sectional and longitudinal methods. For example, the cross-sectional comparison considered the difference between the two classes on the post-test scores.

1) Horizontal Analysis

This analysis uses post-test data from the experimental and control groups to compare differences in mathematical thinking after the intervention. The post-intervention results are presented in Figure 5. The mean score of the experimental group was 83.22, whereas the corresponding value for the control group was 74.09.

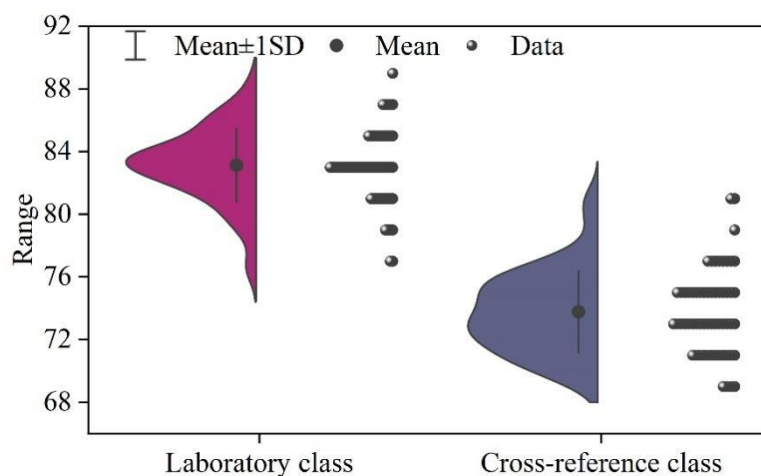


Figure 5: Posttest score for Math thinking of experiment class and comparison class

To find out if there is a difference between the development of mathematical thinking levels of the experiment and control groups after the teaching method, an independent-samples t test

was carried out using the post-test scores of the two groups. These results are provided in Table 4. Levene's test of Equality of Variances indicated $F = 1.225$ and $P = 0.163 > 0.05$, implying that the assumption of homogeneity of variances holds. Thus, the test was conducted on the basis of the equality of variances. Under this assumption, the value of t was calculated to be $t = -2.523$ and $P = 0.011 < 0.05$, indicating the significance of the difference in mathematical thinking development levels.

Table 4: Test of test of independent samples after the study of the experiment

	Levene variance equivalence test		Average equivalent t test						
	F	Sig.	t	Freedom	Sig(2-tail)	MD	SED	The difference is 95% confidence interval	
								Lower limit	Upper limit
Assumed equal variance	1.225	0.163	-2.523	66	0.011	0.801	3.216	-6.523	7.963
Unassuming equal variance	-	-	-2.523	63.354	0.011	0.801	3.216	-6.519	7.942

Two sample independent t-test analysis was used for comparison between the two groups on the sub-dimensions of mathematical thinking following the implementation of the instruction. Statistical analyses are shown in Table 5 below. The results show that, following the intervention, learners in the experimental group performed better than learners in the control group in creative thinking, algorithmic thinking, and problem-solving skills. Significantly, there was a significant difference between learners in algorithmic thinking ($P = 0.002$). However, there were no significant differences in cooperative communication and critical thinking skills ($P > 0.05$).

Table 5: The independent sample t test was tested after each dimension

Dimension	Class	N	Mean	SD	t	P
Creativity	Laboratory class	45	17.31	3.096	-2.631	0.023
	Cross-reference class	45	14.89	3.194		
Algorithm thinking	Laboratory class	45	16.23	3.259	-2.523	0.002
	Cross-reference class	45	13.52	3.282		
Cooperative communication	Laboratory class	45	16.93	3.283	0.063	0.362
	Cross-reference class	45	16.93	3.29		
Critical thinking	Laboratory class	45	14.23	3.326	-1.214	0.074
	Cross-reference class	45	13.52	3.471		
Problem solving	Laboratory class	45	18.52	3.534	-2.312	0.012
	Cross-reference class	45	15.23	3.622		

2) Longitudinal Analysis

In the analysis of the longitudinal study, the unit of analysis will be pre- and post-test scores from the experimental group that can allow longitudinal analysis on the development of mathematical thinking skills for the students of the experimental group both before and after the learning process. In Figure 6, the scores from the pre-test and post-test of the development of mathematical thinking skill in the experimental group are shown. For example, the average score for the pre-test is 72.96, whereas the average score for the post-test is 83.22.

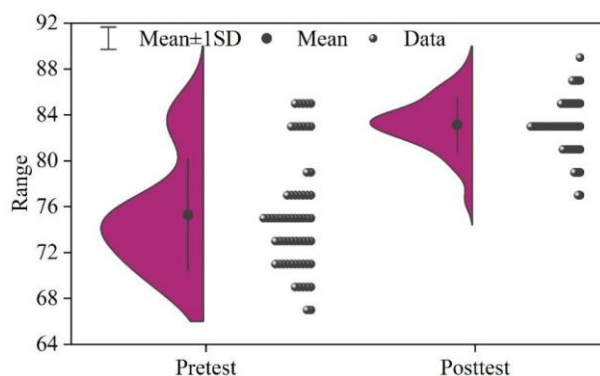


Figure 6: Experimental class mathematical thinking of pretest and posttest

The purpose of this experiment is to determine whether the existence of significant differences in the development of mathematical thinking among experimental groups exists prior to and following teaching practice. This was achieved using paired samples t-test of pre- and post-test scores, whose results are presented in Table 6. From the table, the average difference between post-test and pre-test scores of mathematical thinking is found to be 10.26; meaning the post-test score is 10.26 units more than the pre-test score.

Table 6: Matching sample t test results of experimental class pretest and posttest

Pairing	Pair difference					t	Freedom	Sig(2-tail)
	Mean	SD	SEM	The difference is 95% confidence interval				
				Lower limit	Upper limit			
Pretest-Posttest	-10.26	13.624	3.521	-16.523	-3.521	-3.006	36	0.002

To find the difference between the sub-components of mathematics thinking among the experimental group prior to and after intervention, two-sample t-tests were carried out based on pre- and post-test scores among the participants for the sub-components. Results from the test are summarized in Table 7. After intervention, the experimental group showed increased competence in all the sub-components of creativity, algorithmic thinking, and problem solving, while algorithmic thinking improved the most among all three sub-components. However, no difference was observed in the sub-component of cooperative communication and critical thinking ($P > 0.05$).

Table 7: T test of each dimensional matching sample of the experimental class

Dimension	Pretest-Posttest	N	Mean	SD	t	P
Creativity	Pretest	45	14.24	3.016	-0.523	0.523
	Posttest	45	17.31	3.017		
Algorithm thinking	Pretest	45	13.31	3.095	1.003	0.263
	Posttest	45	16.23	3.283		
Cooperative communication	Pretest	45	16.72	3.403	1.423	0.256
	Posttest	45	16.93	3.439		
Critical thinking	Pretest	45	13.41	3.44	-1.536	0.231
	Posttest	45	14.23	3.464		
Problem solving	Pretest	45	15.28	3.487	0.523	0.242
	Posttest	45	18.52	3.541		

The AI-powered educational methodology will be able to offer learners appropriate paths and resources, thus improving students' mathematical understanding. It is necessary to state the fact that the best results will be achieved by improving creative thinking, algorithmic skills, and problem-solving skills.

4 Conclusion

The promotion of AI technology provides some possible methods for nurturing mathematical thinking through training. The application of various AI-enabled educational technology tools takes place on the Smart Education Cloud Platform to promote mathematics thinking development based on this platform. Through analyzing the application practice data of Middle School A, it is discovered that 85% of students have a positive impression of this platform. Moreover, it can be found that students are more satisfied with the recommendations for their learning path and learning resources by the platform. After using the Smart Education Cloud Platform, indicators of creativity, algorithmic thinking, and problem-solving ability experience some improvements, as the total score for mathematics thinking reaches 10.26. On the other hand, in the control class adopting the traditional way of teaching, the increase of test scores is 9.13 points. From a combination of lateral and longitudinal comparisons, it can be concluded that the Smart Education Cloud Platform makes use of AI technology in a comprehensive manner and contributes to improving students' mathematical thinking.

In the future, it is necessary to expand the use cases and strengthen the integration between artificial intelligence and teaching practice. These measures can serve as a guide for the teaching approach in intelligent classrooms and even contribute to the development of intelligent classroom teaching in other disciplines. These measures can provide better support for building intelligent education.

Funding

This work was supported by B2024174, Research on data integration algorithms for teaching effectiveness analysis.

About the Author

Na Li was born in Jinmen, Hubei, P.R. China, in 1980. I obtained my master's degree from Wuhan University in China. I am currently working at the School of Mathematics and Statistics, Hubei University of Education. My main research direction is My main research direction is mathematics education and the application of modern educational technology in mathematics education.

Zhiming Liu was born in Huanggang, Hubei, P.R. China, in 1972. I obtained my master's degree from Huazhong University of Science and Technology in China. I am currently working at the School of Computer and Artificial Intelligence, Hubei University of Education. My main research direction is Complex System Modeling and Simulation, Intelligent Information Processing, and Modern Educational Technology.

References

- [1] Cheng, J., Bao, J., & Zhang, D. (2021). From 'two basics', to 'four Basics' to 'core

- mathematics competencies' in mainland China. In *Beyond Shanghai and PISA: Cognitive and non-cognitive competencies of Chinese students in mathematics* (pp. 1-13). Cham: Springer International Publishing.
- [2] Sanjaya, A., Johar, R., Ikhsan, M., & Khairi, L. (2018, September). Students' thinking process in solving mathematical problems based on the levels of mathematical ability. In *Journal of Physics: Conference Series* (Vol. 1088, No. 1, p. 012116). IOP Publishing.
- [3] Leung, F. K. (2021). Mathematics Core Competencies of Chinese Students—What Are They?. In *Beyond Shanghai and PISA: Cognitive and Non-cognitive Competencies of Chinese Students in Mathematics* (pp. 315-332). Cham: Springer International Publishing.
- [4] Madraximovich, K. E., & Ruzimovich, Y. J. (2021). Application of problem-based teaching methods in the development of mathematical thinking skills of students. *Annals of the Romanian Society for Cell Biology*, 25(2), 43-47.
- [5] Soboleva, E. V., Chirkina, S. E., Kalugina, O. A., Shvetsov, M. Y., Kazinets, V. A., & Pokaninova, E. B. (2020). Didactic potential of using mobile technologies in the development of mathematical thinking. *Eurasia Journal of Mathematics, Science and Technology Education*, 16(5), em1842.
- [6] Isoda, M., & Katagiri, S. (2012). *Mathematical thinking: How to develop it in the classroom* (Vol. 1). World Scientific.
- [7] Firdaus, F., Kailani, I., Bakar, M. N. B., & Bakry, B. (2015). Developing critical thinking skills of students in mathematics learning. *Journal of Education and Learning (EduLearn)*, 9(3), 226-236.
- [8] Calao, L. A., Moreno-León, J., Correa, H. E., & Robles, G. (2015, September). Developing mathematical thinking with Scratch: An experiment with 6th grade students. In *European conference on technology enhanced learning* (pp. 17-27). Cham: Springer International Publishing.
- [9] Harjo, B., Kartowagiran, B., & Mahmudi, A. (2019). Development of Critical Thinking Skill Instruments on Mathematical Learning High School. *International Journal of Instruction*, 12(4), 149-166.
- [10] Van Doc, N., Nam, N. H., Thanh, N. T., & Giam, N. M. (2023). Teaching mathematics with the assistance of an AI chatbot to enhance mathematical thinking skills for high school students. *International Journal of Current Science Research and Review*, 6(12), 8574-8580.
- [11] Rongrong, C. (2019). Exploring the education of mathematical thinking in mathematics teaching. In *Proceedings of the 3rd International Conference on Economics Management Engineering and Education Technology* (Vol. 325, pp. 71-74).
- [12] Mustafa, S., & Sari, V. (2019). The Implementation of Mathematical Problem-Based Learning Model as an Effort to Understand the High School Students' Mathematical Thinking Ability. *International Education Studies*, 12(2), 117-123.
- [13] Korkmaz, Ö., Çakir, R., & Özden, M. Y. (2017). A validity and reliability study of the

- computational thinking scales (CTS). *Computers in human behavior*, 72, 558-569.
- [14] Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education*, 46(1), 88-124.
- [15] Liguang, X., & Qiong, S. (2019). The cultivation path of high-level mathematical thinking. *Mathematical Bulletin* (05), 33-36.
- [16] Mateus-Nieves, E., & Díaz, H. R. D. (2021). Development of mathematical thinking skill from the formulation and resolution of verbal arithmetic problems. *Acta Scientiae*, 23(1), 30-52.
- [17] Saragih, S., & Napitupulu, E. E. (2015). Developing student-centered learning model to improve high order mathematical thinking ability. *International Education Studies*, 8(06), 104-112.
- [18] Pathak, S. (2021). Learners creativity through mathematical thinking and teaching. *Journal of Asia Social Science*, 2(3), 1-9.
- [19] Nurmanita, N., Siagian, P., & Sitompul, P. (2019). Development of Learning Device through Problem Based Learning Model Assisted by Geogebra to Improve Students' Critical Mathematical Thinking Ability. *Journal of Mathematical Sciences and Applications*, 7(1), 1-9.
- [20] Atoyebi, O. M., & Atoyebi, S. B. (2022). A Meta-analytic Review of the Relationship between Mathematics Anxiety and the Mathematical Thinking of Africa Students. *Archives of Current Research International*, 22(7), 15-28.
- [21] Samo, D. D., & Kartasasmita, B. (2017). Developing Contextual Mathematical Thinking Learning Model to Enhance Higher-Order Thinking Ability for Middle School Students. *International Education Studies*, 10(12), 17-29.
- [22] Mudrikah, A. (2016). Problem-Based Learning Associated by Action-Process-Object-Schema (APOS) Theory to Enhance Students' High Order Mathematical Thinking Ability. *International Journal of Research in Education and Science*, 2(1), 125-135.
- [23] Rane, N. (2023). Enhancing mathematical capabilities through ChatGPT and similar generative artificial intelligence: Roles and challenges in solving mathematical problems. Available at SSRN 4603237.
- [24] Shen, J., & Wu, Q. (2021, September). Application and cultivation on mathematical thinking in artificial intelligence. In *2021 4th International Conference on Intelligent Robotics and Control Engineering (IRCE)* (pp. 136-140). IEEE.
- [25] Li, W. (2025). The path of cultivating college students' mathematical thinking ability in the new era. *Frontiers in Educational Research*, 8(3).
- [26] e Silva, J. C. (2025). Computational thinking versus artificial intelligence in mathematics teaching. *International Journal of Mathematical Education in Science and Technology*, 1-12.

- [27] Tashtoush, M. A., Qasimi, A. B., Shirawia, N. H., & Hussein, L. A. (2025). The efficacy of utilizing artificial intelligence techniques in developing critical thinking in mathematics among secondary school students and their attitudes toward it. *Iraqi Journal for Computer Science and Mathematics*, 6(1), 3.
- [28] Arifin, M. Z., Zulkarnain, I., & Ansori, H. (2025). The influence of artificial intelligence on critical thinking ability in mathematics: A systematic literature review. *Indonesian Journal of Science and Mathematics Education*, 8(1), 82-92.
- [29] Hamid, N., & Haidar, I. (2025). Integration of Artificial Intelligence in Mathematics Learning: An Effort to Improve Creative Thinking Abilities. *Jurnal Pedagogi dan Inovasi Pendidikan*, 1(1), 10-19.