



## Cooperative clearing model of inter-provincial capacity market and electricity market based on game equilibrium

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**SUMMARY:** *In inter-provincial power trading, the collaborative clearing of capacity market and electricity market needs to simultaneously deal with strategic bidding interaction, transmission coupling and high-dimensional operational constraints. Based on two-layer market interaction and mixed-integer optimization architecture, a cooperative clearing model based on game equilibrium is established. The upper layer is the strategic offer of the provincial seller and buyer in the capacity market and the electricity market, and the lower layer is the joint clearing under tie-line restrictions, reserve requirements, hill climbing boundaries and node balance constraints. Combining KKT reconstruction, binary linearization, and branch-and-cut acceleration, the equalization process is transformed into a solvable mixed-integer framework. The experimental results on a 12-province system containing 168 generating units, 36 inter-provincial channels and 24 periods show that under different load scenarios, the proposed method reduces the total power purchase cost by 11.8%, reduces the new energy power abandonment rate by 14.6%, improves the clearing convergence efficiency by 23.4%, and keeps the equilibrium deviation within 3.1%.*

**KEYWORDS:** *Game equilibrium; Inter-provincial transaction; Capacity-quantity cooperative clearing; Mixed integer optimization*

## 1 Introduction

Under the background of the connection between the new power system and the unified national electricity market, the cross-provincial transaction has shifted from a single electricity exchange to a composite form of parallel configuration of capacity value and electric energy value. The capacity market assumes the function of ensuring the sufficiency, and the electricity market reflects the short-term marginal supply-demand relationship. The two types of markets are coupled in the bidding subject, constraint boundary, income settlement and timing linkage. If the cross-provincial transactions are still processed by segmented clearing or isolated matching, it is difficult to form a calculation link between the quantity quotation of the generation side, the power purchase preference of the power receiving side, the channel capacity occupation and the reserve security demand, and the clearing result is difficult to simultaneously consider the economy, feasibility and strategic stability. Therefore, the collaborative clearing model for inter-provincial capacity market and electricity market has become the key direction of the research on electricity market computing mechanism.

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Existing research has provided a basis for cross-regional market modeling. Chen et al. studied the two-stage clearing framework of China's inter-provincial power transactions, indicating that inter-provincial transactions need to complete electricity matching in multi-stage constraint transfer [1]. Fu et al. proposed a medium - and long-term cross-provincial market transaction arrangement and clearing model considering security constraints, so that cross-regional sections and system security boundaries were included in the solution process [2]. Shao et al. studied the day-ahead energy and reserve joint clearing model considering flexible load, indicating the necessity of synchronous optimization in multi-variety market [3]. Chen et al. analyzed the optimal participation mode of the distribution network in the electric energy and reserve market and revealed the characteristics of uncertainty propagation under the multi-layer market interface [4]. He et al. proposed a two-stage market clearing model based on bilevel optimization to form a linkage relationship between new energy consumption constraints and market decision-making [5].

In terms of multi-agent strategy interaction, Hong et al. constructed a two-layer game decision-making framework in which retailers participate in the local market and the wholesale market, indicating that market players will adjust their strategy set according to price feedback [6]. Wang et al. proposed the market decision-making model of flexible load aggregators, which makes the demand side response behavior enter the quotation and clearing chain [7]. Yu et al. studied the Nash equilibrium solution and distributed bidding method based on reinforcement learning, indicating that equilibrium search in incomplete information environment has computational characteristics [8]. Wu et al. proposed a real-time iterative bidding hybrid model coupled with the dynamic process of the power system, so that the bidding evolution can track the change of the system state [9]. Nokandi et al. constructed a three-stage two-level model for virtual power plants to participate in joint market scheduling, reflecting the timing nested relationship between multi-stage decisions [10].

With the expansion of market coupling objects, joint modeling is no longer limited to a single power transaction. Zhang et al. proposed a multi-agent hybrid game bidding model oriented to the electric-carbon coupling mechanism to characterize the multi-market revenue correlation [11]. Lan et al. studied the joint clearing model of day-ahead energy and auxiliary services at the generation side, which took into account both low carbon and efficiency, and provided a basis for the unification of multi-attribute market objectives [12]. Mei et al. proposed the optimal bidding strategy for virtual power plants to participate in the composite market of electric energy and auxiliary services, and included the dynamic demand response price into the revenue measurement [13]. Jiang et al. studied the strategic behavior of power generation enterprises in the electricity market and carbon market, indicating that the bidding pattern of subjects in the coupled market has the characteristics of linkage evolution [14]. Starting from the equilibrium analysis under the reliability option condition, Feng et al. discussed the balance relationship between the generation-side strategy and the user-side cost [15]. Wu et al. proposed a Stackelberg electricity market clearing method for multiple stakeholders, which made the leader-follower relationship have a more explicit solution structure [16]. Chen et al. further studied the Stackelberg transaction method in the distribution network of small and micro parks, which provided a transferable idea for hierarchical transaction calculation [17]. Jiang et al. proposed a two-stage stochastic electric energy and standby market clearing model considering real-time quotation strategy, which strengthened the integrated expression of stochastic scenario and market decision [18]. Fu et al. constructed a resilient microgrid market clearing framework considering emission reduction targets, which extended the solvable model under multi-objective constraints [19]. Shi et al. proposed a multi-regional joint market clearing model of electric energy and reserve

under uncertain environment, indicating that cross-regional joint clearing requires scenario modeling and multi-regional coordination mechanism [20].

The above results have promoted the deepening of the research on cross-regional market calculation, but the collaborative clearing of capacity market and electricity market still needs to be expressed by computerization. On the one hand, there are high-dimensional coupling among capacity revenue allocation, spot electricity matching, cross-provincial channel occupancy, unit climbing, reserve constraints and new energy fluctuations, and sequential liquidation is easy to cause variable fragmentation. On the other hand, the subjects of inter-provincial transactions have strategic attributes, and the subjects of different provinces and regions will adjust their application portfolios according to capacity compensation, marginal prices and transmission bottlenecks. It is difficult to describe the equilibrium convergence process by only relying on static optimization. Combining the game equilibrium mechanism with the mixed integer optimization framework, the agent behavior, network constraints and market rules can be written into a unified data structure, objective function and constraint matrix. Then, the stable solution is realized by branch and bound, linearization reconstruction and scene parallel computing, and the computability of the model is enhanced.

Based on this, this paper constructs a collaborative clearing model around the coupling scenario of cross-provincial capacity market and electricity market. In the modeling layer, the leader-follower game equilibrium relationship was introduced to describe the strategic linkage between the power transmission side, the power receiving side and the market operation organization. In the calculation layer, the capacity allocation, power matching, channel allocation and system security boundary were incorporated into the mixed integer optimization framework. At the algorithm level, the cooperative clearing process is formed by using equilibrium constraint reconstruction, complementary condition linearization and hierarchical search mechanism. This study aims to provide the calculation accuracy and strategy interpretation foundation for the joint capacity-power allocation in cross-provincial markets.

## 2 Theoretical basis and key technologies

### 2.1 Inter-provincial market interaction mechanism driven by game equilibrium

When the inter-provincial capacity market and the electricity market are cleared jointly, the relationship between the power transmission province, the power receiving province, the power generation enterprise, the power selling entity and the market operation organization is not a one-way matchmaking relationship, but a continuous iterative strategy interaction process. The capacity bidding determines the medium and long-term available supply boundary, and the electricity bidding reflects the marginal allocation direction of the time period. The inter-provincial tie line surplus, reserve demand, unit climbing constraints and new energy output fluctuations will change the declaration method of the subject. In order for dual-market results to be consistent in a unified computing framework, the interaction mechanism needs to organize quote input, status update, boundary check, and return writeback into an iterative data stream rather than separate settlement steps.

In order to uniformly describe the revenue structure of the transmission side under the joint effect of capacity compensation, electricity transaction and cross-zone transmission constraints, this paper expresses its comprehensive utility function as formula (1), so as to construct the multi-agent equilibrium relationship and strategy update process in the following.

$$U_i^s = \sum_t (\pi_t^e q_{it} + \pi_t^c c_{it} - C_i(q_{it}) - \rho_i |q_{it} - \hat{q}_{it}|) - \sum_l \eta_l f_{ilt}^2 \quad (1)$$

Here,  $U_i^s$  represents the comprehensive revenue of subject  $i$  on the power transmission side. Let  $\pi_t^e$  and  $\pi_t^c$  denote the electricity price and capacity price at time period  $t$ , respectively.  $q_{it}$  and  $c_{it}$  represent the transaction quantity and the winning bid capacity respectively.  $C_i(q_{it})$  is the cost of generating electricity. Let  $\rho_i$  denote the deviation penalty coefficient;  $\hat{q}_{it}$  represents the plan declaration value;  $f_{ilt}$  denotes the power occupancy of the subject on section  $l$ ; Let  $\eta_l$  denote the corresponding channel cost weights. This equation puts the dual market revenue and network occupancy cost into the same evaluation framework, which is conducive to directly comparing the marginal revenue differences of different declaration combinations in the subsequent calculation.

In order to illustrate how various subjects in the dual market complete information transmission and state synchronization on a unified platform, this paper organizes capacity declaration, electricity declaration, channel occupation, boundary check and price return into a single round of interactive links, as shown in Fig. 1.

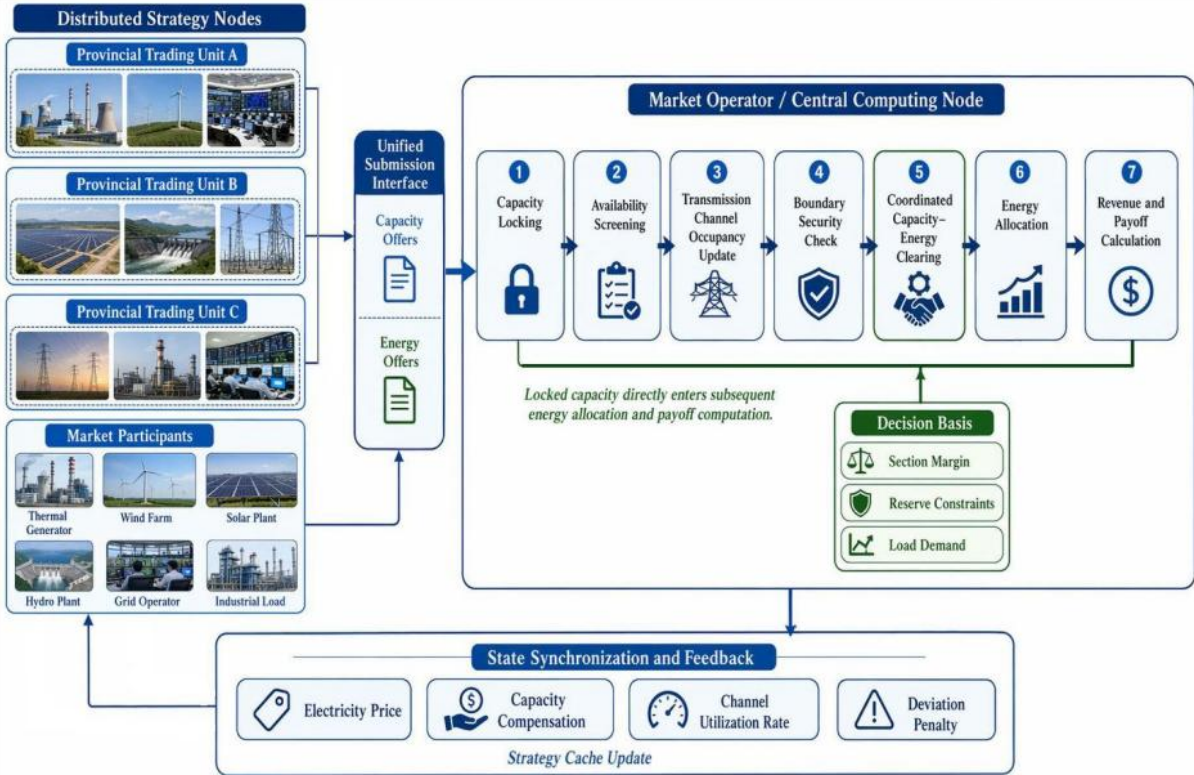


Figure 1: Inter-provincial capacity-electricity market interaction link diagram

In this link, market operators assume the function of central computing nodes, and provincial trading units and market entities constitute distributed strategy nodes. After receiving the capacity quotation and electricity quotation, the central node firstly performs capacity locking and availability screening, and then completes the joint match according to the section allowance, reserve constraint and load demand. Finally, the power price, capacity compensation, channel occupancy rate and deviation penalty are written back to the policy cache of each subject. The chain processing method formed in this way makes the capacity results no longer stay at the static qualification level, but directly enter the power allocation

and revenue calculation process.

In order to describe the response behavior of the power receiving side under the joint influence of power purchasing cost, capacity guarantee and shortage risk, this paper writes its demand side utility into equation (2), which is used to reflect the dynamic power purchasing preference of the power receiving province.

$$U_j^d = \sum_t (V_j(d_{jt}) - \pi_t^e d_{jt} - \pi_t^c r_{jt}) - \omega_j \sum_t (L_{jt} - d_{jt})^2 \quad (2)$$

Here,  $U_j^d$  represents the comprehensive utility of subject  $j$  on the receiving side.  $V_j(d_{jt})$  represents the electricity revenue function;  $d_{jt}$  denotes the power purchased during period  $t$ ;  $r_{jt}$  stands for capacity purchase share;  $L_{jt}$  is the load demand; Let  $\omega_j$  denote the default penalty weight. This formula shows that the power receiving side will not make decisions only based on the electricity price, but will evaluate the capacity guarantee level and the supply shortage cost at the same time, so the capacity procurement and power procurement are naturally coupled in the strategy space.

In order to measure the degree of influence of provincial disjoint surface allowance and reserve support capacity on the dual-market linkage effect, this paper further defines the cross-provincial coupling coefficient, as shown in Equation (3), which is used to describe the effective intensity of transmission from capacity to electricity.

$$\Gamma_{mn,t} = \frac{F_{mn,t}^{\max} - \sum_k x_{kmn,t}}{F_{mn,t}^{\max}} \cdot \frac{R_{m,t}^{\text{ava}}}{R_{m,t}^{\text{req}} + \varepsilon} \quad (3)$$

Here,  $\Gamma_{mn,t}$  denotes the coupling strength from province  $m$  to province  $n$  at time period  $t$ .  $F_{mn,t}^{\max}$  denotes the upper bound of tie-line transmission;  $x_{kmn,t}$  denotes the cross-zone power occupied by principal  $k$ ;  $R_{m,t}^{\text{ava}}$  indicates available spare;  $R_{m,t}^{\text{req}}$  stands for demand reserve; Let  $\varepsilon$  denote the tiny constant that prevents the denominator from being zero. The higher the coefficient is, the more effectively the capacity locking result can be mapped to the electricity transaction space, and the smoother the information connection between the double markets is.

In order to show how each subject continues to revise the declaration vector after the price writeback, this paper represents the state evolution process in the capacity-quantity cooperative clearing as an iterative closed-loop structure, as shown in Fig. 2.

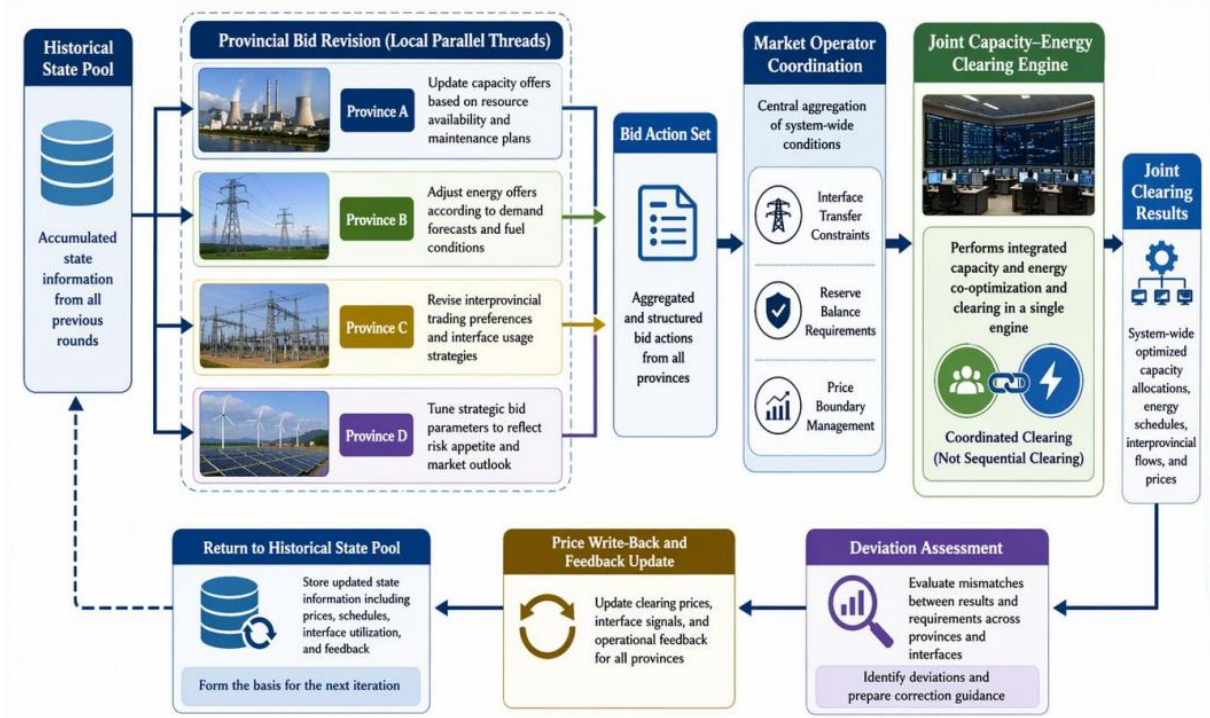


Figure 2: State iteration diagram of inter-provincial capacity-electricity collaborative clearing

In the figure, the historical state vector, the offer action vector, the joint clearing matrix and the deviation evaluation result successively enter the next round of parameter correction process. The provincial subject can update the quotation in the local thread, and the market operator is responsible for summarizing the section constraints, reserve balances and price boundaries, and sending the results back to the state pool. This structure is suitable for parallel iteration on computer platform, and can reduce repeated reading and variable splitting which are common in sequential clearing.

In order to enable market operators to coordinate revenue distribution, supply-demand balance and cross-zone security boundary within a unified framework, this paper defines the upper joint objective function as shown in Equation (4).

$$\max Z = \sum_t \left( \sum_i U_i^s + \sum_j U_j^d - \lambda_t \Delta_t - \mu_t \Phi_t \right) \quad (4)$$

where  $Z$  represents the overall system goal;  $\Delta_t$  represents the amount of power mismatch;  $\Phi_t$  represents the safety loss caused by section over-limit, insufficient reserve and capacity loss. Let  $\lambda_t$  and  $\mu_t$  denote the corresponding penalty weights, respectively. This formula does not simply pursue the expansion of transaction scale, but establishes a unified measurement between profit and security.

In order to characterize the strategy update behavior of each agent with bounded adjustment based on the market results of the previous round, the filing correction process of the follower layer is shown in Equation (5).

$$x_a^{(k+1)} = \arg \max_{x_a \in \Omega_a} \left[ U_a(x_a, x_{-a}^k) - \tau_a \|x_a - x_a^k\|_2^2 \right] \quad (5)$$

Here,  $x_a^{(k+1)}$  represents the strategy vector of agent  $a$  at round  $k + 1$ .  $\Omega_a$  denotes its feasible region; Let  $x_{-a}^k$  denote the set of strategies of the other agents in round  $k$ ; Let  $\tau_a$  denote the stabilizing regulation parameter. The proximity penalty term can restrain the large jump of the offer, which makes the equilibrium search process more suitable for the distributed parallel computing environment.

In order to judge whether the dual market enters a stable joint equilibrium state after multiple rounds of interaction, this paper constructs a residual index in which price, strategy and coupling coefficient are jointly involved, as shown in equation (6).

$$R^k = \left[ \sum_a \left\| x_a^{(k+1)} - x_a^k \right\|_2^2 + \sum_t \left| \pi_t^{(k+1)} - \pi_t^k \right|^2 + \sum_{m,n} \left| \Gamma_{mn,t}^{(k+1)} - \Gamma_{mn,t}^k \right|^2 \right]^{1/2} \quad (6)$$

Here,  $R^k$  denotes the joint residual of round  $k$ ; The first term reflects the strategy change of the subject, the second term reflects the price fluctuation, and the third term reflects the coupling relationship change between provinces. When  $R^k$  is lower than the given threshold, it can be considered that the capacity market and the electricity market have formed a stable linkage relationship, and the subsequent collaborative clearing algorithm can carry out integer constraint decomposition and boundary clipping on this basis.

In summary, the interaction between the cross-provincial capacity market and the electricity market is not a simple superposition of quotes, but a closed-loop calculation process driven by strategic response, section constraints, capacity commitment and price feedback. After embedding the game equilibrium mechanism into the process, the reporting behaviors, revenue changes and network boundaries of various subjects can be expressed synchronously under the same framework, which provides a stable foundation for the subsequent collaborative clearing model construction and algorithm development, and enhances the interpretability support of the results.

## 2.2 Mixed integer optimization and collaborative clearing calculation method

Mixed-integer optimization undertakes two tasks of "rule landing" and "boundary fidelity" in the collaborative clearing of cross-provincial capacity market and electricity market. Capacity winning state, tie-line direction state, unit start and stop state have discrete attributes, while time-of-use electricity, reserve allocation, marginal price and inter-provincial power flow are continuous variables. If only continuous optimization is retained, the linkage relationship between capacity commitment and power allocation will be weakened, and the executable boundary in cross-provincial transactions will be difficult to preserve completely. When the integer variables and continuous variables are put into a unified solution space, the capacity quotation, electricity quotation, section allowance, reserve demand and penalty settlement can be encoded as different variable blocks in the same matrix, so as to provide a consistent data basis for subsequent branching, relaxation and backchecking.

In order to show how capacity variables, electricity variables and network boundaries can be assembled and written back in the unified solver, the collaborative clearing calculation chain is organized into five steps: variable encoding, constraint generation, relaxation solving, node screening and result return, as shown in Fig. 3.

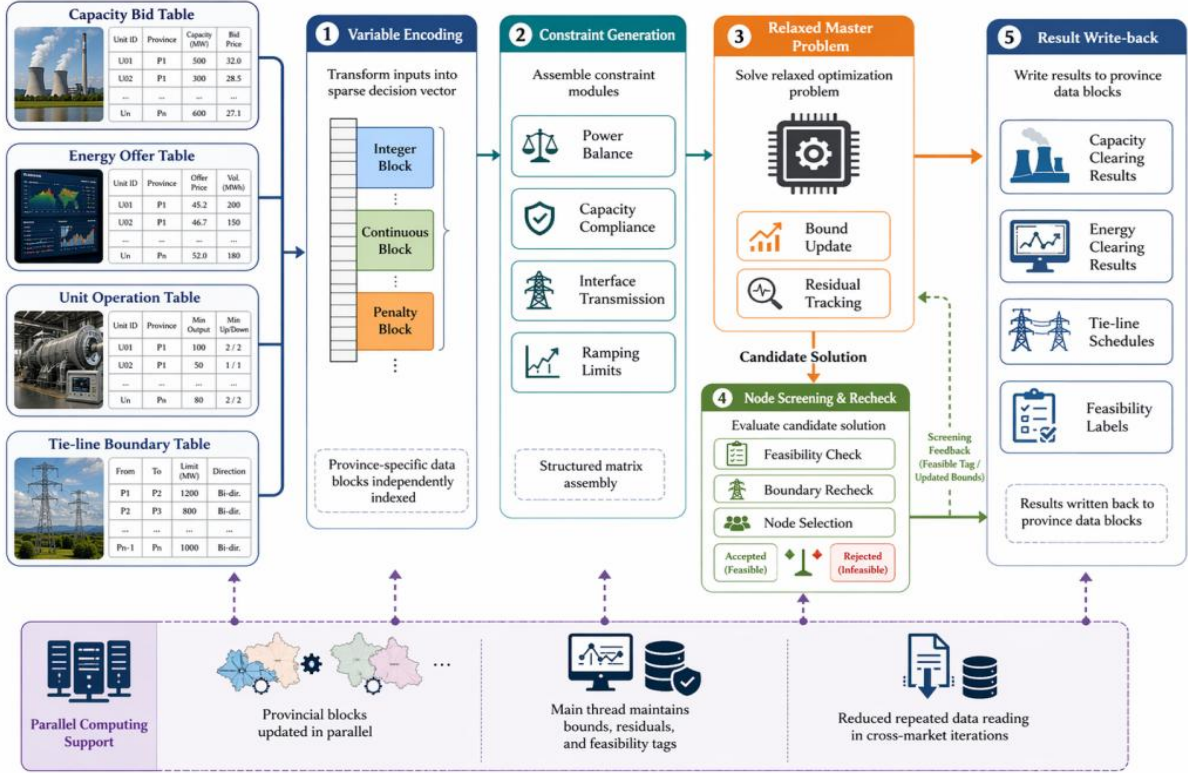


Figure 3: Flowchart of mixed-integer optimization for capacity-quantity collaborative clearing

In Fig. 3, the capacity declaration form, the electricity quotation table, the unit operation table and the tie line boundary table first enter the variable coding layer to form a sparse vector composed of integer blocks, continuous blocks and penalty blocks. Then, the constraint generation layer sequentially assembled power balance, capacity performance, section transmission and hill climbing boundaries. The solver completes the bound update on the relaxed master problem, and then passes the candidate solution to the back-checking module. With this organization, the provincial data blocks can be independently numbered and updated in parallel in the program, and the main thread only needs to maintain the bounds, residuals, and feasible labels, thereby reducing repeated reads in multiple rounds of calculation across markets.

In order to unify the objective relationship between capacity transaction, electricity allocation and section constraints, the collaborative clearing master problem is shown in Equation (7).

$$\min J = \sum_t \left[ \sum_i (\alpha_i p_{it} + \beta_i u_{it} + \gamma_i r_{it}) + \sum_{m,n} \delta_{mn} f_{mn,t} + \kappa_t \xi_t \right] - \sum_t (\omega_t^c C_t + \omega_t^e E_t) \quad (7)$$

where  $J$  represents the system synthesis cost;  $p_{it}$  represents the transaction quantity of unit  $i$  in time period  $t$ ;  $u_{it}$  represents the start-stop state variable;  $r_{it}$  stands for reserve commitment;  $f_{mn,t}$  denotes the transmission power from  $m$  to  $n$  province; Let  $\xi_t$  denote the safety deviation;  $C_t$  and  $E_t$  represent the capacity transaction scale and electricity transaction scale respectively. The corresponding weights are  $\alpha_i, \beta_i, \gamma_i, \delta_{mn}, \kappa_t, \omega_t^c, \omega_t^e$ . This equation compresses the capacity revenue, electricity revenue, channel occupancy cost and security cost in the same objective function, which enables the solver to directly compare the

global costs of different declaration combinations.

In order to ensure that the provinces still meet the power conservation and capacity performance boundary under the time-division transaction results, the core equilibrium condition of collaborative clearing is shown in Equation (8).

$$\sum_{i \in \Omega_m} p_{it} + \sum_n f_{nm,t} - \sum_n f_{mn,t} + b_{m,t} - s_{m,t} = D_{m,t}, \quad \sum_{i \in \Omega_m} y_i^c \bar{q}_i \geq Q_m^c \quad (8)$$

Here,  $\Omega_m$  denotes the set of units corresponding to province  $m$ ;  $b_{m,t}$  and  $s_{m,t}$  represent electricity purchased and sold respectively.  $D_{m,t}$  denotes the time period load demand;  $y_i^c$  represents the capacity winning state;  $\bar{q}_i$  represents the capacity that the unit can provide;  $Q_m^c$  represents the provincial capacity demand lower bound. The first part of the constraint is used to maintain the time balance of the electricity market, and the second part of the constraint is used to ensure the sufficiency of the capacity market, thus forming the basic boundary of the two-market codomain calculation.

Table 1 shows the mapping relationship between capacity market, electricity market and inter-provincial network at the program level.

*Table 1: Corresponding table of mixed-integer calculation structure for capacity-quantity collaborative clearing*

Computational Dimension	Variable Form	Constraint Source	Solver Interface
Capacity clearing	Binary variables, continuous capacity variables	Capacity demand, fulfillment boundary	Master problem variable block
Energy clearing	Continuous variables	Power balance, bid upper bound	Linear relaxation block
Tie-line occupancy	Binary direction variables, continuous power flow variables	Corridor limit, transaction direction	Network constraint block
Unit operation	Commitment variables, ramping variables	Minimum output, ramping boundary	Unit constraint block
Deviation penalty	Non-negative continuous variables	Security verification, breach settlement	Check-and-correction block

In order to simultaneously express the line safety, unit climbing and transmission direction constraints, the linkage boundary reconstruction between network and unit is shown in Equation (9).

$$H p_t + T f_t \leq b_t, \quad -\underline{\Delta}_i \leq p_{it} - p_{i,t-1} \leq \bar{\Delta}_i, \quad 0 \leq f_{mn,t} \leq y_{mn,t}^{tr} \bar{F}_{mn} \quad (9)$$

where  $H$  is the unit injection mapping matrix,  $T$  is the section transmission mapping matrix, and  $b_t$  is the network boundary vector.  $\underline{\Delta}_i$  and  $\bar{\Delta}_i$  denote the downslope and upslope climbing ability of the unit, respectively.  $y_{mn,t}^{tr}$  represents the tie line direction state variable;  $\bar{F}_{mn}$  represents the upper limit of channel transmission. In this formula, power flow boundary, unit dynamic boundary and line activation state are uniformly written into linear constraints, so that the branch and bound process can gradually cut the infeasible region according to node.

In order to measure the convergence degree of branch and bound and the effect of

node clipping, the stopping criterion of the collaborative clearing algorithm is shown in Equation (10).

$$\Theta^{(h)} = \frac{|UB^{(h)} - LB^{(h)}|}{|UB^{(h)}| + \epsilon} + v_1 \sum_t \xi_t^{(h)} + v_2 \sum_{m,n,t} |f_{mn,t}^{(h)} - f_{mn,t}^{(h-1)}| \quad (10)$$

Here,  $\Theta^{(h)}$  represents the comprehensive residual error after the  $h$  search round.  $UB^{(h)}$  and  $LB^{(h)}$  denote the current upper and lower bounds, respectively. The  $\epsilon$  is a small constant that prevents the denominator from becoming zero. Let  $\xi_t^{(h)}$  denote the current safety deviation; The last term represents the fluctuations of the sectional power in adjacent rounds.  $v_1$  and  $v_2$  are the weighting coefficients. This formula not only measures the convergence of the target value, but also synchronizes the safety deviation and the stability of the transmission path, so that the stopping condition is more in line with the calculation characteristics of cross-provincial dual market collaborative clearing.

At the implementation level, mixed-integer solving doesn't just pass variables directly to the solver. The capacity declaration form, electricity quotation table, tie line matrix and unit status table need to be uniformly numbered first, and then transcribed into sparse coefficient matrix and right-end item vector by the program. After this process, the data blocks of different provinces can be updated independently in the shared memory, and the main thread is only responsible for reading the bounds, residuals, and feasible labels. The resulting collaborative clearing calculation method not only retains the dual-market linkage boundary, but also enhances the solution stability of the model in large-scale cross-provincial scenarios.

### 3 Construction of collaborative clearing model of inter-provincial capacity market and electricity market

#### 3.1 Modeling framework design of capacity-electricity market collaborative clearing

The core of modeling the collaborative clearing of inter-provincial capacity market and electricity market is to write the capacity commitment, electricity matching, inter-provincial transmission boundary, unit availability state and price feedback into the same state space. The transmission province, receiving power province, power generation enterprises and market operators form continuous interactions on different time scales. The capacity results determine the subsequent electricity allocation boundaries, and the electricity transaction results will reverse modify the capacity declaration intensity and cross-regional trading direction. Therefore, the modeling framework cannot adopt a separate expression, but needs to organize the capacity quote flow, electricity quote flow, section state flow and settlement write back flow into a unified calculation chain.

In order to translate the heterogeneous input of provincial subject in capacity market and electricity market into computable state vector, the market state of a single provincial node in time period  $t$  is shown in Equation (11) in this paper.

$$s_{m,t} = \left[ \hat{C}_{m,t}, \hat{E}_{m,t}, D_{m,t}^f, R_{m,t}^{req}, \sum_{n \in \mathcal{N}_m} \bar{F}_{mn,t}, \omega_{m,t} \right]^T \quad (11)$$

where  $s_{m,t}$  denotes the state vector of provincial node  $m$  at time period  $t$ .  $\hat{C}_{m,t}$  denotes the declared value of capacity;  $\hat{E}_{m,t}$  denotes the declared value of electricity;  $D_{m,t}^f$  stands for predicted load;  $R_{m,t}^{req}$  are standby requirements;  $\bar{F}_{mn,t}$  denotes the available tie-line capacity with neighboring provinces; Let  $\omega_{m,t}$  denote the state weight parameters. This formula uniformly encodes capacity, electricity, load, reserve and network boundary into a single vector, so that the program can be indexed, cached and updated in parallel by provincial node.

In order to clearly describe the organizational relationship of capacity declaration, electricity matching, tie line verification and result writing back in the system, this paper designs the cross-provincial collaborative clearing framework as a hierarchical node linkage structure, and its state flow is shown in Fig. 4.

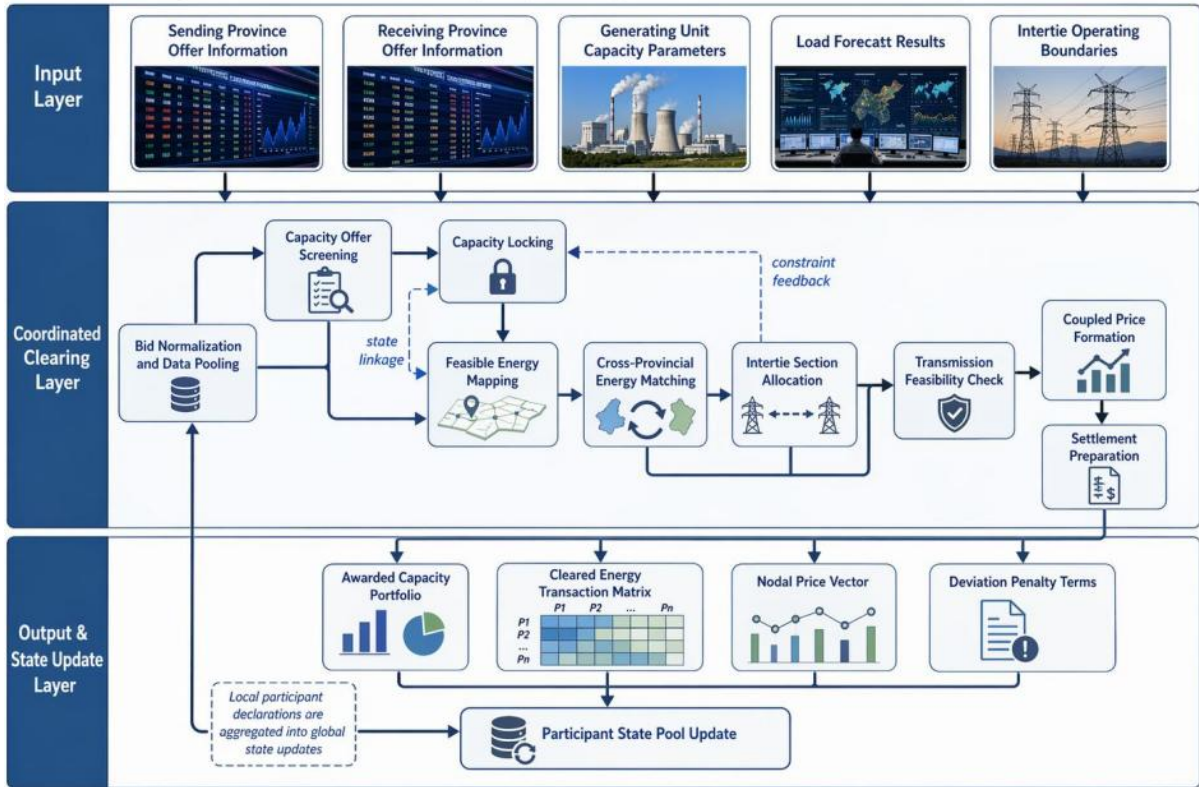


Figure 4: Modeling framework diagram of inter-provincial capacity-electricity market collaborative clearing

In Fig. 4, the input layer receives the quotation information of power saving, unit capacity parameters, load forecasting results and tie-line operating boundaries. The middle layer completed the capacity locking, feasible electricity mapping, provincial gap allocation and price formation. The output layer generates the capacity winning bid set, the electricity transaction matrix, the node price vector and the deviation penalty term, and writes them back to each subject state pool. In this way, the local declaration behavior of market players can be transformed into the global status update under a unified structure, which provides a stable interface for the subsequent game equilibrium constraints and integer optimization solutions.

In order to write the constraint relationship between capacity commitment and subsequent electricity transaction space into the same modeling framework, the feasible mapping between unit capacity and electricity quantity is shown in Equation (12).

$$0 \leq e_{i,t} \leq \min\{\kappa_i c_{i,t}, \bar{p}_i u_{i,t}, \bar{p}_i - \Delta_i^{up}(1 - u_{i,t-1})\} \quad (12)$$

where  $e_{i,t}$  denotes the tradable electricity quantity of unit  $i$  in time period  $t$ .  $c_{i,t}$  denotes the medium value of capacity; Let  $\kappa_i$  denote the cashability coefficient from capacity to quantity;  $\bar{p}_i$  represents the maximum output of the unit;  $u_{i,t}$  denotes the running state of the unit;  $\Delta_i^{\text{up}}$  represents the amount of climbing constraints in the start-up phase. This equation shows that the upper bound of electricity transaction is not only determined by the upper limit of output, but also constrained by the winning bid result of capacity and the dynamic operation boundary. Therefore, the results of capacity market will directly enter the feasible region of electricity market in this framework.

In order to keep the inter-provincial transaction direction, electricity balance and network occupancy relationship consistent in the unified matrix, the inter-provincial transaction mapping relationship is written as shown in Equation (13).

$$g_{mn,t} = \sum_{i \in \Omega_m} \phi_{i,mn} e_{i,t} - \sum_{j \in \Omega_n} \psi_{j,nm} d_{j,t}, \quad a_{mn,t} = \mathbb{I}(g_{mn,t} > 0) \quad (13)$$

Here,  $g_{mn,t}$  denotes the net trading volume between province  $m$  and province  $n$  in time period  $t$ .  $\phi_{i,mn}$  and  $\psi_{j,nm}$  denote the mapping coefficients of transmission units and receiving loads to inter-provincial channels, respectively.  $d_{j,t}$  denotes the demand response amount of the receiving side;  $a_{mn,t}$  denotes the state variable of inter-province transaction direction;  $\mathbb{I}(\cdot)$  is the indicator function. This equation relates the net inter-provincial trading volume to the direction state, so that the channel enabled state, the direction of the transaction and the size of the transaction are synchronized in the unified link.

In order to depict the comprehensive performance of capacity compensation, electricity settlement, deviation penalty and cross-regional purchase and sale revenue at the provincial level, the settlement function of provincial nodes is shown in Equation (14).

$$\Pi_m = \sum_t (\pi_t^c C_{m,t}^{\text{clr}} + \pi_{m,t}^{\text{e,exp}} E_{m,t}^{\text{exp}} - \pi_{m,t}^{\text{e,imp}} E_{m,t}^{\text{imp}} - \vartheta_m \epsilon_{m,t}) \quad (14)$$

Here,  $\Pi_m$  denotes the settlement revenue of provincial node  $m$ .  $C_{m,t}^{\text{clr}}$  stands for capacity winning scale;  $E_{m,t}^{\text{exp}}$  and  $E_{m,t}^{\text{imp}}$  represent the outgoing power and incoming power respectively. Let  $\pi_{m,t}^{\text{e,exp}}$  and  $\pi_{m,t}^{\text{e,imp}}$  denote the settlement prices on the sending and receiving sides, respectively. Let  $\epsilon_{m,t}$  denote the amount of bias; Let  $\vartheta_m$  denote the deviation penalty coefficient. This formula combines the capacity revenue, electricity revenue and deviation correction in the dual market into a unified settlement expression, which is conducive to comparing the revenue changes of different provincial strategy combinations in the subsequent equilibrium calculation.

In order to make the subject of each province complete the state writeback and form a new offer input after multiple rounds of clearing, the node state update process is shown in Equation (15) in this paper.

$$s_{m,t}^{(k+1)} = \Pi_{S_m} \left( s_{m,t}^{(k)} + \alpha P_m z_t^{(k)} + \beta Q_m \lambda_t^{(k)} - \gamma r_{m,t}^{(k)} \right) \quad (15)$$

Here,  $s_{m,t}^{(k+1)}$  represents the provincial state after the  $k+1$  update round. The projection operator  $\Pi_{S_m}(\cdot)$  is used to ensure that the state falls into the feasible set.  $z_t^{(k)}$  represents the transaction result of current round capacity-electric quantity; Let  $\lambda_t^{(k)}$  denote the price vector;

$r_{m,t}^{(k)}$  represents the bias and constraint residual;  $\alpha, \beta, \gamma$  are the update weights;  $P_m, Q_m$  are mapping matrices. The formula shows that the framework is not a static calculation structure, but a closed-loop system containing result writeback and state correction. After the completion of a round of settlement, each province will enter the next round of declaration and update according to the transaction results and price feedback.

In the computer implementation layer, the capacity quotation table, the electricity quotation table, the unit parameter table and the tie line boundary table will be uniformly numbered, and then mapped to the sparse matrix structure composed of the state block, the transaction block and the constraint block. After this process, the provincial nodes can independently call the local state in the shared memory, and the market operator only needs to maintain the transaction boundary, the price vector and the residual label to complete a unified assembly and writeback. Therefore, the framework has strong structural extensibility, and can adapt to the data update requirements with the increase of the number of cross-province subjects, the expansion of tie-line relationships and the refinement of time scales.

### 3.2 Cooperative capacity-electric quantity clearing algorithm under game equilibrium constraint

After modeling the capacity-electricity collaborative clearing framework, the solution phase needs to transform the leader's offer, the follower's power purchase response, the inter-provincial channel boundary and the capacity performance constraints into a unified computable process. If the sequential trial calculation method is still adopted, it is difficult to form a stable feedback between the capacity price, the electricity price, the transmission and receiving power and the section margin. To this end, this paper adopts the cooperative algorithm structure of "game equilibrium reconstruction - single-layer transliteration - integer pruning search", so that the two-layer interaction process can be completed in the same solver.

In order to write the leader's bidding decision and the follower's optimal response into the same solution domain, the upper objective of the two-layer model is expressed as Equation (16).

$$\begin{aligned} \max \mathcal{L} = & \sum_t \left[ \sum_i (\pi_t^c z_{i,t} \bar{q}_i + \pi_t^e p_{i,t} - C_i(p_{i,t})) \right. \\ & \left. - \chi_1 \|f_t - f_t^{\text{ref}}\|_1 - \chi_2 \|r_t - r_t^{\text{req}}\|_2^2 - \chi_3 \sum_m \xi_{m,t} \right] \end{aligned} \quad (16)$$

Here  $\mathcal{L}$  represents the leader master problem objective;  $\pi_t^c$  and  $\pi_t^e$  are the capacity price and the electricity price, respectively.  $z_{i,t}$  and  $p_{i,t}$  are the capacity bid winning state and electricity transaction volume respectively.  $\bar{q}_i$  is the available capacity of the unit;  $C_i(\cdot)$  is the generation cost function;  $f_t^{\text{ref}}$  represents the reference section assignment vector;  $r_t^{\text{req}}$  denotes the standby demand vector. Let  $\xi_{m,t}$  denote the amount of provincial deviation;  $\chi_1, \chi_2, \chi_3$  are penalty weights. In this equation, the dual market revenue, network security and equilibrium error are incorporated into the upper master problem, so that the search is always centered on the global profit.

In order to transform the local optimal behavior of followers in capacity purchase, electricity declaration and inter-provincial electricity receiving into computable conditions, this paper gives a response model, as shown in Equation (17).

$$\mathbf{x}_m^{k+1} = \arg \min_{\mathbf{x}_m \in \Omega_m} \left[ \sum_t (\lambda_t d_{m,t} + \mu_t r_{m,t} + \varrho_m \epsilon_{m,t}^2) + \frac{\tau_m}{2} \|\mathbf{x}_m - \mathbf{x}_m^k\|_2^2 \right] \quad (17)$$

Here,  $\mathbf{x}_m$  represents the joint decision vector of  $m$  receiving power.  $d_{m,t}$  denotes the power purchased during the period;  $r_{m,t}$  is the volume purchased;  $\lambda_t$  and  $\mu_t$  are the electricity price and capacity price signals, respectively.  $\epsilon_{m,t}$  is the load deviation; Let  $\varrho_m$  denote the deviation penalty coefficient;  $\tau_m$  denotes the stability parameter;  $\Omega_m$  is the provincial feasible region. This formula limits the declaration offset of adjacent rounds with a quadratic stability term, which not only suppresses the parallel update oscillation, but also retains the response sensitivity to price changes.

In order to embed the follower's optimal response into the leader's master problem and translate the two-layer structure into a single-layer constraint system, this paper writes the KKT condition, as shown in Equation (18).

$$\begin{cases} \nabla_{\mathbf{x}_m} \mathcal{F}_m(\mathbf{x}_m, \lambda) + \nabla \mathbf{g}_m(\mathbf{x}_m)^\top \alpha_m + \nabla \mathbf{h}_m(\mathbf{x}_m)^\top \beta_m = 0, \\ \mathbf{g}_m(\mathbf{x}_m) \leq 0, \quad \mathbf{h}_m(\mathbf{x}_m) = 0, \\ \alpha_m \geq 0, \quad \alpha_m^\top \mathbf{g}_m(\mathbf{x}_m) = 0 \end{cases} \quad (18)$$

where  $\mathcal{F}_m$  represents the follower objective function;  $\nabla_{\mathbf{x}_m}$  is the gradient operator with respect to the decision variables;  $\mathbf{g}_m$  and  $\mathbf{h}_m$  denote inequality and equality constraints, respectively.  $\alpha_m$  and  $\beta_m$  are the corresponding dual variables. The condition group translates the optimal behavior of the lower level into explicit algebraic constraints, so that the capacity-quantity cooperative clearing can complete assembly and back inspection in the same matrix system.

In order to eliminate the bilinear structure in the complementary relaxation term and maintain the solvability of the integer programming, the relevant conditions are linearly reconstructed as shown in Equation (19).

$$0 \leq \alpha_{j,t} \leq M_j y_{j,t}, \quad 0 \leq -g_{j,t}(\mathbf{x}) \leq M_j(1 - y_{j,t}), \quad y_{j,t} \in \{0,1\} \quad (19)$$

Here,  $\alpha_{j,t}$  denotes the  $j$  complementary dual variable.  $g_{j,t}(\mathbf{x})$  denotes the relaxation function of the corresponding constraint;  $y_{j,t}$  are the new 0-1 auxiliary variables.  $M_j$  is the linearized upper bound parameter. This set of constraints decomposes the complementarity into two parts: the mutually exclusive state and the bound value control, so that the original nonlinear system is transformed into a mixed integer linear structure that can be directly handled by the solver.

In practical computation, uniform matrix assembly does not mean that all constraints must be activated simultaneously. Section states, capacity gaps and unit boundaries have obvious spatio-temporal sparsity in different scenarios, so the algorithm only assembles the subset of boundaries that the current candidate solution may hit in the node expansion stage. In this way, the dimension of the master problem matrix can change dynamically with the search path, avoiding loading a large number of redundant restrictions in the initial stage.

In order to improve the search efficiency in large-scale cross-provincial examples, this paper further proposes a node selection criterion based on constraint activation and scene pruning as shown in Equation (20).

$$S^h = \omega_1 \text{Gap}^h + \omega_2 \|v^h\|_1 + \omega_3 D^h + \omega_4 \frac{n_{\text{int}}^h}{n_{\text{all}}^h} \quad (20)$$

Here,  $S^h$  represents the priority value of the  $h$  search node.  $\text{Gap}^h$  is the gap between the upper and lower bounds corresponding to the node.  $v^h$  represents constraint violation vector;  $D^h$  represents the deviation coefficient between the scene and the current main path;  $n_{\text{int}}^h$  and  $n_{\text{all}}^h$  denote the number of undetermined integer variables and the total number of variables, respectively. Let  $\omega_1$  to  $\omega_4$  be the weights. The smaller the priority value, the more the node enters the expansion sequence first. In this design, the convergence of boundary value, feasible degree and scene similarity are taken into account at the same time, which is more suitable for cross-provincial multi-scene parallel clearing calculation.

The algorithm is implemented by alternating mechanism of "master problem - back check problem". The main problem gives the capacity price, electricity price and cross-area transaction structure, and the back-check problem checks the section occupancy, capacity performance and unit dynamic boundary. When the backchecking results are out of bounds or deviations, the system will rewrite the corresponding cut plane into the main problem, and return the updated bound information. The process is completed by sparse matrix caching and index mapping, and it does not need to construct all constraint coefficients repeatedly, so it can still maintain good solving efficiency in large-scale examples.

In order to judge whether the algorithm reaches a stable cooperative clearing state, and simultaneously examine the objective convergence, constraint satisfaction and equilibrium fluctuation, the termination criterion is defined, as shown in Equation (21).

$$R^k = \frac{|UB^k - LB^k|}{|UB^k| + \varepsilon} + v_1 \|x^{k+1} - x^k\|_{\infty} + v_2 \|\lambda^{k+1} - \lambda^k\|_1 + v_3 E^k + v_4 Q^k \quad (21)$$

Here,  $R^k$  represents the comprehensive residual of round  $k$ ;  $UB^k$  and  $LB^k$  are the current upper and lower bounds, respectively.  $\varepsilon$  is a small constant to prevent the denominator from being zero.  $x^k$  denotes the joint decision vector; Let  $\lambda^k$  denote the price vector;  $E^k$  denotes the total amount of constraint violations;  $Q^k$  represents the equilibrium fluctuation range of each province. Let  $v_1$  to  $v_4$  be the weights. The criterion checks the convergence of bound value, physical feasibility and strategy stability at the same time. Only when the three conditions are satisfied at the same time, the algorithm outputs the final cooperative clearing result.

Through the above design, the capacity-electric quantity collaborative clearing algorithm under the game equilibrium constraint no longer stays at the abstract two-level description level, but is translated into a structured calculation process that can be directly entered into the unified solver. Capacity bidding, electricity declaration, section allocation, reserve check and price update complete linkage in the same iterative chain. KKT reconstruction, linearization of complementary conditions, constraint activation and node pruning jointly ensure the computability and convergence stability of the algorithm in large-scale cross-provincial scenarios. Therefore, the algorithm provides a unified calculation basis for the following experimental part of the clearing cost comparison, price fluctuation analysis, section utilization statistics and equilibrium deviation test.

## 4 Experiment and result analysis

In order to verify the applicability of the proposed model in the joint clearing of inter-provincial capacity market and electricity market, the experimental platform was built with Python 3.11, Gurobi 10.0 and Ubuntu 22.04 LTS. The server is configured with an Intel Xeon Gold 6338 processor, 256GB memory, and 1TB SSD. The test system contains 12 provincial nodes, 168 generating units, 36 inter-provincial contact channels, and 24 time-sharing segments. The capacity declaration, electricity quotation, reserve demand, tie line boundary and new energy output sequence are written into the solver through a unified interface, and the coefficient assembly is completed by sparse matrix. In order to reduce the influence of the difference of quotation scales in different provinces on numerical stability, capacity declaration, electricity quotation, reserve demand and channel allowance are normalized before the experiment. The main problem and the back-check problem are called alternately in the same memory pool, and the single round of matrix assembly time is controlled within 0.41s, which can support the continuous iteration of multi-scene joint clearing.

In order to investigate the operation performance of the proposed model under different system load intensities, this paper sets up four scenarios: low load, benchmark load, high load and peak load, and makes statistics on capacity turnover rate, electricity turnover rate, average convergence time and constraint satisfaction rate. The results are shown in Table 2. This table directly reflects the adaptability of the model under different running intensities and provides a unified scenario basis for subsequent horizontal comparisons.

*Table 2: Joint clearing results of the proposed model under different load scenarios*

Load Scenario	Capacity Clearing Rate / %	Energy Clearing Rate / %	Average Convergence Time / s	Constraint Satisfaction Rate / %
Low-load scenario	91.4	93.8	121	99.1
Base-load scenario	93.7	95.6	144	98.4
High-load scenario	95.2	96.9	158	97.8
Peak-load scenario	96.1	97.4	173	96.9

Table 2 shows that as the system load level rises, the volume turnover rate and electricity turnover rate increase synchronously, indicating that the joint clearing framework is able to release more cross-provincial trading capacity under high demand conditions. The average convergence time is increased from 121s to 173s, and the increase remains within the controllable interval, which indicates that the algorithm still maintains good search stability after the constraint scale is expanded. The constraint satisfaction rate always maintains above 96.9%, indicating that the linkage relationship between capacity locking, time-sharing transaction and section boundary is not unstable due to load enhancement.

In order to observe the change of transaction intensity in the joint market at different times throughout the day, this paper further draws the heat map of the inter-provincial capacity-electricity joint transaction. Fig. 5 shows that the receiving provinces form the most intensive joint transaction area from 19:00 to 21:00, and the capacity winning coefficient of the eastern receiving group is stable above 0.82, and the corresponding electricity transaction peak reaches 6.8GW. The marginal output of the delivery province is more active from 10 to 13 hours, and the capacity conversion rate of the Northwest transmission group reaches 0.87. The results show that the capacity commitment does not stay at the static guarantee level, but directly restricts the power flow direction in peak hours, and a continuous transmission

relationship is formed between the two markets.

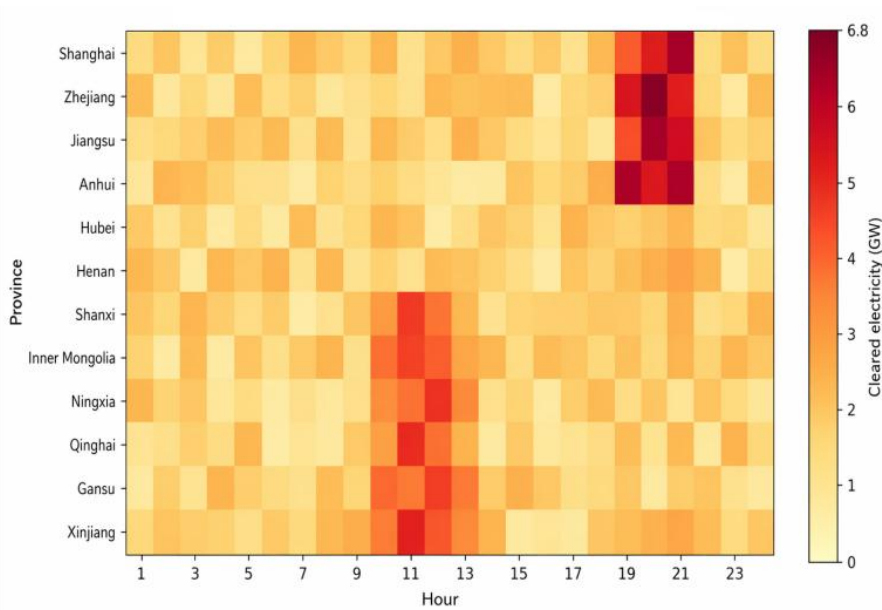


Figure 5: Heat map of inter-provincial capacity - electricity volume joint transaction

In order to compare the differences of different methods in comprehensive cost, wind and light curtailment rate, solution convergence time, equilibrium deviation and feasible rate, this paper selects the sequential clearing model, the no-game joint model and the proposed model for comparison, and the main results are shown in Table 3. The total power purchase cost of the proposed model is 22.82 billion yuan, which is 11.8% lower than that of the sequential clearing model. The rate of new energy power abandonment dropped from 9.6% to 8.2%, a decrease of 14.6%; The average convergence time was reduced from 188s to 144s, with an improvement of 23.4%. The equalization deviation under 500 perturbation experiments is controlled at 3.1%. These results show that when the game equilibrium constraint and the capacity-quantity coordination structure are introduced at the same time, the price writeback and boundary check can be completed in a unified iterative chain, so as to reduce redundant transactions and cross-zone deviation.

Table 3: Comparison of results for different models

Model	Total Power Purchase Cost / 10 <sup>8</sup> CNY	Renewable Curtailment Rate / %	Average Convergence Time / s	Equilibrium Deviation / %	Feasibility Rate / %
Sequential clearing model	258.7	9.6	188	6.7	93.2
Joint model without game equilibrium	252.1	8.9	171	4.8	95.1
Proposed model	228.2	8.2	144	3.1	98.4

In order to characterize the occupancy intensity of critical channels in all-day scheduling, this paper presents a 3D surface plot of the utilization of 36 tie lines in 24 periods. Fig. 6 shows that the channel utilization forms two high-value platforms from 8 to 11 hours and from 18 to 21 hours, where the utilization of 21 channels is distributed between 70% and 90%,

and only 3 channels exceed 95% in individual periods. Compared with the sequential clearing results, the proposed model still maintains a smooth transmission surface during the noon trough, which indicates that the pre-allocation in the capacity market weakens the phenomenon of sudden channel crowding in the electricity market, and makes the cross-region resource allocation more continuous.

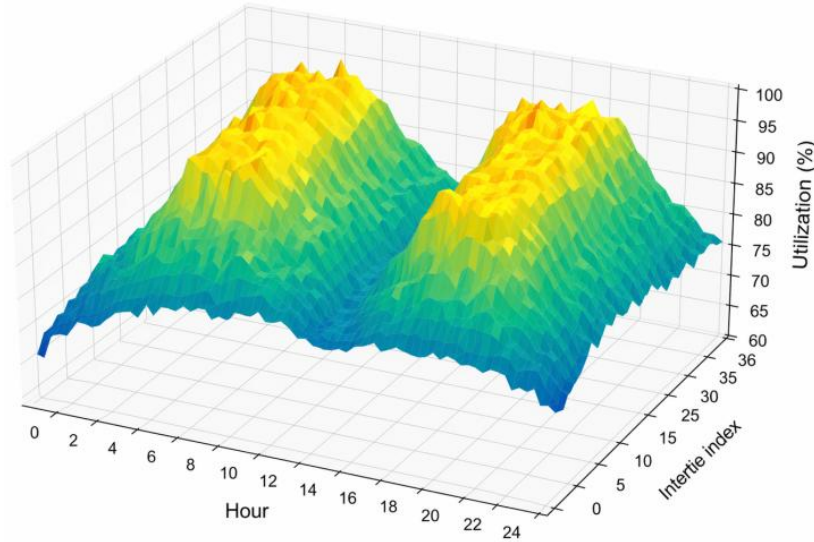


Figure 6: 3D surface plot of typical tie-line utilization

In order to analyze the price distribution characteristics of different provincial nodes in the joint market, this paper further constructs the boxplot of node electricity price and capacity compensation. Fig. 7 shows that the median electricity price under the proposed model is concentrated around 286.4 yuan /MWh, and the inter-provincial interquartile range is contracted to 31.7 yuan /MWh, which is significantly lower than 48.9 yuan /MWh of the sequential clearing model. The discrete range of capacity compensation is also contracted from 17.6 yuan /kW to 11.2 yuan /kW. The shortening of the length of the box indicates that the capacity and electricity form a more stable price linkage after the joint clearing, the marginal clearing price no longer fluctuates sharply because of the local channel squeeze, and the price signal guides the pricing strategy of the provinces more smoothly.

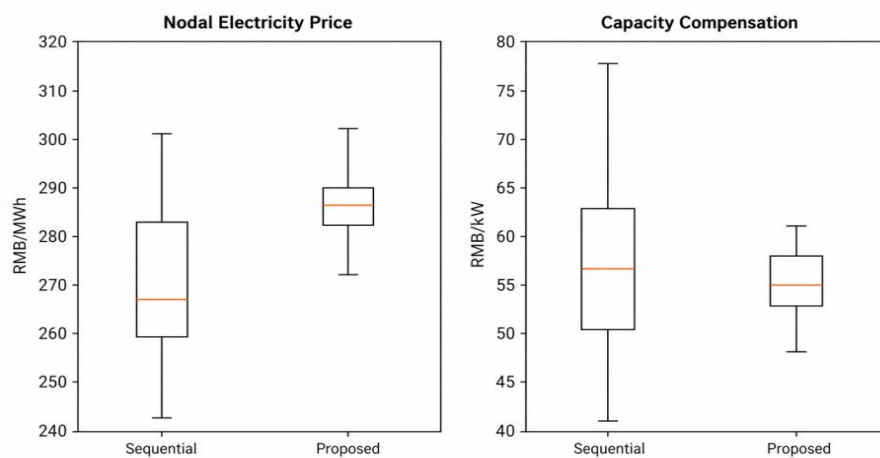


Figure 7: Boxplot of node electricity price versus capacity compensation

In order to identify the degree of influence of each module inside the model on the final result, this paper designed an independent ablation experiment to remove the game equilibrium constraint, the capacity-electricity coupling term and the dynamic section check term respectively. The results are shown in Table 4. After removing the game equilibrium constraint, the total electricity purchasing cost rose to 24.16 billion yuan, and the equilibrium deviation increased to 5.4%. After removing the capacity-quantity coupling term, the power abandonment rate of new energy rose to 9.1%. After removing the dynamic section check term, the feasible rate decreases to 94.6%. The full model remains optimal in all four metrics, indicating that the performance of the proposed method does not come from a single module, but from the combined effect of strategic constraints, market coupling, and network boundaries.

Table 4: Results of model ablation experiments

Model Structure	Total Power Purchase Cost / $10^8$ CNY	Renewable Curtailment Rate / %	Equilibrium Deviation / %	Feasibility Rate / %
Without game-equilibrium constraints	241.6	8.7	5.4	96.1
Without capacity-energy coupling term	236.9	9.1	4.2	97.0
Without dynamic corridor checking term	233.8	8.5	3.9	94.6
Full model	228.2	8.2	3.1	98.4

In order to further observe the fluctuation distribution of the equilibrium results in the perturbed scenario, this paper draws the equilibrium deviation violin plot on 500 Monte Carlo samples. Fig. 8 shows that the deviation distribution of the proposed model mainly concentrates in the range of 2.4% to 3.6% with a median of 3.0%, while the upper bound of the distribution of the sequential clearing model exceeds 6.5%, and the non-game joint model mainly falls in the range of 3.8% to 5.2%. At the same time, the violin form of the proposed model is more convergent, and the long tail part is significantly shortened, which indicates that the joint correction of price, volume and power flow allocation by the algorithm is more sufficient and the equilibrium state is more stable when the load disturbance and channel disturbance exist at the same time.

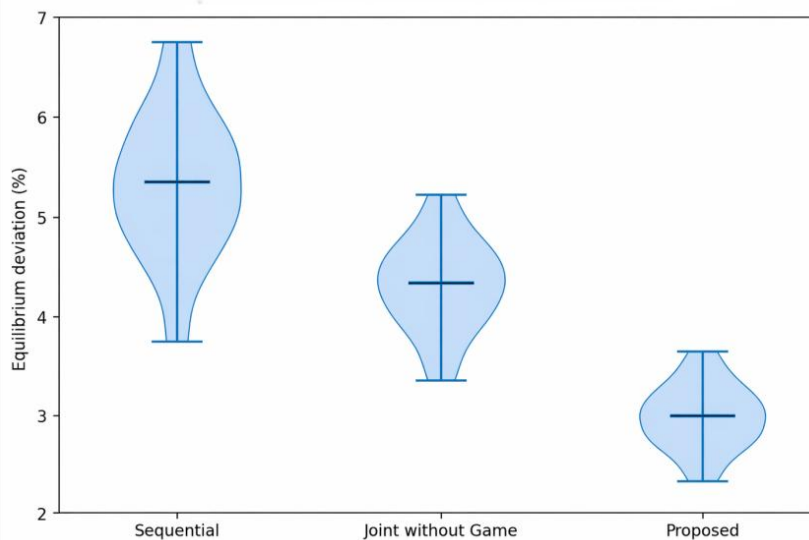


Figure 8: Violin plot of equalization deviation in the scenario with 500 perturbations

Based on the above charts, it can be seen that the proposed model shows more balanced advantages in cost control, power abandonment suppression, convergence efficiency, price stability and scenario robustness. The capacity market provides the inter-time supply boundary, and the electricity market deals with the time-sharing transaction results. After the joint calculation of the two markets is completed under the unified solver, they are no longer separated from each other. The quotation correction of provincial nodes will be directly reflected in the changes of section utilization and node prices, which is also an important reason why the proposed model is superior to the comparison method. Among the 500 groups of perturbation samples, 492 groups reach the termination threshold within 100 rounds, and the other samples do not appear infeasible solution diffusion, which indicates that the algorithm maintains good numerical stability in the high-dimensional constraint environment, and provides a reliable basis for the mechanism analysis in the subsequent discussion section.

## 5 Discussion

The cross-provincial capacity-electricity collaborative clearing model based on game equilibrium and mixed integer optimization proposed in this study shows strong comprehensive advantages in the scenarios of 12 provincial nodes, 36 contact channels and 24 periods of joint transactions. Compared with the sequential clearing model, the total power purchase cost is reduced by 11.8%, the new energy power abandonment rate is reduced by 14.6%, the average convergence time is shortened by 23.4%, and the equilibrium deviation is stable at 3.1%. The key to performance improvement is that capacity commitment, time-sharing transaction, section constraint and price feedback are written into the same calculation chain, and the provincial subject's offer correction can be written back and rebalanced in a unified solver. Compared with the joint model without game constraints, the proposed method is more stable in price volatility control and channel utilization smoothness. However, the current discussion is still based on day-ahead scenarios and standardized pricing rules, and has not yet included more fine-grained real-time disturbances and heterogeneous market behavior. This shows that the game equilibrium does not only improve the local declaration strategy, but enhances the information transmission efficiency between the capacity market and the electricity market, so that the cross-provincial trading boundaries, reserve demand and node prices maintain higher consistency. In the experiment, the convergence of the scene is completed within 100 rounds, which also shows that the method has numerical stability and strong realizability under large-scale constraint matrices.

## 6 Conclusions

In order to realize the integrated configuration of cross-provincial capacity market and electricity market, this paper constructs a collaborative clearing model with game equilibrium constraints, and incorporates provincial subject quotation, capacity commitment, electricity transaction, tie line boundary and settlement feedback into the unified solution framework. Through state encoding, matrix assembly and single-layer transcription, the model reconstructs the original staged processing method into a computation chain that can be executed in a unified solver. The experimental results show that the proposed method is superior to the sequential clearing model and the game-free joint model in terms of cost control, power abandonment suppression, price stability and equilibrium convergence, indicating that the pre-constraint of the capacity market and the time-sharing settlement of the electricity market can form a more efficient information transmission under the unified

framework.

(1) The unified expression of game equilibrium, capacity-quantity coupling constraints, and mixed-integer optimization is completed at the method level, so that the interaction between the transmission side, the power side, and the market operation organization no longer remains in the abstract description, but can be translated into computable state vectors, constraint matrices, and iterative residuals. This process enhances the reproducibility of the model and provides a stable interface for parallel computing in large-scale inter-provincial transaction scenarios.

(2) There are still some limitations in this paper. The current experiment mainly focuses on the day-ahead clearing process, real-time rolling correction, extreme weather shock, strong nonlinear operation cost of the unit and cross-market behavior learning have not been fully included, and some parameters still depend on regular setting, so the generalization ability in large-scale dynamic scenarios still needs to be verified.

(3) The follow-up research will focus on three directions: first, the real-time measurement and online prediction module is introduced to construct the day-ahead, intra-day and real-time linkage hierarchical solution mechanism; Secondly, graph computing and reinforcement learning methods are combined to improve the fast approximation ability in complex network states. The third is to expand the joint modeling of carbon market, auxiliary service market and multi-type transaction varieties between provinces, so that the collaborative clearing results are closer to the real market operating environment and demand.

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## References

- [1] Chen Y, Wang H, Yan Z, et al. A two-phase market clearing framework for inter-provincial electricity trading in Chinese power grids[J]. *Sustainable cities and society*, 2022, 85: 104057.
- [2] Fu X, Yang K, Li G, et al. Research on the trading arrangement and clearing model of medium-and long-term inter-provincial markets considering security constraints[J]. *Frontiers in Energy Research*, 2022, 9: 839108.
- [3] Shao Y, Hao L, Cai Y, et al. Day-ahead joint clearing model of electric energy and reserve auxiliary service considering flexible load[J]. *Frontiers in Energy Research*, 2022, 10: 998902.
- [4] Chen H, Wang D, Zhang R, et al. Optimal participation of ADN in energy and reserve markets considering TSO-DSO interface and DERs uncertainties[J]. *Applied Energy*, 2022, 308: 118319.
- [5] He Q, Lin Z, Chen H, et al. Bi-level optimization based two-stage market clearing model considering guaranteed accommodation of renewable energy generation[J]. *Protection and Control of Modern Power Systems*, 2022, 7(3): 1-13.
- [6] Hong Q, Meng F, Liu J, et al. A bilevel game-theoretic decision-making framework for strategic retailers in both local and wholesale electricity markets[J]. *Applied Energy*, 2023, 330: 120311.
- [7] Wang X, Su Y, Xu W, et al. A market decision-making model for load aggregators with flexible load[J]. *Frontiers in Energy Research*, 2023, 10: 1030076.
- [8] Yu L, Wang P, Chen Z, et al. Finding Nash equilibrium based on reinforcement learning for bidding strategy and distributed algorithm for ISO in imperfect electricity market[J]. *Applied Energy*, 2023, 350: 121704.
- [9] Wu C, Gu W, Yi Z, et al. A multi-rate hybrid model for real-time iterative bidding coupled with power system dynamics[J]. *Applied Energy*, 2023, 337: 120864.
- [10] Nokandi E, Vahedipour-Dahraie M, Goldani S R, et al. A three-stage bi-level model for joint energy and reserve scheduling of VPP considering local intraday demand response exchange market[J]. *Sustainable Energy, Grids and Networks*, 2023, 33: 100964.
- [11] Zhang X, Guo X, Zhang X. Bidding modes for renewable energy considering electricity-carbon integrated market mechanism based on multi-agent hybrid game[J]. *Energy*, 2023, 263: 125616.
- [12] Lan L, Zhang X, Zhang Y. Low carbon and efficiency oriented day-ahead joint electrical energy and ancillary services market clearing model for generation-side in China[J]. *Energy Economics*, 2023, 121: 106686.

- [13] Mei S, Tan Q, Liu Y, et al. Optimal bidding strategy for virtual power plant participating in combined electricity and ancillary services market considering dynamic demand response price and integrated consumption satisfaction[J]. *Energy*, 2023, 284: 128592.
- [14] Jiang K, Liu N, Yan X, et al. Modeling strategic behaviors for GenCo with joint consideration on electricity and carbon markets[J]. *IEEE Transactions on Power Systems*, 2022, 38(5): 4724-4738.
- [15] Feng Y, Feng D, Zhou Y, et al. Generation side strategy and user side cost based on equilibrium analysis of the power market under the reliability option[J]. *Energy*, 2024, 287: 129721.
- [16] Wu X, Ye Q, Chen L, et al. Electricity market clearing for multiple stakeholders based on the Stackelberg game[J]. *Frontiers in Energy Research*, 2024, 12: 1342516.
- [17] Chen L, Ye Q, Wu X, et al. Stackelberg game-based optimal electricity trading method for distribution networks with small-micro industrial parks[J]. *Frontiers in Energy Research*, 2024, 12: 1348823.
- [18] Jiang Z, Xiao Y, Xu T, et al. Two-stage stochastic energy and reserve market clearing model considering real-time offering strategy of renewable energy[J]. *IET Generation, Transmission & Distribution*, 2024, 18(22): 3713-3731.
- [19] Fu X, Guo C, Yang K. Market-clearing framework of a resilient microgrid with renewable energy considering emission reduction targets[J]. *IET Renewable Power Generation*, 2024, 18(7): 1304-1317.
- [20] Shi J, Guo Y, Wu W, et al. Scenario-oriented multi-area joint market clearing of energy and reserve under uncertainty[J]. *Applied Energy*, 2024, 361: 122873.