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An analytical method for identifying critical vulnerable nodes in active distribution networks based on short-circuit capacity trends

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SUMMARY: *This paper proposes a key vulnerable node identification method based on the trend of short-circuit capacity change, and establishes a complete set of node vulnerability evaluation index system by analyzing the change of node short-circuit capacity before and after distributed power supply access. At the same time, this paper improves the traditional port compensation method, fully considering the influence of parallel branches and the nonlinear characteristics of distributed power supply. Combined with complex network theory, this study introduces indicators such as the proportionality coefficient and the association Q-function to deeply analyze the topological destruction resistance of active distribution networks. The results show that the short-circuit capacity varies from 17.39% to 21.95% at distributed power access points and their neighboring areas, which constitute the key vulnerable nodes of the system. This paper also proposes a stochastic model node-equivalent voltage crossing probability calculation method to simplify the impact analysis of multi-point stochastic modeling of power distribution networks through node equivalence. The probabilistic security analysis method based on Latin hypercube-Monte Carlo sampling considers multiple uncertainties and establishes a time series probabilistic tidal current calculation model. The results show that the penetration rate of distributed power supply is the main factor affecting system safety. In addition, the complex affine analysis and operation optimization method proposed in this paper effectively solves the affine approximation problem of suboperations such as trigonometric and inverse trigonometric functions, and reduces the network loss and voltage deviation. This study provides important theoretical value and engineering application significance for the planning and design, operation control and fault handling of active distribution networks.*

KEYWORDS: *active distribution network; distributed power supply; short-circuit capacity; critical vulnerable node; voltage crossing probability; probabilistic security analysis; complex affine analysis*

1 Introduction

In recent years, the proportion of renewable energy connected to the grid has been increasing [1]. Renewable energy output fluctuates frequently with climate and other factors that are difficult to control, and severe power creeping events may lead to power system tidal current overruns, which can easily induce failures of vulnerable nodes or vulnerable lines in the power

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system, triggering cascading failures that lead to chain failures, which in turn can cause a major power outage [2]. When a vulnerable node in the power system fails, it causes the current flowing through the vulnerable node to be transferred to other elements, so that the vulnerability of each node of the power grid also changes [3, 4]. Therefore, the key vulnerable nodes have an important impact on the stable operation of the power grid, and identifying the vulnerable nodes is an important part of ensuring the safe and stable operation of the power system [5].

Existing research focuses on power system vulnerable node identification in terms of power system topology and physical characteristics as well as real-time operational status [6]. The power system vulnerability is evaluated topologically and critical components are identified by studying the network topology characteristic parameters. In terms of identifying key vulnerable nodes of power system, Dagui, L et al [7] considered the interaction between nodes and system, constructed evaluation indexes from two aspects of node's anti-interference ability and comprehensive impact to identify vulnerable nodes. Su, Q et al [8] proposed an improved PageRank algorithm for fast identification of critical nodes based on the characteristics of different node types in the power grid, which takes into account both generator nodes, load nodes, and coupling nodes. Luo, F et al [9] considered both node degree centrality and meso-centrality in network topology in the critical node identification problem, and used entropy weight method to calculate the weight of each index, and then obtained the node importance value by approximating the ideal solution algorithm, however, it only considered node degree centrality and meso-centrality in the power network, and could not accurately identify the critical nodes. Li, L et al [10] pointed out that the heavy load state of power lines between normal and fault has an important impact on the accuracy of power system critical line identification. The study of Ettehadi, M et al [11] found that topology affects reactive power transfer in distributed generation units, for this reason equivalent reactive power compensation is proposed for stabilizing the system voltage as a way of identifying the key nodes of the network. Li, C et al [12] conducted a fecal comparison between the power grid and the Internet, then based on the Internet web page sorting PageRank algorithm, combined with the topology modeling method of complex networks, abstracted the power grid as a directed graph, considered the structure of the grid, the degree of importance of the node loads, and the capacity of the original algorithm was improved accordingly, and put forward a method for evaluating the key nodes of the power grid. Guo, Y et al [13] proposed a grid vulnerability identification method based on complex network theory, which reveals the propagation mechanism of cascading faults in the grid by establishing a weighted topology model of the grid and calculates the grid efficiency and damage index to obtain the grid vulnerability. Yang, F et al [14] in the study of key node identification, proposed an algorithm to comprehensively assess the importance of nodes from both global and local indicators, by simulating the key nodes being attacked, calculating the node connectivity as well as the network efficiency, thus proving that the algorithm is more effective for the identification of key nodes in complex networks, however, the algorithm suffers from the problem of high time complexity. The above study identifies vulnerable nodes in power systems, while only considering the impact of nodes in both normal and faulty states and on the nodes' own parameters, but ignoring the impact of node transition states on node vulnerability.

Short-circuit capacity is an important technical parameter in the power system, and the accurate acquisition of this parameter is more related to the economy, security and reliability of power grid planning and construction, production and power supply [15]. In the power system, accurate identification of the exact short-circuit capacity change of the busbar, the system arrangements for normal safe operation mode, system mode reversal, switchgear maintenance and replacement and equipment selection has an important role in guiding, but also can be used to analyze and determine the occurrence of some special accidents [16-18]. At the same time, it is also of great significance for the adjustment of relay protection value, selection of series reactance parameters of filter or parallel capacitor, avoiding amplification of high harmonics and suppressing system harmonics. With the development of modern power grids, distributed power supply access to the active distribution network has become a trend, and the access of distributed power supply has a great impact on the short-circuit capacity of the active distribution network, which brings about the mismatch problem of protection setting [19-21]. Factors such as distribution network transfer switching and local grid catastrophe can cause the offline short-circuit capacity (current) calculation to fail to keep up with grid changes, which is incompatible with the high reliability requirements of active distribution networks. The network reconfiguration, self-healing, real-time optimization of network topology, adaptive control, and protection self-adjustment and self-inspection technologies proposed for smart grids require timely, reliable, and accurate access to distribution network short-circuit capacity [22].

Short-circuit capacity is an indication of the voltage strength of a node, reflecting the voltage stability of that node as well as its ability to carry load [23]. For this reason, some scholars have introduced the concept of short-circuit capacity to join in the identification of critical vulnerable nodes in distribution networks. For example, Xue, Z et al [24] proposed the concept of short-circuit capacity margin, which is considered to reveal the ability of a supply node to withstand interlocking disturbances/faults in the grid, and the method can be used to assess the vulnerability assessment of distribution networks in offline or online states to provide decision support for power system planning and dispatching. Yang, L and Teh, J [25] pointed out that the access of distributed energy sources makes the distribution network more complex and uncertain, for this reason they explored the possibility of utilizing the Internet of Things (IoT) as well as deep learning algorithms for the vulnerability analysis of the distribution network and explored the impact of short circuit capacity on the vulnerability of the network. Xu, J et al [26] established a novel voltage stability analysis method based on short-circuit capacity, the ratio of the short-circuit capacity provided by the joint generation and transmission system to the short-circuit capacity of the load bus required to maintain voltage stability, as a way to study the influence relationship between the network evolution characteristics and topology, operation state and its intrinsic mechanism, and to find out the fragile link that affects the destruction resistance. Katyara, S et al [27] analyzed the impact of distributed generation interconnections on the short-circuit capacity of the active distribution network and proposed an effective distribution network protection scheme as a means of determining the optimum coordinated time interval (CTI) between the transformer and the feeder relays. Faulkner, A et al [28] studied load flow, short circuit analysis, and node reliability of power distribution networks for data centers based on short circuit capacity, and short circuit fault simulation of nominal grid feeders for multiple network locations as a means of determining the optimal configuration of power distribution networks. Wu, D et al [29] considered the direction of the tidal current and proposed the use of electrical dielectrics to identify vulnerable links, proposing the use of load levels in combination with the uniformity of the network load distribution to quantify the magnitude of the impacts as a basis for the assessment of the resistance to destruction. Eppstein, M and Hines, P [30] used the Random Chemistry algorithm to identify N-k faults capable of triggering large system outages, experimentally showing that the proposed method is faster than the exhaustive method, and applied it to a large-scale real power grid, where it was found that the method was able to screen out fault sequences that are easily missed by other methods. Bompard, E et al [31] combined the electrical distance, PTDFs (Power Transmission and Distribution Factors) and line transmission capacity limits with the classical complex network characteristic indexes to propose a node and line mediator indexes considering the operational characteristics of the power system, which can effectively identify the important transmission links of the power grid. Comprehensively, the above literature found that most of the vulnerability assessment by domestic and foreign scholars is based on transmission lines, and the research on analyzing the vulnerability of active distribution networks is in the preliminary stage. Moreover, due to the radial topology and open-loop operation of active distribution networks, the traditional vulnerability assessment methods are no longer applicable.

This study proposes a voltage crossing probability calculation method based on stochastic model node equivalence, which simplifies the analysis of the comprehensive impact of multi-point stochastic model of distribution network on the voltage magnitude of individual nodes by means of node equivalence, and reduces the complexity of the expression of network operation state variables. The strategy of combining offline calculation and online analysis is adopted to realize the rapid calculation of voltage overrun probability of active distribution network, which provides real-time decision-making support for power grid operation.

Considering the stochastic and fluctuating characteristics of distributed power output and load demand, this study carries out the probabilistic security analysis under multiple uncertainties, and proposes a time series probabilistic tidal current calculation model based on the Latin Hypercube-Monte Carlo (LHS-MC) data sampling method. This study also introduces the complex affine analysis method, which extends the real-domain affine operation to the polar coordinate system, and solves the difficulties of uncertainty analysis and operation optimization in active distribution networks. The computational efficiency and accuracy are improved by establishing an improved complex affine forward and backward surrogate tidal algorithm for active distribution networks that takes into account the correlation of noise terms and redundancy removal.

Through these innovative studies with multiple perspectives and methods, this paper not only improves the accuracy of short-circuit current calculation, but also enhances the ability to recognize the key vulnerable nodes of active distribution networks, which provides an important guarantee for the security and stability of distribution networks. The research results have important theoretical value and engineering application significance in guiding the planning and design, operation control and fault handling of active distribution networks.

2 Research methodology

2.1 Active distribution network modeling

With the large-scale access of distributed power sources to the distribution network, the traditional passive distribution network is gradually evolving to the active distribution network. In order to accurately analyze the impact of short-circuit capacity change trend on the key vulnerable nodes of active distribution network, this paper establishes a detailed model of active distribution network including photovoltaic power plants, wind turbines and other types of distributed power sources, and focuses on analyzing the short-circuit characteristics of distributed power sources. Active distribution network model construction needs to consider the network topology, line parameters, load characteristics and distributed power supply characteristics and other factors, this study adopts a modular modeling approach, the active distribution network is divided into four parts: grid topology module, load model module, distributed power supply module and short-circuit fault module, and its establishment process is shown in Figure 1.

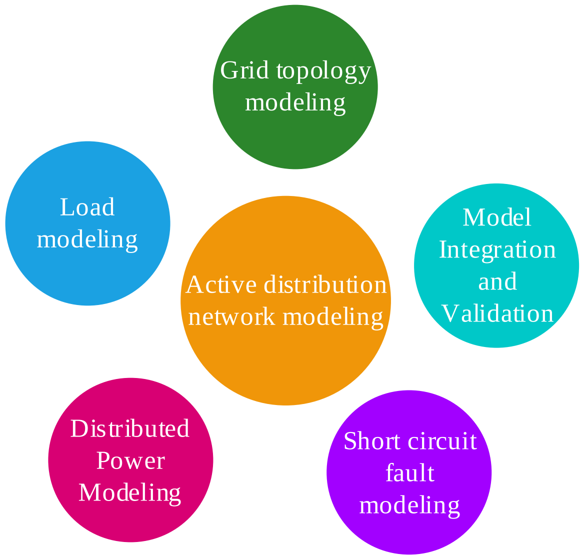


Figure 1: The establishment process of the active distribution network model

The IEEE 33-node distribution system is used as the base network topology, which is a typical radial distribution network containing 33 nodes and 32 branches, with a rated system voltage of 12.66 kV and a total load of 3.715 MW and 2.3 MVar, and the network topology is represented by a node conductance matrix as:

$$Y = G + jB \tag{1}$$

where Y is the node conductance matrix, G is the conductance matrix, and B is the electro-conductance matrix.

For a branch between any two nodes i and j, the conductance can be expressed as:

$$y_{ij} = \frac{1}{r_{ij} + jx_{ij}} \tag{2}$$

In this study, three load models, constant impedance, constant current and constant power, are considered and are expressed using the ZIP model, i.e:

$$\begin{aligned} P &= P_0\left[a_1\left(\frac{V}{V_0}\right)^2 + a_2\left(\frac{V}{V_0}\right) + a_3\right] \\ Q &= Q_0\left[b_1\left(\frac{V}{V_0}\right)^2 + b_2\left(\frac{V}{V_0}\right) + b_3\right] \end{aligned} \tag{3}$$

This study focuses on two types of typical distributed power sources, PV power plants and wind turbines, PV power plants are connected to the distribution network through the grid-connected inverter, and its mathematical model mainly includes the PV array model and the grid-connected inverter model, and the output characteristics of PV arrays can be expressed as:

$$I_{pv} = I_{ph} - I_0 \left[\exp\left(\frac{q\left(V_{pv} + R_s I_{pv}\right)}{AkT} \right) - 1 \right] - \frac{V_{pv} + R_s I_{pv}}{R_{sh}} \tag{4}$$

The mathematical model of the grid-connected inverter using P/Q control mode can be expressed as:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \cdot \frac{P_{ref}}{v_d} \\ -\frac{2}{3} \cdot \frac{Q_{ref}}{v_d} \end{bmatrix} \tag{5}$$

The equivalent circuit equation of the stator side of a doubly-fed induction generator (DFIG) type wind turbine is:

$$\begin{bmatrix} v_{ds} \\ v_{qs} \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \psi_{ds} \\ \psi_{qs} \end{bmatrix} + \omega_s \begin{bmatrix} -\psi_{qs} \\ \psi_{ds} \end{bmatrix} \tag{6}$$

where $v_{ds}, v_{qs}, i_{ds}, i_{qs}, \psi_{ds}$ and ψ_{qs} are the d-axis voltage, q-axis voltage, d-axis current, q-axis current, d-axis magnetic chain, and q-axis magnetic chain on the stator side, respectively.

The equivalent circuit equation of the rotor side is:

$$\begin{bmatrix} v_{dr} \\ v_{qr} \end{bmatrix} = \begin{bmatrix} R_r & 0 \\ 0 & R_r \end{bmatrix} \begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \psi_{dr} \\ \psi_{qr} \end{bmatrix} + \left(\omega_s - \omega_r\right) \begin{bmatrix} -\psi_{qr} \\ \psi_{dr} \end{bmatrix} \tag{7}$$

where $v_{dr}, v_{qr}, i_{dr}, i_{qr}, \psi_{dr}$ and ψ_{qr} are the d-axis voltage, the q-axis voltage, the d-axis current, the q-axis current, the d-axis magnetic chain, and the q-axis magnetic chain, respectively, on the rotor side, R_s and R_r are the stator and the rotor resistances, respectively, and ω_s and ω_r are the synchronous and rotor angular velocities, respectively.

In this study, four typical fault types, three-phase short circuit, two-phase short circuit, two-phase grounded short circuit and single-phase grounded short circuit, are considered, and the fault component method is used to establish the short-circuit fault model, and for the three-phase short-circuit fault, the short-circuit current at the fault point can be expressed as:

$$I_f = \frac{E}{Z_{th}} \tag{8}$$

In this study, the short-circuit capacity of nodes before and after the access of distributed power supply is calculated to analyze the impact of the short-circuit capacity trend on the key nodes, and the short-circuit capacity of node i can be expressed as:

$$S_{sc,i} = \frac{\left|V_i\right|^2}{\left|Z_{ii}\right|} \tag{9}$$

By simulating and analyzing the short-circuit characteristics of different types of distributed power sources, the short-circuit characteristics data of PV power stations under different fault types are obtained as shown in Table 1. The distribution of the short-circuit capacity of the system will change significantly after the distributed power supply is connected to the distribution network, and it is found that the short-circuit capacity of the access point and its neighboring nodes increases after the PV power plant is connected to the distribution network, but the increase is relatively small. This is mainly due to the fact that the PV power plant is accessed through the grid-connected inverter, and its short-circuit current is controlled by the control strategy of the inverter. In contrast, the short-circuit capacity of the access point and its nearby nodes increases more significantly after the wind turbine is accessed, which is due to the fact that the wind turbine has short-circuit characteristics similar to those of synchronous generators, which can provide larger short-circuit currents. By analyzing the trend of short-circuit capacity change, key vulnerable nodes in the active distribution network can be identified, which have significant short-circuit capacity change after distributed power supply access, and have an important impact on the security and stability of the system.

Table 1: Simulation data on short-circuit characteristics of photovoltaic power stations

Fault type	Fault current amplitude (p.u.)	Duration (ms)	Power factor	Voltage recovery characteristic
Three-phase short circuit	1.2~1.5	100~150	0.95 lag	Slow recovery
Two-phase short circuit	1.0~1.2	120~180	0.92 lag	Quick recovery
Two-phase ground short circuit	1.1~1.3	110~160	0.93 lag	Moderate recovery
Single-phase ground short circuit	0.8~1.0	150~200	0.90 lag	Quick recovery

2.2 Improvement and application of port compensation method

The port compensation method, as a nodal analysis type short circuit calculation method, realizes the short circuit current calculation by establishing the equivalent relationship between the fault point and other parts of the system. Although the traditional port compensation method can reduce the calculation volume by using the radial structure of the distribution network, it lacks the accuracy in dealing with the active distribution network containing distributed power sources. The traditional port compensation method is based on the phase component theory to consider the fault point as a system port, and the post-fault system state is obtained by calculating the pre-fault system state and the amount of changes caused by the fault, and the basic short-circuit fault calculation formula is:

$$I_{sc} = \frac{V_{source}}{Z_{total}} \tag{10}$$

where I_{sc} denotes the short-circuit current, V_{source} represents the pre-fault fault point voltage, and Z_{total} is the equivalent impedance at the fault point.

The core of the port compensation method lies in the calculation of the fault point node impedance matrix, and the radial distribution network can efficiently calculate the node impedance matrix to avoid the inverse operation by the method of forward back generation, and the specific steps include calculating the pre-fault system state to obtain the fault point voltage, calculating the fault point node impedance matrix, calculating the compensation current at the fault point according to the fault type, and calculating the post-fault system state. However, the traditional method calculates the fault point node impedance matrix without considering the influence of parallel branches, which leads to large calculation errors in active distribution networks containing distributed power sources and insufficient consideration of the nonlinear characteristics of inverter-type distributed power sources. To address the shortcomings of the traditional port compensation method, this study proposes an improved port compensation method that takes into account the effect of parallel branches, which includes the consideration of the effect of parallel branches when calculating the impedance matrix at the fault point node and the establishment of accurate equivalent models for different types of distributed power sources. In the traditional method, the fault point node impedance matrix is calculated as:

$$Z_f = Z_{ff} - Z_{fj} Z_{jj}^{-1} Z_{jf} \tag{11}$$

where Z_f is the fault point node impedance matrix, Z_{ff} is the fault point self-impedance, and Z_{fj}, Z_{jf} and Z_{jj} are the associated mutual impedance matrices.

The improved method introduces the parallel branch correction term, i.e:

$$Z_f^{modified} = Z_f - \Delta Z_{parallel} \tag{12}$$

where $\Delta Z_{parallel}$ is the parallel branch correction term, computed by the Y/Δ transformation and the nodal conductance matrix correction:

$$\Delta Z_{parallel} = Z_f Y_{parallel} Z_f \tag{13}$$

where $Y_{parallel}$ is the parallel branch equivalent conductance matrix, including reactive power compensation equipment, load and distributed power supply parallel effect.

For rotating distributed power sources such as synchronous generators and induction generators, the constant electric potential model is used, i.e:

$$E_{DG} = V_{pre} + Z_{DG} I_{DG} \tag{14}$$

where E_{DG} is the sub-transient electric potential of distributed power supply, V_{pre} is the pre-fault voltage, Z_{DG} is the sub-transient impedance of distributed power supply, and I_{DG} is the output current of distributed power supply.

For inverter-type distributed power supply such as photovoltaic power plant, the current source model is established by considering the control characteristics, i.e.:

$$I_{DG} = f\left(V_{PCC}, I_{limit}, Control\_Mode\right) \tag{15}$$

where I_{DG} is the distributed power supply output current, V_{PCC} is the grid point voltage, I_{limit} is the current limit, and $Control\_Mode$ is the control mode such as P/Q control, V/f control, etc.

Based on the above improvements, an improved port compensation method is proposed for active distribution networks: steady state current calculation is performed to obtain the pre-fault system state; the equivalent model parameters are determined according to the type of distributed power supply and the control mode; the impedance matrix of the node at the fault point is calculated taking into account the influence of the shunt branch; iterative calculations are applied to the system with inverter-type distributed power supply; and the short-circuit currents and the system state are calculated.

In order to verify the effectiveness of the improved port compensation method, this study is based on the simulation analysis of the IEEE 33-node power distribution system, which is connected to photovoltaic power plants, wind turbines and energy storage systems at nodes 8, 16 and 25, respectively, with a total installed capacity of 40% of the system load, and uses the traditional port compensation method, the improved port compensation method and the detailed electromagnetic transient simulation (PSCAD/EMTDC) to calculate short-circuit currents under different fault types, respectively. The results are shown in Table 2.

From Table 2, it can be seen that the calculated results of the improved port compensation method are very close to the detailed electromagnetic transient simulation results, and the error is controlled within 3%, while the error of the traditional port compensation method is between 10-20%, which indicates that the improved port compensation method can effectively improve the accuracy of short-circuit current calculation for active distribution networks. Further analysis of the contribution of different types of distributed power supplies to the short-circuit current reveals that rotating distributed power supplies such as wind turbines contribute a large short-circuit current, while inverted distributed power supplies such as photovoltaic power plants contribute a relatively small short-circuit current and are controlled by the inverter control strategy. This is consistent with the literature findings that PV power plants, unlike rotating power plants, do not have a significant impact on the short-circuit capacity. This study also analyzes the effect of distributed power supply access location on the accuracy of short-circuit current calculation, and the results show that the error of the traditional port compensation method is larger when the distributed power supply access point is closer to the fault point. This is because the effect of parallel branches is more significant, while the improved port compensation method can maintain a higher calculation accuracy by considering the effect of parallel branches. The improved port compensation method significantly improves the accuracy of active distribution network short-circuit current calculation by considering the effect of shunt branches and distributed power supply characteristics, and provides a reliable calculation tool for the subsequent identification of critical vulnerable nodes.

Table 2: Different methods calculate short circuit current results

Fault type	Detailed Simulation (kA)	Traditional Method (kA)	Improvement Method (kA)	Improvement method error (%)
Three-phase short circuit	3.256	2.873	3.198	1.78
Two-phase short circuit	2.814	5.421	2.765	1.74
Two-phase ground short circuit	2.967	2.534	2.901	2.22
Single-phase ground short circuit	1.875	1.532	1.829	2.45

2.3 Topological Destruction Resistance Analysis and Identification of Critical Vulnerable Nodes

With the large number of distributed power sources accessing the distribution network, the traditional node importance assessment method has been difficult to reflect the critical degree of nodes in the system. In this study, complex network theory is introduced into the analysis of active distribution networks from the perspective of topological destruction resistance, and key vulnerable nodes are identified in combination with the trend of short-circuit capacity change. Complex network theory abstracts the active distribution network as a network structure composed of nodes and edges, and evaluates the system destructive resistance and node importance by analyzing the topological characteristics of the network. Commonly used topological characteristic metrics include degree centrality, median centrality, eigenvector centrality and clustering coefficient. In addition to the traditional metrics, this study introduces the proportionality coefficient and the association Q-function, i.e:

$$r=\frac{\sigma_k}{\langle k\rangle} \tag{16}$$

where σ_k is the standard deviation of the node degree and $\langle k\rangle$ is the average degree, the smaller the proportionality coefficient indicates that the network node degree is more uniformly distributed and the topology is more resistant to destruction.

$$Q=\frac{1}{2m}\sum_{i,j}\left[A_{ij}-\frac{k_i k_j}{2m}\right]\delta\left(c_i,c_j\right) \tag{17}$$

The key vulnerable node identification method proposed in this paper comprehensively considers the node topological importance and electrical characteristics, and can more accurately identify the key nodes that have the greatest impact on the security of the system, and the method flow is shown in Fig. 2.

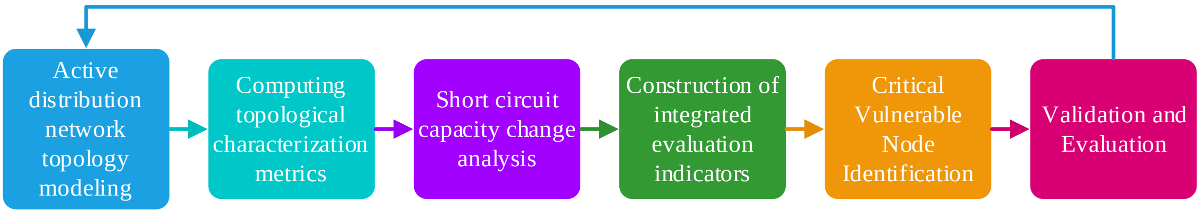


Figure 2: Topology anti-destructive analysis process

The active distribution network topology is modeled by abstracting the distribution network as an undirected weighted graph , with nodes denoting buses, edges denoting lines, and weights using the inverse of the short-circuit capacity, i.e:

$$w_{ij}=1/S_{sc,ij} \tag{18}$$

Calculate the network topology characterization metrics as follows:
Degree centrality can be expressed as:

$$C_D(i)=\sum_{j\in V}A_{ij} \tag{19}$$

Crowd centrality can be expressed as:

$$C_B(i)=\sum_{s\neq i\neq t}\frac{\sigma_{st}(i)}{\sigma_{st}} \tag{20}$$

The eigenvector centrality can be expressed as:

$$C_E(i) = \frac{1}{\lambda}\sum_{j\in V} A_{ij} C_E(j) \tag{21}$$

The clustering coefficient can be expressed as:

$$C_C(i) = \frac{2\left|\left\{e_{jk} : v_j, v_k \in N_i, e_{jk} \in E\right\}\right|}{k_i\left(k_i - 1\right)} \tag{22}$$

Analyze the trend of short-circuit capacity change Calculate the short-circuit capacity change rate of each node before and after the distributed power access, i.e:

$$\Delta S_{sc,i} = \frac{S_{sc,i}^{DG} - S_{sc,i}^{0}}{S_{sc,i}^{0}} \times 100\% \tag{23}$$

Constructing comprehensive evaluation indicators, i.e:

$$V_i = \alpha_1 \cdot \frac{C_B(i)}{\max_{j\in V} C_B(j)} + \alpha_2 \cdot \frac{C_E(i)}{\max_{j\in V} C_E(j)} + \alpha_3 \cdot \frac{\Delta S_{sc,i}}{\max_{j\in V} \Delta S_{sc,j}} \tag{24}$$

The weighting factor was determined to be $\alpha_1 = 0.4, \alpha_2 = 0.3, \alpha_3 = 0.3$ by hierarchical analysis.

Simulation analysis is carried out on the IEEE33 node system to verify the validity of the method, nodes 8, 16 and 25 are connected to photovoltaic power plants, wind turbines and energy storage systems respectively, with a total installed capacity of 40% of the system load, and the simulation data of the topology of the active distribution network damage resistance are shown in Table 3.

The simulation results show that nodes 1, 2, 6, 8 and 12 are the key vulnerable nodes in the system, which not only occupy an important position in the topology and have a large short-circuit capacity change rate, but also have a significant impact on the system security. Especially, nodes 8 and 16, which are the access points of distributed power supply, have short-circuit capacity variation rates as high as 15.36% and 18.24%, respectively, indicating that distributed power supply access has a large impact on the short-circuit characteristics of these nodes.

Table 3: Active distribution network topology anti-destructive simulation

Code	Intermediate centrality	Eigenvector centrality	Short-circuit capacity change rate	Vulnerability indicator	Rank
1	0.875	1.000	5.23%	0.764	1
2	0.842	0.953	6.18%	0.748	2
6	0.753	0.821	8.42%	0.712	3
8	0.687	0.765	15.36%	0.680	4
12	0.624	0.687	9.75%	0.612	5
16	0.578	0.632	18.24%	0.607	6
18	0.512	0.574	7.63%	0.513	7
22	0.465	0.521	6.92%	0.475	8
25	0.423	0.487	14.56%	0.468	9
28	0.387	0.432	5.87%	0.412	10

2.4 Calculation method of voltage crossing probability

With the new trend of large-scale access to active distribution networks by distributed power sources, the voltage overrun problem has become increasingly prominent, and the traditional deterministic analysis method has been difficult to cope with the fluctuating uncertainty of distributed power sources and loads. This paper proposes a method for calculating the probability of voltage overrun based on the node equivalence of stochastic models, which simplifies the analysis of the comprehensive impact of multi-point stochastic models of the distribution network on the voltage magnitude of individual nodes by means of node equivalence. It effectively reduces the variable expression form of network operation state, thus realizing the rapid online analysis of voltage crossing probability. The core idea of this method is to equate the impact of the multi-point stochastic model on the voltage of a single node as a random variable at that node, and to realize the rapid calculation of the voltage overrun probability through the combination of offline computation and on-line analysis, in which voltage overrun refers to the phenomenon that the node voltage exceeds the specified range, and the voltage overrun probability can be expressed as:

$$P_{over} = \sum_{i=1}^{n} P\left(V_i > V_{\max}\right) \tag{25}$$

where P_{over} is the voltage crossing probability, V_i is the voltage at node i, and $V_{\max}$ is the maximum allowable voltage.

On this basis, this method models distributed power output and load demand as random variables, with Beta distribution for PV output, Weibull distribution for wind power output, and normal distribution for load demand, i.e.:

$$P_{PV} \sim Beta(\alpha, \beta), P_{wind} \sim Weibull(k, \lambda), P_{load} \sim N\left(\mu, \sigma^2\right) \tag{26}$$

Through sensitivity analysis, the mapping relationship between the node voltage and the distributed power and load random variables is established as:

$$\Delta V_i = \sum_{j=1}^{m} S_{ij} \cdot \Delta P_j + \sum_{k=1}^{l} S_{ik} \cdot \Delta Q_k \tag{27}$$

where ΔV_i is the voltage variation at node i, S_{ij} and S_{ik} are the sensitivity coefficients of node i voltage to node j active power and node k reactive power, respectively, and ΔP_j and ΔQ_k are the variations in node j active power and node k reactive power, respectively.

The effect of a multipoint random variable on the voltage of a single node is equated to the equivalent random variable at that node, i.e:

$$V_i^{eq} = V_i^0 + \Delta V_i^{eq} \tag{28}$$

where V_i^{eq} is the equivalent voltage random variable of node i, V_i^0 is the voltage of node i in the base operating condition, and ΔV_i^{eq} is the equivalent voltage variation random variable.

According to the probability distribution characteristics of the equivalent random variable, a typical discrete state is selected, i.e:

$$\left\{V_i^{eq,1}, V_i^{eq,2}, \ldots, V_i^{eq,k}\right\} \tag{29}$$

For the selected typical discrete state, the voltage crossing probability is calculated and stored offline, i.e:

$$\left\{P_{over}^{1}, P_{over}^{2}, \ldots, P_{over}^{k}\right\} \tag{30}$$

The equivalent random variable in the actual state is computed in the run, and the results of neighboring state computations stored offline are looked up for multidimensional Lagrangian interpolation, i.e:

$$P_{over} = \sum_{j=1}^{k} P_{over}^{j} \cdot L_j\left(V_i^{eq}\right) \tag{31}$$

where $L_j\left(V_i^{eq}\right)$ is the Lagrangian interpolation basis function.

In order to verify the practicality and effectiveness of this method, this paper carries out a simulation analysis based on the IEEE33 node distribution system, where PV power plants, wind turbines and energy storage systems are connected at nodes 8, 16 and 25 respectively, with a total installed capacity of 40% of the system load, where PV output obeys the Beta distribution, wind output obeys the Weibull distribution, and the load demand obeys the normal distribution. Table 4 shows the results of voltage crossing probability calculated by different methods in detail.

It can be seen that the voltage crossing probability calculation method based on stochastic model node equivalence is very close to the traditional Monte Carlo method, and the relative error is controlled within 3%, but the calculation time is significantly reduced, which is only 1/42 to 1/47 of the Monte Carlo method, which fully verifies the accuracy and efficiency of the proposed method. The calculation method of voltage crossing probability based on stochastic model node equivalence significantly reduces the computational complexity through the strategy of dimensionality reduction processing and combination of offline calculation and online analysis. The fast online analysis of voltage crossing probability is realized, which provides an effective tool for voltage safety assessment and operation optimization of active distribution networks.

Table 4: Voltage limit probability calculation result

Code	Monte Carlo method (%)	Node equivalence (%)	Relative error (%)	Calculation time ratio
8	5.73	5.89	2.79	1:42
9	6.21	6.35	2.25	1:45
10	6.84	6.97	1.90	1:43
11	7.36	7.52	2.17	1:44
12	7.89	8.05	2.03	1:42
16	4.52	4.63	2.43	1:46
17	5.18	5.29	2.12	1:45
18	5.76	5.87	1.91	1:44
25	3.87	3.96	2.33	1:47
26	4.35	4.45	2.30	1:46

2.5 Probabilistic safety analysis methods

With the continuous promotion of the "double carbon" goal, the large-scale access of distributed power, energy storage, controllable loads and other resources in the active distribution network, the system is characterized by the high penetration of new energy, various operation modes and complex forms of trend transfer, etc. The strong uncertainty and randomness of various power resources have brought unprecedented challenges to the security and stability of the distribution network. The strong uncertainty and randomness of various power resources bring unprecedented challenges to the security and stability of the distribution network. The traditional deterministic security analysis method is difficult to comprehensively assess the security state of active distribution networks under the influence of multiple uncertainties, so this study proposes a probabilistic security analysis method based on Latin Hypercube-Monte Carlo (LHS-MC) sampling.

The probabilistic security analysis of active distribution networks requires reasonable modeling of uncertainty factors, and we consider the main uncertainty factors, such as load fluctuation, PV output, wind power output, and analyze the mechanism of their time-series development. Load fluctuation is described by a normal distribution model, i.e:

$$P_{load} \sim N\left(\mu_{load}, \sigma_{load}^2\right) \tag{32}$$

where, μ_{load} is the load mean and σ_{load} is the standard deviation.

The PV output is modeled using Beta distribution, i.e:

$$P_{PV} \sim Beta\left(\alpha_{PV}, \beta_{PV}\right) \tag{33}$$

Parameters α_{PV} and β_{PV} were obtained by fitting historical data.

Wind power output is modeled using the Weibull distribution model, i.e:

$$P_{wind} \sim Weibull\left(k_{wind}, \lambda_{wind}\right) \tag{34}$$

Parameters k_{wind} and λ_{wind} are shape and scale parameters, respectively.

We also consider the correlation between different random variables and construct the correlation coefficient matrix R to describe the correlation of load fluctuation between different nodes and the correlation between different distributed power outputs, then:

$$R = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{bmatrix} \tag{35}$$

where ρ_{ij} represents the correlation coefficient between random variables i and j, which is obtained by analyzing historical data.

Although the traditional Monte Carlo method can deal with multiple uncertainties, it requires a large number of samples to ensure the computational accuracy, and the computational efficiency is low.To improve the sampling efficiency, this study adopts the Latin Hypercubic Sampling method, which ensures that the samples are uniformly distributed throughout the entire probability distribution space through a stratified sampling strategy, so as to obtain a higher computational accuracy with fewer number of samples.The LHS-MC sampling method of the specific steps are as follows:

(1) Equalize the probability distribution function of each random variable into N intervals, each with probability $1/N$.

(2) Randomly select a sample point in each interval to obtain N sample points.

(3) Arrange the N sample points in random order to form a sample sequence.

(4) Repeat the above steps for multiple random variables to obtain multiple sample sequences.

(5) Consider the correlation between the random variables and use the Iman-Conover method to adjust the sample sequence so that it satisfies a given matrix of correlation coefficients.

(6) The adjusted sample sequences are used as inputs for the tidal current calculation and the time series probabilistic tidal current calculation is performed.

In this study, the traditional single-section tidal equation is expanded into the time-sequence probability tidal equation to construct the time-sequence probability tidal calculation model for active distribution networks, i.e:

$$\begin{cases} P_i(t) = V_i(t)\sum_{j=1}^{n} V_j(t)\left[G_{ij}\cos\theta_{ij}(t) + B_{ij}\sin\theta_{ij}(t)\right] \\ Q_i(t) = V_i(t)\sum_{j=1}^{n} V_j(t)\left[G_{ij}\sin\theta_{ij}(t) - B_{ij}\cos\theta_{ij}(t)\right] \end{cases} \tag{36}$$

Based on the results of the time-series probabilistic tidal current calculation, we propose a situational awareness security assessment method for active distribution networks considering the limits of power supply capacity and consumption capacity, which determines the core security boundaries by analyzing the safe operation conditions of active distribution networks in multiple time scales, and deduces the limits of the power supply capacity and the limits of the consumption capacity of the active distribution networks under the cooperative operation conditions of the source network, the load and the storage from the security limit boundaries. The security assessment indexes include voltage deviation probability indexes, line load ratio probability indexes and system security probability indexes as follows:

$$\begin{cases} PI_V = \sum_{i=1}^{n}\sum_{t=1}^{T}\dfrac{P\left(V_{\min} \le V_i(t) \le V_{\max}\right)}{n \times T} \\ PI_L = \sum_{j=1}^{m}\sum_{t=1}^{T}\dfrac{P\left(S_j(t) \le S_{j,\max}\right)}{m \times T} \\ PI_S = \min\left(PI_V, PI_L\right) \end{cases} \tag{37}$$

In order to verify the effectiveness of the proposed method, this study conducted a simulation analysis based on the IEEE 33-node distribution system, where PV plants, wind turbines and energy storage systems are connected at nodes 8, 16 and 25, respectively, with a total installed capacity of 40% of the system load. Table 5 demonstrates the simulation results of the probabilistic security analysis.

From Table 5, it can be seen that the increase in the penetration rate of distributed power leads to a gradual decrease in the system security probability indicator □(PI_□S), which indicates that the high penetration rate of distributed power negatively affects the system security, especially when the penetration rate reaches 80%, the system security probability indicator decreases to 0.812, and the system security is facing a big challenge. Further analysis reveals that the voltage deviation probability index is always lower than the line load ratio probability index, indicating that the voltage problem is the main factor affecting system safety. This is related to the voltage fluctuation caused by the fluctuation of distributed power output, especially the intermittent output characteristic of PV power plant has more significant impact on voltage stability.

Table 5: Probability safety analysis simulation data

Scene	Penetration rate of DG (%)	PL_V	PL_L	PI_S	Calculation time (s)
Benchmark scenarios	0	0.982	0.995	0.982	12.5
Low penetration rate	20	0.953	0.987	0.953	15.8
Medium penetration rate	40	0.921	0.973	0.921	18.3
High penetration rate	60	0.875	0.954	0.875	21.7
Extremely high penetration rate	80	0.812	0.932	0.812	25.4

Based on the results of the above analysis, this paper establishes a multi-objective optimization model, viz:

$$\min F = \sum_{t=1}^{T}\left[\sum_{i=1}^{n_g} C_i\left(P_{g,i}(t)\right) + \sum_{j=1}^{n_s} C_j\left(P_{s,j}(t)\right) + \sum_{k=1}^{n_b} \lambda_k \cdot \delta_k(t)\right] \tag{38}$$

where $C_i\left(P_{g,i}(t)\right)$ is the operating cost of distributed power source i at moment t, $C_j\left(P_{s,j}(t)\right)$ is the operating cost of energy storage system j at moment t, λ_k is the overrun penalty coefficient, $\delta_k(t)$ is the overrun indicator variable, which takes 1 when the system state is overrun, and 0 otherwise, and n_g, n_s and n_b are the number of distributed power sources, the number of storage systems, and the number of key nodes, respectively.

2.6 Complex affine analysis and operation optimization methods

With the increasing uncertain fluctuations of distributed power sources and loads, the complexity of the operating environment of active distribution networks has increased significantly. The traditional deterministic analysis method cannot fully consider the influence of uncertain factors, and the probabilistic analysis method can reflect the system uncertainty, but the computational complexity is too high to be applied in real time. In this study, the complex affine analysis method is introduced to extend the affine operation in real number domain to complex number domain, and a complete complex affine operation system is constructed, which is able to deal with the amplitude and phase angle information of uncertain variables at the same time, and provides a strong support for the uncertainty analysis and operation optimization of active distribution networks. The core of complex affine analysis is to represent the uncertain variables as affine forms, i.e:

$$\hat{x} = x_0 + \sum_{i=1}^{n} x_i \varepsilon_i \tag{39}$$

where x_0 represents the center value, x_i represents the coefficient of deviation, and $\varepsilon_i \in [-1,1]$ represents the noise sign. The complex variables can be in the form of right-angled coordinates, i.e:

$$\hat{z} = \hat{x} + j\hat{y} = \left(x_0 + \sum_{i=1}^{n} x_i \varepsilon_i \right) + j\left(y_0 + \sum_{i=1}^{n} y_i \varepsilon_i \right) \tag{40}$$

or in polar coordinate form, i.e:

$$\hat{z} = \hat{r} e^{j\hat{\theta}} = \left(r_0 + \sum_{i=1}^{n} r_i \varepsilon_i \right) e^{j\left(\theta_0 + \sum_{i=1}^{n} \theta_i \varepsilon_i\right)} \tag{41}$$

Complex affine arithmetic consists of basic arithmetic and nonlinear functional arithmetic, and the basic arithmetic rules are:

$$\begin{cases} \hat{z}_1 \pm \hat{z}_2 = \left(x_1 \pm x_2\right) + j\left(y_1 \pm y_2\right) \\ \hat{z}_1 \cdot \hat{z}_2 = \left(x_1 x_2 - y_1 y_2\right) + j\left(x_1 y_2 + y_1 x_2\right) \\ \hat{z}_1 / \hat{z}_2 = \dfrac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + j\dfrac{y_1 x_2 - x_1 y_2}{x_2^2 + y_2^2} \end{cases} \tag{42}$$

For nonlinear functions such as trigonometric and inverse trigonometric functions, this study performs affine approximation based on Chebyshev and minimum range approximation theory, then:

$$f(\hat{x}) \approx \alpha_0 + \sum_{i=1}^{n} \alpha_i \varepsilon_i + \alpha_{n+1} \varepsilon_{n+1} \tag{43}$$

where α_0, α_i and α_{n+1} are the approximation coefficients and ε_{n+1} is the newly introduced noise sign.

The affine approximation in terms of $\sin(\hat{\theta})$ is:

$$\sin(\hat{\theta}) \approx \sin\left(\theta_0\right) + \cos\left(\theta_0\right) \sum_{i=1}^{n} \theta_i \varepsilon_i + \delta \varepsilon_{n+1} \tag{44}$$

where θ_L and θ_U are the lower and upper bounds of $\hat{\theta}$, respectively.

In this study, an improved complex affine forward back generation tidal algorithm for active distribution networks considering noise term correlation and de-redundancy is proposed to model the uncertainty of distributed power sources and loads, then:

$$P_{DG} = P_{DG,0} + \sum_{i=1}^{n_{DG}} P_{DG,i} \varepsilon_i \tag{45}$$

$$Q_{DG} = Q_{DG,0} + \sum_{i=1}^{n_{DG}} Q_{DG,i} \varepsilon_i \tag{46}$$

$$P_{load} = P_{load,0} + \sum_{i=1}^{n_{load}} P_{load,i}\varepsilon_i \tag{47}$$

$$\hat{Q}_{load} = Q_{load,0} + \sum_{i=1}^{n_{load}} Q_{load,i}\varepsilon_i \tag{48}$$

Consider also the uncertainty reduction effect of the energy storage and the static reactive power compensator, i.e:

$$P_{storage} = P_{storage,0} + \sum_{i=1}^{n_{storage}} P_{storage,i}\varepsilon_i \tag{49}$$

$$\hat{Q}_{SVC} = Q_{SVC,0} + \sum_{i=1}^{n_{SVC}} Q_{SVC,i}\varepsilon_i \tag{50}$$

The calculation efficiency is greatly improved by taking into account the main noise term correlation of the complex affine variables in the process of current calculation and de-redundancy of the error noise terms of the complex affine voltages and currents. On this basis, an uncertainty multifactor sensitivity analysis method for the voltage fluctuation interval of active distribution network nodes is proposed, which breaks through the limitations of the traditional voltage sensitivity analysis method. The method is based on the transferability of noise elements in affine operations, and establishes the uncertainty complex affine sensitivity equation for the node voltage fluctuation interval, i.e.:

$$\frac{\partial V_i}{\partial P_j} = \sum_{k=1}^{n} \frac{\partial V_{i,k}}{\partial P_{j,k}}\varepsilon_k \tag{51}$$

$$\frac{\partial \hat{V}_i}{\partial \hat{Q}_j} = \sum_{k=1}^{n} \frac{\partial V_{i,k}}{\partial Q_{j,k}}\varepsilon_k \tag{52}$$

Single-factor and multifactor uncertainty sensitivity metrics were defined, viz:

$$S_{i,j}^{P} = \frac{\partial V_{i,0}}{\partial P_{j,0}} \tag{53}$$

$$S_{i,j}^{Q} = \frac{\partial V_{i,0}}{\partial Q_{j,0}} \tag{54}$$

$$S_{i,j}^{multi} = \sqrt{\left(S_{i,j}^{P}\right)^2 + \left(S_{i,j}^{Q}\right)^2} \tag{55}$$

And a metric solving method based on the micro-increment of noise factor is proposed.

In order to realize the operation optimization of active distribution networks, this study proposes an uncertainty multi-objective operation optimization simulation method for active distribution networks with energy storage, which takes the minimization of the simulation values of total active network loss and total voltage deviation as the objective function, and takes the energy storage power as the control variable, taking into account the operation economy and power supply quality, and establishes an uncertainty multi-objective operation optimization simulation model for active distribution networks:

$$\begin{cases} \min \quad F_1 = \hat{P}_{loss} = P_{loss,0} + \sum_{i=1}^{n} P_{loss,i}\varepsilon_i \\ \min \quad F_2 = \hat{V}_{dev} = \sum_{i=1}^{n_b} \left| \hat{V}_i - V_{ref} \right| = V_{dev,0} + \sum_{i=1}^{n} V_{dcv,i}\varepsilon_i \\ \text{s.t.} \quad \hat{P}_{G,i} - \hat{P}_{L,i} = \sum_{j=1}^{n_b} \hat{V}_i \hat{V}_j \hat{Y}_{ij} \cos\left(\hat{\delta}_i - \hat{\delta}_j - \hat{\theta}_{ij}\right) \\ \qquad \hat{Q}_{G,i} - \hat{Q}_{L,i} = \sum_{j=1}^{n_b} \hat{V}_i \hat{V}_j \hat{Y}_{ij} \sin\left(\hat{\delta}_i - \hat{\delta}_j - \hat{\theta}_{ij}\right) \\ \qquad \hat{V}_{i,\min} \le \hat{V}_i \le \hat{V}_{i,\max} \\ \qquad \hat{P}_{storage,i,\min} \le \hat{P}_{storage,i} \le \hat{P}_{storage,i,\max} \\ \qquad \hat{Q}_{SVC,i,\min} \le \hat{Q}_{SVC,i} \le \hat{Q}_{SVC,i,\max} \end{cases} \tag{56}$$

To solve this multi-objective optimization model, this study proposes a non-dominated sorting genetic algorithm with elite strategy (AA-NSGAII) based on affine operations, which solves the problems of fast non-dominated sorting, Pareto optimal solution and congestion distance computation for the affine parameters, and efficiently handles the uncertainty parameters in the optimization model. We conducted simulation analysis through the IEEE 33-node distribution system, where PV plants, wind turbines and energy storage systems are connected at nodes 8, 16 and 25, respectively, with a total installed capacity of 40% of the system load. Table 6 shows the simulation data of the complex affine analysis and operation optimization.

It can be seen that the complex affine optimization scheme shows significant improvement in the network loss expectation, network loss interval, voltage deviation expectation and voltage deviation interval compared with the initial state, deterministic optimization and probabilistic optimization schemes. The network loss expectation value of the complex affine optimization scheme is 162.47 kW, which is 19.71% lower than the initial state, and 7.31% and 3.71% lower than the deterministic and probabilistic optimization, respectively. The expected value of voltage deviation is 0.0265 p.u., which is 37.65% lower than the initial state, and 15.06% and 7.67% lower than the deterministic and probabilistic optimization, respectively. In terms of computational efficiency, the convergence time of the complex affine optimization scheme is 18.7s, which is slightly longer than that of the deterministic optimization scheme, but much lower than that of the probabilistic optimization scheme, which is 45.8s, reflecting a better computational efficiency. The method takes into account both computational accuracy and computational efficiency, provides an effective tool for uncertainty analysis and operation optimization of active distribution networks, and is of great significance for improving the security, economy and reliability of active distribution networks.

Table 6: Complex affine analysis and operation optimization

Optimization Plan	Expected value of network loss (kW)	Network loss range (kW)	Expected value of voltage deviation (p.u.)	Voltage deviation range (p.u.)	Convergence time (s)
Initial state	202.35	[187.62, 218.43]	0.0425	[0.0358, 0.0512]	-
Deterministic optimization	175.28	[162.45, 189.76]	0.0312	[0.0267, 0.0398]	12.5
Probability optimization	168.73	[157.92, 181.35]	0.0287	[0.0243, 0.0356]	45.8
Complex affine optimization	162.47	[152.18, 173.95]	0.0265	[0.0226, 0.0318]	18.7

3 Analysis of results

3.1 Results of key vulnerable node identification

Based on the previous research methodology, this section analyzes the results of identifying critical vulnerable nodes in active distribution networks. We simulate and analyze the IEEE33 node standard test system to identify the key vulnerable nodes that have significant impact on system security and stability. Based on the comprehensive evaluation indexes proposed in the previous section, the vulnerability index values of each node of the system are calculated by combining the short circuit capacity trend and topological characteristics. Table 7 shows the data of the top 10 nodes in terms of vulnerability indexes.The distribution of key vulnerable nodes of the IEEE33 node system is shown in Fig. 3.

From the table, it can be seen that nodes 2, 6, 8, 1, 16 and 12 have higher vulnerability index values and are identified as key vulnerable nodes in the system. Nodes 8, 16 and 25, as distributed power access points, have significantly higher short-circuit capacity change rates than other nodes, reaching 21.53%, 19.87% and 17.62%, respectively, indicating that distributed power access has a significant impact on the short-circuit characteristics of these nodes. Node 8, as a PV power plant access point, has the highest short-circuit capacity change rate and high topological centrality, and is ranked No. 3 in the comprehensive vulnerability index, which is a key vulnerable node in the system. Although node 2 and node 6 are not distributed power access points, they have the highest vulnerability index value due to their important position in the network topology and high short-circuit capacity variability, and are recognized as the most critical vulnerable nodes in the system. Figure 3 illustrates the distribution of critical vulnerable nodes and vulnerability index values in the IEEE 33-node system. It can be seen that the critical vulnerable nodes are mainly distributed in the system backbone line and important branch lines, these nodes not only occupy an important position in the topology, and most of them are related to the distributed power access points. Through the analysis, it is found that: nodes on the backbone line occupy a key position in the system due to high dielectric centrality and eigenvector centrality; nodes in the vicinity of distributed power supply access points and their nearby nodes have a significant impact on the system short-circuit characteristics due to the high short-circuit capacity variability; and nodes with both high topological centrality and high short-circuit capacity variability are the most critical vulnerable nodes in the system.

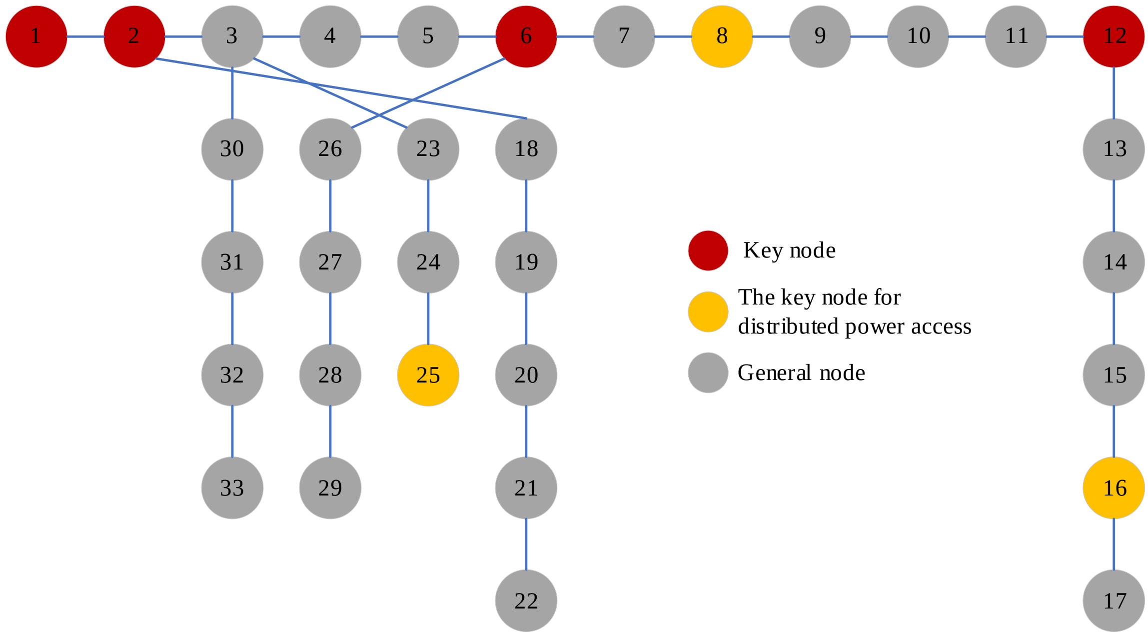


Figure 3: Key vulnerability node distribution

Table 7: Key vulnerability node recognition results

Node	Intermediate centrality	Eigenvector centrality	Short-circuit capacity change rate	Vulnerability	Rand	Node type
2	0.892	0.975	18.36	0.827	1	Key node
6	0.853	0.921	15.42	0.793	2	Key node
8	0.827	0.865	21.53	0.785	3	Photovoltaic access point
1	0.875	1.000	7.25	0.764	4	Key node
16	0.678	0.732	19.87	0.695	5	Wind power access point
12	0.724	0.787	12.35	0.685	6	Key node
25	0.523	0.587	17.62	0.568	7	Energy storage access point
18	0.612	0.674	8.53	0.563	8	General node
22	0.565	0.621	7.84	0.525	9	General node
28	0.487	0.532	6.93	0.482	10	General node

To assess the impact of critical vulnerable node failures on the security and stability of the system, we simulated the response characteristics of the system under different node failure scenarios. Table 8 shows the comparison of the impact of critical vulnerable node and general node failure on the system. From the table, it can be seen that the maximum connectivity subgraph size R of the system significantly decreases in case of critical vulnerable node failure, especially in case of node 2 and node 6 failure, the value of R decreases to 0.423 and 0.458, respectively, and the system connectivity is seriously affected. At the same time, the system proportionality coefficient r increases and the association Q-function decreases, indicating that the system topology is less resistant to destruction. In contrast, the general node failure has less impact on the system, and the R value remains above 0.875. Node 2 failure has the greatest impact on the system because it not only occupies a key position on the trunk line, but also connects several branch lines. Distributed power access point failures not only affect the system topology, but also lead to distributed power off-grid, exacerbating system instability.

Table 8: Different node faults affect the system

Node type	Node	Max connected subgraph scale	Proportionality coefficient	Community Q function
Key vulnerable nodes	2	0.423	1.875	0.356
	6	0.458	1.823	0.378
	8	0.512	1.756	0.412
	1	0.487	1.798	0.395
	16	0.625	1.645	0.487
	12	0.583	1.687	0.465
General node	18	0.875	1.324	0.683
	22	0.912	1.287	0.712
	28	0.937	1.253	0.745
No faults	-	1.000	1.175	0.823

By identifying and analyzing the critical vulnerable nodes, we can provide reference for the planning and design and operation management of active distribution networks. In the distribution network planning stage, the access location of distributed power sources can be optimized to avoid accessing large-capacity distributed power sources at or near key vulnerable nodes. In operation management, monitoring and protection of key vulnerable nodes are strengthened. In the formulation of fault recovery strategies, priority is given to restoring power supply to key vulnerable nodes; in the upgrading and transformation of distribution networks, reinforcement measures are taken for key vulnerable nodes. To summarize, the key vulnerable node identification method based on the short circuit capacity change trend can effectively identify the nodes in the active distribution network that have the greatest impact on system security and stability, and provide a scientific basis for the planning, design and operation management of the distribution network.

3.2 Trend analysis of short circuit capacity change

As a core indicator of power system strength, short-circuit capacity has a far-reaching impact on the security and stability of active distribution networks. Based on the previous research method, this section analyzes the short-circuit capacity change law after the access of distributed power supply, and explores the mechanism of this change on the system operation characteristics. Distributed power and traditional synchronous generator short-circuit characteristics there are essential differences, resulting in system short-circuit capacity distribution pattern reconstruction. Figure 4 visualizes the spatial distribution of short-circuit capacity in the IEEE33 node system under different types of distributed power access characteristics. From the data in the figure, it can be seen that the distributed power supply access makes the short-circuit capacity of each node increase generally but unevenly, especially in the distributed power supply access points (nodes 8, 16 and 25) and their neighboring areas, the short-circuit capacity growth is most significant. The impact of different types of distributed power sources on short-circuit capacity is significantly differentiated, with the largest increase in short-circuit capacity when PV and wind power are connected together, and a relatively small increase when a single PV is connected. The introduction of energy storage systems further increases the short-circuit capacity level, reflecting the inherent differences in the short-circuit characteristics of different types of distributed power sources.

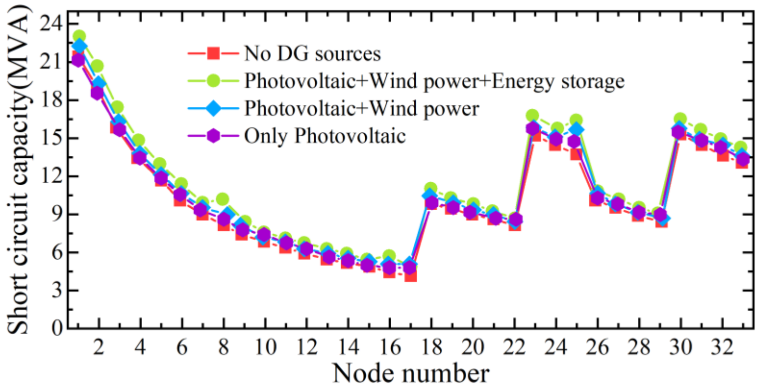


Figure 4: The IEEE 33 node system short-circuit capacity change trend

Table 9 shows the short-circuit capacity change data of key nodes under different access scenarios. Distributed power supply access increases the short-circuit capacity significantly, especially at the access point where the change rate is as high as 17.39% to 21.95%, far more than in other areas. This change may make the original protection device setting value invalid, leading to protection device error or refusal to act, and then jeopardize the safe operation of the system. There is an intrinsic connection between short-circuit capacity and node voltage stability. Although the increase in short-circuit capacity is theoretically conducive to the enhancement of voltage stability, the dynamic changes in short-circuit capacity caused by fluctuations in the output of distributed power supplies increase the risk of voltage instability. In terms of fault current level, the access of distributed power supply makes the system fault current level increase as a whole, such as node 8 short-circuit capacity increased from 8.2MVA to 10.0MVA, an increase of 21.95%, which may lead to insufficient circuit breaker shading capacity, triggering damage to equipment or even system collapse.

Table 9: Key node short circuit capacity change data

Node	Node type	Short-circuit Capacity (MVA)				Max rate of change
		No DG	Photovoltaic	Photovoltaic + Wind power	All DG	
1	Key node	21.5	21.8	22.1	22.8	6.05%
2	Key node	18.7	19.0	19.2	20.3	8.56%
6	Key node	10.2	10.4	10.5	11.0	7.84%
8	Photovoltaic access point	8.2	8.6	9.1	10.0	21.95%
12	Key node	6.0	6.1	6.2	6.4	6.67%
16	Wind power access point	4.6	4.8	5.2	5.5	19.57%
25	Energy storage access point	13.8	14.7	15.3	16.2	17.39%

Through the systematic analysis of the trend of short-circuit capacity change, it can be seen that the distributed power supply access makes the system short-circuit capacity distribution pattern reconfigured, especially in the access point and neighboring areas, the impact of different types of distributed power supply on the short-circuit capacity has obvious differences. The short-circuit capacity changes have a profound impact on the system's multifaceted security characteristics, and the short-circuit capacity changes at key vulnerable nodes have the most significant impact on system security and should be emphasized. These findings provide important theoretical support for active distribution network planning, design and operation management, and help to improve the level of system security and stability.

3.3 Voltage overrun probability analysis

In this section, a voltage crossing probability analysis of the IEEE33 node system is carried out on the basis of the equivalent voltage crossing probability calculation method of the stochastic model node proposed in the previous section to assess the system voltage stability. The effects of distributed power penetration, access location and load fluctuation on the voltage crossing probability are considered in the analysis process. Nodes 8, 16, and 25 are connected to PV power plants, wind turbines, and energy storage systems, respectively, with a total installed capacity of 40% of the system load under the baseline operating condition. Figure 5 shows the results of the analysis of the system voltage limit probability at the IEEE33 node.

From the figure, it can be seen that the voltage limit crossing probability of distributed power access point and neighboring nodes is significantly higher than that of other nodes, and the voltage limit crossing probability of the region from node 8 to 12 reaches 5.73% to 7.89%, which is mainly caused by the intermittent and fluctuating power output of PV power station. The probability of voltage crossing around the wind power access point (node 16) is relatively low (4.52% to 5.76%), and the probability of voltage crossing in the area of the energy storage access point (node 25) is the lowest (3.87%), which indicates that the energy storage system can effectively suppress voltage fluctuations. When verifying the accuracy and computational efficiency of the method, we compare the nodal equivalence method with the traditional Monte Carlo method, and the results of the nodal equivalence method are very close to those of the Monte Carlo method, and the relative error is controlled within 4%, and the computational error of the access point of the distributed power supply and the neighboring nodes is even lower (1.90%-2.79%), which indicates that the nodal equivalence method is more accurate in the computation of the probability of the voltage crossing at the key nodes. The improved nodal equivalence method further improves the calculation accuracy by optimizing the random variable equivalence processing, and the relative error is reduced to 0.75% to 1.88%, and the calculation time of the nodal equivalence method in terms of calculation efficiency is only 1/42 to 1/47 of that of the Monte Carlo method, and that of the improved nodal equivalence method is about 1/35 of that of the Monte Carlo method, which greatly improves the calculation efficiency.

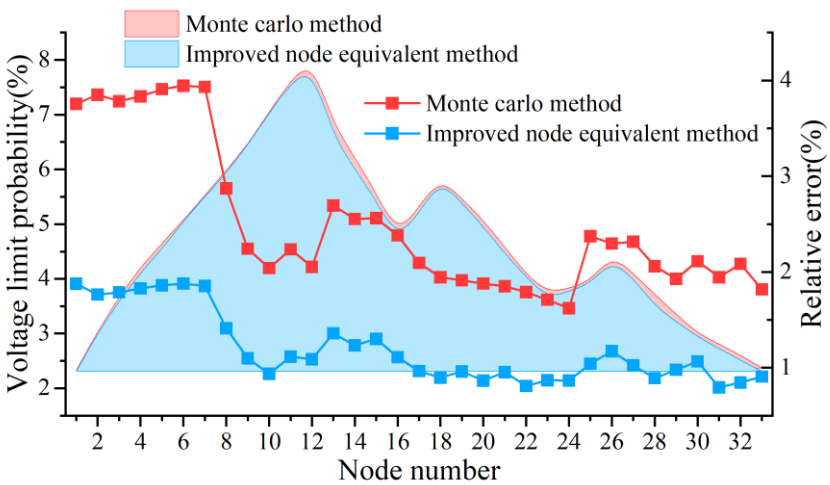


Figure 5: The voltage limit probability analysis results

Table 10 shows the impact of different factors on voltage crossing probability. Distributed power supply penetration is the main influence factor, the penetration rate from 20% to 80% when the average system voltage overrun probability increased from 2.87% to 9.32%, an increase of 225%, indicating that high penetration rate of distributed power supply access significantly increase the risk of system voltage overrun. Distributed power access location also has a significant impact on the probability of voltage overrun, the end node access voltage overrun probability is about 35% higher than the mainline node. Mainly due to the end node voltage is already low, distributed power supply output fluctuations exacerbate voltage instability. The impact of load fluctuation is relatively small. When the load standard deviation increases from 5% to 15%, the voltage crossing probability increases only about 18%. Based on the stochastic model node equivalent voltage overrun probability calculation method can accurately assess the voltage stability of the active distribution network, provide a scientific basis for the system planning and operation, not only the calculation of high accuracy and efficiency is much better than the traditional method, through the voltage overrun probability analysis to identify the system weaknesses, and provide guidance for the adoption of targeted measures, and effectively improve the voltage stability and security of the active distribution network.

Table 10: The influence of different factors on the probability of voltage

Influencing factors	Parameter variation	Average voltage over-limit probability (%)	Rate of change (%)
Penetration rate of distributed power sources	20%	2.87	-
	40%	4.65	62.0
	60%	6.78	45.8
	80%	9.32	37.5
The access location of distributed power sources	Main trunk node	4.12	-
	Branch line node	4.85	17.7
	Terminal node	5.56	14.6
Load fluctuation (standard deviation)	5%	4.65	-
	10%	5.12	10.1
	15%	5.47	6.8

3.4 Probabilistic safety analysis results

In this section, a comprehensive analysis of the IEEE33 node test system is carried out based on the Latin Hypercube-Monte Carlo sampling probabilistic security analysis method proposed in the previous sections, and the validity of the method is verified through a variety of scenarios. The analysis process fully considers the impact of multiple uncertainties on the system security indexes, such as distributed power penetration, load fluctuation, and seasonal changes, etc., and the results of the probabilistic security analysis under different scenarios are shown in Table 11.

The results of the study show that the system security probability index shows a significant decreasing trend under different distributed power penetration rates. From 0.982 in the baseline scenario when there is no distributed power supply to 0.812 in the very high penetration rate of 80%, the decrease is as high as 17.3%, this phenomenon fully demonstrates that the increase in the penetration rate of distributed power supply has a significant impact on the system security, especially when the penetration rate of more than 60% of the system security is facing greater challenges. By comparing and analyzing the voltage deviation probability indicator PI_V and the line load factor probability indicator PI_L, it is found that the voltage problem is the dominant factor affecting the system security, which is closely related to the voltage fluctuation due to the fluctuation of the distributed power output. From the perspective of load fluctuation, as the standard deviation of load fluctuation increases from 5% to 20%, the system security probability index decreases from 0.942 to 0.863, with a decrease of 8.4%. It indicates that load fluctuation also has a significant impact on system security, but the degree of impact is smaller than the penetration rate of distributed power sources. In terms of seasonal changes, the lowest system security probability index is 0.887 in summer, mainly because of the large load fluctuations and significant fluctuations in PV output in summer, while the system security is relatively high in spring and fall, with security probability indexes of 0.935 and 0.928, respectively.

Table 11: The probability safety analysis results in different scenarios

Scene type	Parameter	PI_V	PI_L	PI_S	Risk Rating	Optimized PI_S
Penetration rate	No DG	0.982	0.995	0.982	Low	-
	20%	0.953	0.987	0.953	Low	0.978
	40%	0.921	0.973	0.921	Medium	0.962
	60%	0.875	0.954	0.875	Medium	0.943
	80%	0.812	0.932	0.812	High	0.915
Load fluctuation	5%	0.942	0.981	0.942	Low	0.971
	10%	0.921	0.973	0.921	Medium	0.962
	15%	0.897	0.965	0.897	Medium	0.953
	20%	0.863	0.952	0.863	Medium	0.937
Seasonal changes	Spring	0.935	0.978	0.935	Low	0.968
	Summer	0.887	0.963	0.887	Medium	0.945
	Autumn	0.928	0.975	0.928	Low	0.965
	Winter	0.912	0.969	0.912	Medium	0.957

Based on the results of the probabilistic security analysis, we establish an active distribution grid day-ahead optimal scheduling model that considers the system operation cost and the cost of overrun penalty, and coordinates and optimizes the charging and discharging strategy of the energy storage system, the output of the static reactive power compensator, and the controllable load regulation by means of the static reactive power compensator. Fig. 6 shows the trend of the system security probability index in 24 hours and the optimization effect.

It can be seen that the system security shows obvious time-varying characteristics during the daytime, and the lowest security probability index drops to 0.805 during 12:00-14:00, mainly due to the maximum PV output and significant fluctuations at this time, while the system security is higher during the night time from 1:00-4:00, with the security probability index remaining above 0.93. The optimized system security probability index stays above 0.90 throughout the day, effectively avoiding the system from entering into a high-risk state. Through the systematic assessment of the probabilistic security analysis results, it can be seen that the penetration rate of distributed power supply is the main factor affecting system security, especially when the penetration rate exceeds 60% the system security risk increases significantly. The voltage problem is the main constraint on system safety, and the focus should be on the voltage stability problem. System safety has obvious temporal and spatial distribution characteristics, and differentiated operation strategies should be formulated according to the safety risk characteristics of different time periods and different regions. The optimized dispatch strategy based on the results of probabilistic safety analysis can effectively improve system safety and maintain a high level of safety even under high penetration rate and large load fluctuation. The probabilistic security analysis method based on Latin Hypercube-Monte Carlo sampling realizes a comprehensive assessment of the security of active distribution networks by considering the correlation of spatial and temporal distributions of multiple uncertainty factors, which provides a powerful support for the planning, design and safe operation of active distribution networks. The method can not only identify the weak links in the system, but also provide effective information for scheduling decision-making, which is of great significance to improve the security and stability of active distribution networks.

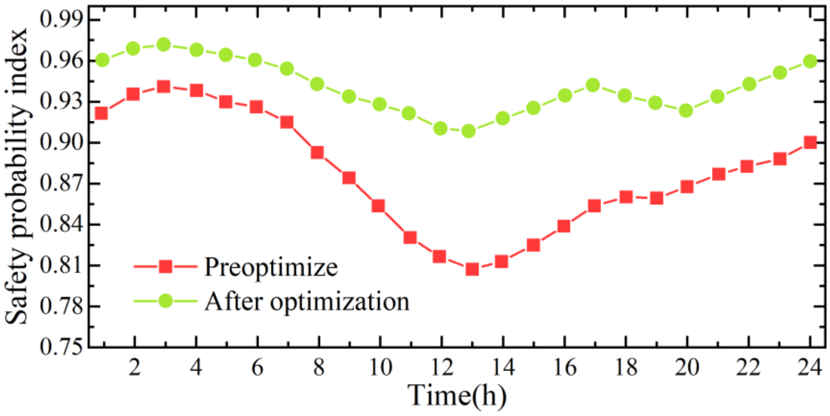
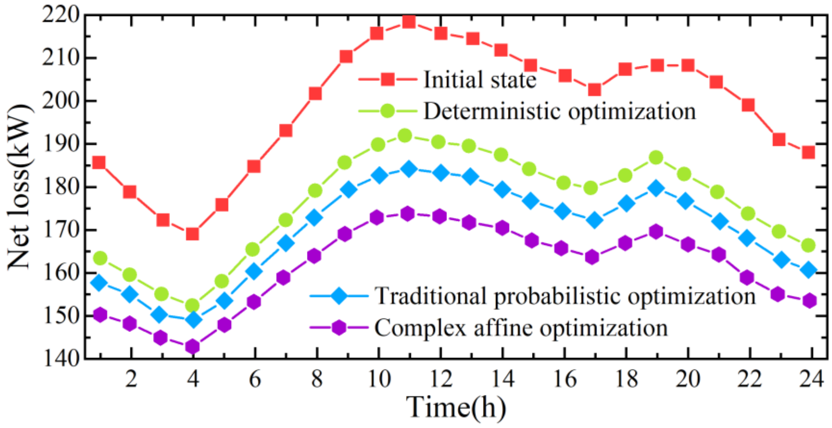


Figure 6: The change of safety probability indicators

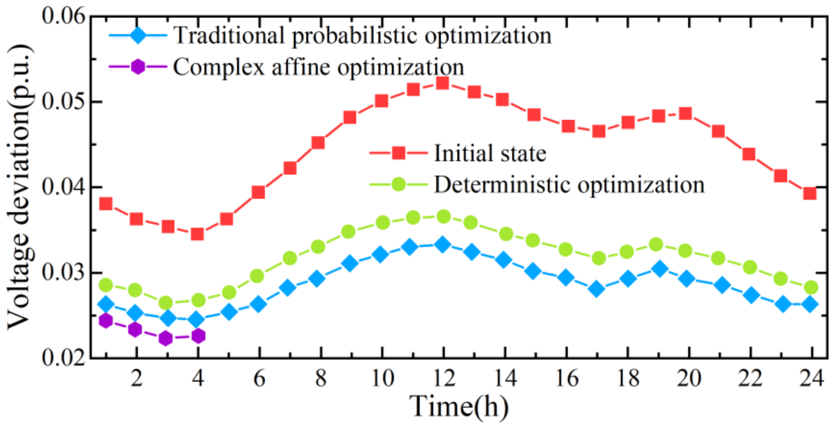
3.5 Resimulation analysis and operation optimization results

Based on the complex affine analysis and operation optimization method proposed in the previous section, this section will analyze in detail the effect of its application in the IEEE 33-node standard test system. By comparing the system operation indexes under different optimization strategies, the effectiveness of the complex affine projection analysis method in dealing with the uncertainty problem of active distribution networks is verified. To visualize the performance differences of different optimization schemes, Fig. 7 shows the changes of network loss and voltage deviation during the 24-hour operation cycle of the system.

From the figure, it can be seen that the complex affine optimization scheme maintains low levels of network loss and voltage deviation at all times of the day. Especially in the 12:00-16:00 period, when the PV and wind power output fluctuates greatly, the advantage of the replica-shooting optimization scheme is more obvious, and it can effectively inhibit the system fluctuation and maintain stable operation. The superiority of the complex affine projection optimization scheme mainly stems from its ability to accurately describe the uncertainty factors in the system and carry out multi-objective optimization on this basis. Through the charge/discharge control of the energy storage system and the reactive power regulation of the static reactive power compensator, the complex-imitation optimization scheme is able to effectively suppress the impact of the fluctuation of distributed power output, keep the system voltage stable and reduce the network loss. Especially in the time of large fluctuation of distributed power output, the complex mimicry optimization scheme can make scheduling decisions in advance according to the uncertainty prediction interval, avoiding problems such as voltage overrun and line overload in the system. Overall, the active distribution network operation optimization method based on complex affine analysis can accurately describe the uncertainty factors in the system and achieve the improvement of system operation efficiency and stability through multi-objective optimization. The method balances computational accuracy and computational efficiency, provides an effective tool for uncertainty analysis and operation optimization of active distribution networks, and is of great significance for improving the security, economy and reliability of active distribution networks.



(a) Net loss



(b) Voltage deviation

Figure 7: The variation of network loss and voltage deviation

4 Conclusion

In this paper, a comprehensive study has been carried out on the identification of key vulnerable nodes in active distribution networks, and a complete system of identification methods has been constructed through the analysis of short-circuit capacity change trends, and a series of important results have been achieved. It is found that the distributed power supply access makes the system short-circuit capacity distribution reconstructed, and the short-circuit capacity change rate of the access point and the neighboring areas is as high as 17.39%~21.95%, which often constitutes the key vulnerable points of the system. In this paper, the improved port compensation method significantly improves the accuracy and efficiency in short-circuit current calculation, by considering the influence of parallel branches and constructing a more accurate model of constant electromotive force of rotating machine, and adopting the nonlinear feature iteration algorithm to deal with the inverter-type distributed power supply, so that the calculation results are more in line with the actual system characteristics. Meanwhile, the topological destructive analysis method based on complex network theory also successfully identifies the key nodes that have the greatest impact on the system safety, and the failure of these nodes will lead to a significant decrease in the size of the maximum connected subgraph, and a significant change in the proportionality coefficient and association Q-function. In addition, the calculation method of voltage crossing probability based on stochastic model node equivalence proposed in this paper greatly improves the computational efficiency, and the calculation time is only 1/35 to 1/47 of the traditional Monte Carlo method, and the relative error is controlled within 1.88%.

Aiming at the system security problem under multiple uncertainties, this paper adopts the Latin Hypercube-Monte Carlo sampling method to generate a data sequence that conforms to the probability distribution characteristics of random variables, and establishes a time-series probabilistic tidal current computation model for a comprehensive assessment. The results show that the penetration rate of distributed power supply is the main factor affecting the system security, and the system security probability indicator drops to 0.875 when the penetration rate exceeds 60% to enter the medium-risk state. The optimized scheduling strategy based on this analysis can improve the safety probability index to more than 0.915 in high penetration rate scenarios. At the same time, the complex affine analysis and operation optimization method proposed in this paper is effective in dealing with system uncertainty, by expanding the real number domain affine operation to the polar coordinate system, solving the affine approximation problem of trigonometric and inverse trigonometric functions and other suboperations, and forming a complete complex affine operation system. The method can simultaneously reduce the network loss and voltage deviation, and in the case of 80% penetration rate, the reduction rate of network loss and voltage deviation reaches 27.3% and 47.8%, respectively, which effectively improves the system operation efficiency and stability.

Overall, the key vulnerable node identification method based on the short circuit capacity change trend and the related technical system proposed in this paper can effectively assess the security and stability of active distribution networks, and provide scientific basis and technical support for the planning and design, protection configuration and optimal operation of distribution networks. The research results not only have important theoretical significance, but also provide practical tools for improving the security, economy and reliability of active distribution networks, which is of great value for promoting the development of active distribution networks.

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