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Cooperative Decision-Making and Control of Multiple Drones Under Electromagnetic Interference

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SUMMARY: *This paper focuses on the decision-making and control problem for multi-drone systems operating under electromag- netic interference. Specifically, for a uniquely modeled multi-drone swarm, the objective is to design a controller that ensures the drones' positions converge accurately to a Nash equilibrium. To account for the effects of electromagnetic interference, a random graph is used to model communication uncertainties and packet loss between drones. Employ- ing a "decision-control" two-layer architecture, we propose a new distributed game-theoretic Nash equilibrium (NE) seeking approach that guarantees asymptotic convergence to the NE point. Unlike existing studies on distributed NE seeking, our work incorporates the influence of electromagnetic interference, resulting in a time-varying and random communication graph, which is more representative of real-world scenarios. Furthermore, the dynamics considered in this study include a more comprehensive model of drone mobility, encompassing orientation. Finally, the effectiveness of the proposed method is demonstrated through a simulation example.*

KEYWORDS: *Multi-drones system; Electromagnetic interference; Distributed Nash equilibrium seeking; Random graph.*

1 Introduction

In recent years, the deployment of multi-drone systems in applications such as surveillance, disaster response, and envi- ronmental monitoring has grown significantly [1-3]. How- ever, these applications often face environments with unpre- dictable electromagnetic interference [4], which can disrupt reliable communication between drones and complicate their control. Ensuring that drones can collaborate effectively and achieve specific objectives under such adverse conditions re- mains a critical challenge. This study addresses this issue by focusing on the distributed Nash equilibrium (NE) seeking problem for multi-drone systems subjected to electromag- netic interference.

Game theory has been increasingly applied across fields such as economics, computer science, and mobile ad hoc networks [5-8], thanks to its universal theoretical methods. The NE-seeking problem offers a game-theoretic approach that allows each drone to achieve an optimal strategy in a decentralized manner, which is particularly valuable in dy- namic and uncertain environments. As a fundamental prob- lem in game theory, NE seeking has been extensively stud- ied. Recent advancements propose distributed approaches to NE seeking, leveraging the benefits of consensus techniques. Unlike traditional centralized methods, distributed NE seek- ing enables each player to exchange information only with neighboring players over a network

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topology, eliminating the need for global information access. This decentralized frame- work is more practical for real-world applications.

To model the effects of electromagnetic interference on drone communication, we adopt a random undirected com- munication graph. This graph captures the random and time-varying connectivity arising from electromagnetic in- terference, providing a more realistic representation of com- plex real-world scenarios compared to static or determinis- tic graphs. However, early works on distributed NE seek- ing often assume fixed communication graphs among drones. In [9], a consensus-based distributed NE-seeking algorithm was proposed. Each player updated its strategy estimate by interacting locally with neighboring players using consen- sus protocols, while actions were updated via gradient de- scent. The algorithm was shown to achieve asymptotic con- vergence to the NE in undirected connected graphs. In [10], the authors extended their work to dynamic games with time- varying cost functions, proposing a hybrid consensus-based NE-seeking strategy. Using Lyapunov stability analysis, they theoretically proved the stability of the NE under this hy- brid strategy. Despite these advancements, works such as [9-12] generally required communication graphs among agents to be undirected and fixed.

To address these limitations, more recent studies have ex- plored NE-seeking algorithms under directed graphs. For in- stance, [13] investigated aggregative games on strongly con- nected, weighted, and balanced digraphs. They employed a dynamic average consensus protocol to estimate aggregate player actions and developed an NE-seeking algorithm us- ing local estimation and gradient strategies. The conver- gence of this algorithm was analyzed through small-gain the- ory. These advancements form the foundation for our study, which introduces a robust distributed NE-seeking framework that incorporates the randomness and time-varying character- istics of communication under electromagnetic interference, making it more applicable to complex real-world scenarios. In summary, existing studies on distributed NE seek- ing typically assume a fixed communication graph, whether undirected or directed. However, in real-world scenarios, network instability caused by electromagnetic interference can lead to temporary drone failures, resulting in a time- varying communication topology. Some researchers have explored distributed NE seeking over directed switching graphs. For instance, [14] demonstrated that players switch- ing among strongly connected communication graphs can achieve asymptotic convergence to the NE under specific residence time and parameter conditions. Similarly, [15] re- quired the graph sequence to be uniformly jointly bipartite for NE-seeking convergence. However, these approaches rely on deterministic switching graphs. Addressing NE seek- ing over random topologies, which better reflects practi- cal challenges, remains relatively unexplored. In [16], the authors tackled NE seeking in strongly monotonic games over time-varying communication networks using a fixed- step approach. They proposed an algorithm incorporating projection-based pseudo-gradient dynamics and a consensus term, achieving linear convergence to the NE. While dis- tributed optimization problems on random graphs have been extensively studied [17, 18], game-theoretic approaches for distributed NE seeking in such settings are relatively scarce. Furthermore, most existing work on distributed NE seek- ing focuses on single integrator systems [19, 21]. However, in multi-drone systems, each drone possesses specific com- putational capabilities, necessitating more advanced models.

Recent research has begun to address distributed optimization for linear systems [22], nonlinear systems [23, 24], and other systems with realistic physical dynamics. This study extends beyond the single integrator paradigm by incorporating both position and orientation into the drone dynamics, providing a more comprehensive representation of drone behavior and enabling precise spatial orientation control.

To address these challenges, we propose a two-layer algo- rithm designed to ensure the drone output strategies asymp- totically converge to the NE, solving the distributed NE- seeking problem for multi-drone systems under electromag- netic interference. The decision layer facilitates information exchange with neighboring agents, enabling local estimation based on both local and neighboring data to formulate op- timal strategies. The control layer designs feedback track- ing control laws to implement the decision layer's instruc- tions. Our approach differs from existing works by directly addressing the unique challenges posed by electromagnetic interference and random communication topologies. This novel framework improves the robustness and applicability of distributed NE-seeking methods in complex and dynamic environments.

The main contributions of this paper are stated as follows:

(1) the existing distributed Nash equilibrium seeking for agent systems is mostly focused on the first-order integrator system [14, 21, 25], while this paper fo- cuses on the optimization and game research of the mobility model of the drone.;

(2) Due to the electromagnetic interference, the com- munication map considered in this paper is ran-dom and has more practical significance compared to [9, 12].

The remaining sections of this paper are organized as fol- lows: The necessary notations are given in the Section 2. In Section 3, we establish the problem of distributed NE seeking for multi-drones system under electromagnetic interference and give some definitions. In addition, Section 4 provides the main result. Then, Section 5 provides numerical simulations for illustration. Finally, conclusion is made in Section 6.

2 Notation

Let $\mathbb{R}$ represent the set of real numbers. The Kronecker product is denoted by $\otimes$, the Euclidean norm of a vector by $|\cdot|$, the mathematical expectation by $E\cdot$, and the symbol of multi- plicative by $\prod$. The vectors $0_n \in \mathbb{R}^n$, $1_n \in \mathbb{R}^n$ and $1_n \in \mathbb{R}^{n\times n}$ denote the all-zeros vector, the column vector with all ele- ments being 1, and the *n*-dimensional identity matrix, respec- tively. For $x_i \in \mathbb{R}^{n_i}$ where $i \in N^+$, the notation $col(x_i)_{i\in N^+}$ or $[x_i]_{i\in N^+}$ refers to the stacked vector obtained from vectors

x_i. The matrix $diag(A_1, A_2, \ldots, A_n)$ is a block diagonal ma-trix that has main diagonal blocks A1, . . . , An and off-diagonal blocks that are zero matrices. For $A \in \mathbb{R}^{n\times n}$, $\lambda_{\min}(A)$ denotes the minimum eigenvalue of matrix A. $atan2(y, x)$ calculates the angle from the positive x-axis to the point (x, y), correctly identifying the quadrant for values from $-\pi$ to π. For a con-tinuously differentiable function $f(x): \mathbb{R}^n \to \mathbb{R}$, the gradient at $x \in \mathbb{R}^n$ is denoted by $\nabla f(x) \triangleq col(\frac{\partial f(x)}{\partial x_i})_{i\in 1,\cdots,n}$. We use $(\Omega, \Pi, \mathcal{P})$ to represent the probability space.

3 Problem Formulation

In this section, we give the relevant background and some necessary assumptions, and then formulate the problem in detail.

Due to the impact of electromagnetic interference on com- munication between drones, the electromagnetic interference can be modeled by a random undirected graph. The com- munication random graph caused by electromagnetic inter- ference refers to the changes in the communication network structure of a robot or drone swarm under the influence of electromagnetic interference. This change can be modeled using random graphs to capture the uncertainty and dynamic characteristics of the communication network. In an elec- tromagnetic interference environment, communication links between drones may become intermittent due to signal at- tenuation, interference, noise, and other factors, leading to changes in the communication topology.

Thus, the communication among drones can be described by a random undirected graph (of no self-edges) $\mathcal{G}_k = \{\mathcal{V}, \mathcal{E}_k\}, k = 0,1,\cdots$, where $\mathcal{V} \triangleq \{1,\cdots,N\}$ is the drone set and $\mathcal{E}_k$ is the bidirectional edge set. The edge $(j,i) \in \mathcal{E}_k$ represents that drone i and j can exchange information with a positive weight w_{ij}^k at sampling time k. Note that $w_{ij}^k = 0$ if $(j,i) \notin \mathcal{E}_k$. Define $\mathcal{W}_k \triangleq [w_{ij}^k]_{N\times N}$ as the weight matrix of the Graph $\mathcal{G}_k$. The $\mathcal{L}_k \triangleq I_N - \mathcal{W}_k$ is defined as the Laplacian ma- trix of the $\mathcal{G}_k$. The eigenvalues of $\mathcal{L}_k$ can be arranged in ascending order as $0 = \lambda_1(\mathcal{L}_k) < \lambda_2(\mathcal{L}_k)\cdots \le \lambda_N(\mathcal{L}_k)$. By disk theorem, $\lambda_N(\mathcal{L}_k) \le 2$ almost surely. Furthermore, the graph $\overline{\mathcal{G}} := (\mathcal{V}, \overline{\mathcal{E}})$ denotes the expected graph of the random graph sequence, where the edge set is given by $\overline{\mathcal{E}} \triangleq \mathbb{E}\{\mathcal{E}_k\}$, and is associated with the expected weight matrix $\overline{\mathcal{W}} = \mathbb{E}\{\mathcal{W}_k\}$.

The assumption below regarding the weight matrix of the random graph is equivalent to stating that the expected graph $\overline{\mathcal{G}}$ is connected.

Assumption 1 [26] The weight matrices $\mathcal{W}_k, k = 0,1,\cdots$ are independent and identically distributed (i.i.d.).

Each $\mathcal{W}_k$ is doubly stochastic, i.e., $W_k 1_N = 1_N$ and $1_N^T W_k = 1_N^T$

The second-largest eigenvalue of $E\{\mathcal{W}_k^T \mathcal{W}_k\}$ is less than

Remark 1 Assumption 1 imposes a mild requirement on random graphs and is commonly employed in the literature on distributed optimization over random networks [14, 16, 27].

For $i = 1, \ldots, N$, the kinematic model of drone i can be described as follows:

$$\dot{x}_i = v_i \cos\theta_i \; \dot{y}_i = v_i \sin\theta_i \; \dot{\theta}_i = \omega_i \; Y_i = [x_i, y_i]^T$$

where $[x_i, y_i]^T \in \mathbb{R}^2$ represents the Cartesian coordinates of the center of mass (i.e., the absolute position), and $\theta_i \in \mathbb{R}$ represents the heading angle (i.e., orientation) with respect to the inertial frame. The linear and angular velocities, denoted by $v_i \in \mathbb{R}$ and $\omega_i \in \mathbb{R}$ respectively, are treated as control in- puts. The state vector of the *i*-th drone is denoted by Y_i *i* and is considered as the system output.

It is worth noting that, in practice, the orientations θ_i and $\theta_i + 2K_i\pi$, where $K_i \in \mathbb{Z}$, are equivalent, as they represent the same physical direction.

Let $\Lambda(\mathcal{V}, \mathcal{Q}_i, J_i)$ be the multi-drone game, where $\mathcal{V} = \{1,\cdots,N\}$ is the drone set, $\mathcal{Q}_i$ stands for the local output strat- egy set of drone *i*, and J_i represents the cost function of drone *i*.

In the multi-drones game $\Lambda(\mathcal{V}, \mathcal{Q}_i, J_i)$, the communication random graph $\mathcal{G}_k$ caused by electromagnetic interference is used to describe the drone involved in the game. The objec- tive of each drone is to minimize its own cost function, which has the following form:

$$\min_{Y_i \in \mathbb{R}^3} \; J_i(Y_i, Y_{-i}) \tag{2}$$

$$s.t. \; Y_i \in \mathcal{Q}_i, \forall i \in \mathcal{V}.$$

Where $Y_i \in \mathcal{Q}_i$ represents the output of the drone *i*, the $\mathcal{Q}_i \in \mathbb{R}^3$ stands for the local set of values for the drone *i*. Y_{-i} represents the output set of all drones except drone *i*.

Definition 1 In the game $\Lambda(\mathcal{V}, \mathcal{Q}_i, J_i)$, the strategy profile $Y^* = (Y_i^*, Y_{-i}^*) \in \mathcal{Q}$ is said to be an NE if

$$J_i(Y_i^*, Y_{-i}^*) \leq J_i(Y_i^i, Y_{-i}^*), \forall i \in \mathcal{V}, Y_i^i \in \mathcal{Q}_i, \tag{3}$$

where $Y_{-i}^* = col(Y_1^*, ..., Y_{i-1}^*, Y_{i+1}^*, ..., Y_N^*)$, $\mathcal{Q} = \prod_{i \in \mathcal{V}} \mathcal{Q}_i \subset \mathbb{R}^{nN}$. $n = \sum_{i=1}^{N} n_i$ denotes the output strategies set of all drones, and $\prod$ represents the Cartesian product.

This paper aims to design a distributed NE seeking algorithm for multi-drones under electromagnetic interference such that all output of each drone Y_i converge to the NE point Y_i^* almost surely, that is,

$$P\{\lim_{k \to \infty} \| Y_i(k) - Y_i^* \| = 0\} = 1. \tag{4}$$

Remark 2 Different from the existing works on dis- tributed NE seeking which focus on signal-integrator MASs, the dynamic considered in this work is the standard mobile drone model, which makes both the design and the conver- gence analysis of the distributed NE seeking algorithm much more complicated.

To solve the problem, the following basic assumptions are required.

Assumption 2 For each drone *i*, the strategy set $\mathcal{Q}_i$ is nonempty, compact, and convex. Moreover, for any fixed Y_{-i}, the cost function $J_i(\cdot)$ is continuously differentiable and convex with respect to Y_i. Specifically, for any $Y, z \in \mathbb{R}^{n_i}$ and $Y_{-i} \in \mathbb{R}^{n-n_i}$, the following inequality holds:

$$J_i(z, Y_{-i}) \geq J_i(Y, Y_{-i}) + \langle z - Y, \nabla_i J_i(Y, Y_{-i}) \rangle, \tag{5}$$

where $\nabla_i J_i(Y, Y_{-i}) = \frac{\partial J_i}{\partial Y}(Y, Y_{-i})$ denotes the gradient of J_i respect to *Y*.

Assumption 3 Let $\Gamma : \mathbb{R}^n \to \mathbb{R}^n$ denote the pseudo- gradient mapping of the game (2), defined as $\Gamma(Y) = [\nabla_{Y_i} J_i(Y_i, Y_{-i})]$ for all $i \in \mathcal{V}$. The pseudo-gradient Γ is assumed to be μ-coercive and θ_0-Lipschitz continuous, i.e., there exist positive constants μ and θ_0 such that for all $Y, z \in \mathbb{R}^n$,

$$\langle Y - z, \Gamma(Y) - \Gamma(z) \rangle \geq \mu \| Y - z \|^2,$$

$$| \Gamma(Y) - \Gamma(z) \| \leq \theta_0 \| Y - z \|. \tag{6}$$

Remark 3 Assumptions 2 and 3 are standard in the liter- ature on distributed Nash equilibrium (NE) seeking, as they ensure the existence and uniqueness of the NE in the game. These assumptions are commonly adopted in existing works on NE computation, such as [12, Assumptions 1 and 2], [11, Assumptions 2 and 3], and [23, Assumptions 1 and 2].

In context of distributed NE seeking problem, each drone i cannot obtain all other drones' output information. Hence, an estimator $Y^i = (Y_i^i, Y_{-i}^i)$ is implemented by drone i to estimate the strategy of all participants. In this case, the cost function (2) can be reformulated as follows:

$$\min_{Y_i^i \in Q_i} \; J_i(Y_i^i, Y_{-i}^i) \tag{7}$$

$$s.t.\; Y^i = Y^j, \forall i, j \in \mathcal{V}$$

where $Y_i^i = Y_i$ represents the output of the i-th drone, Y_{-i}^i represents estimation of all other drones' outputs by the i-th drone.

4 Main Result

This section, we develop a distributed NE seeking algo- rithm for general multi-drones (1) under electromagnetic in- terference. We use the "decision-control" two-layer architec- ture: the reference signal is generated in the decision layer; while at the control layer a tracking controller is designed to enforce the output of each drone to track the reference signal. Finally, the asymptotic convergence analysis of distributed NE seeking algorithm is established.

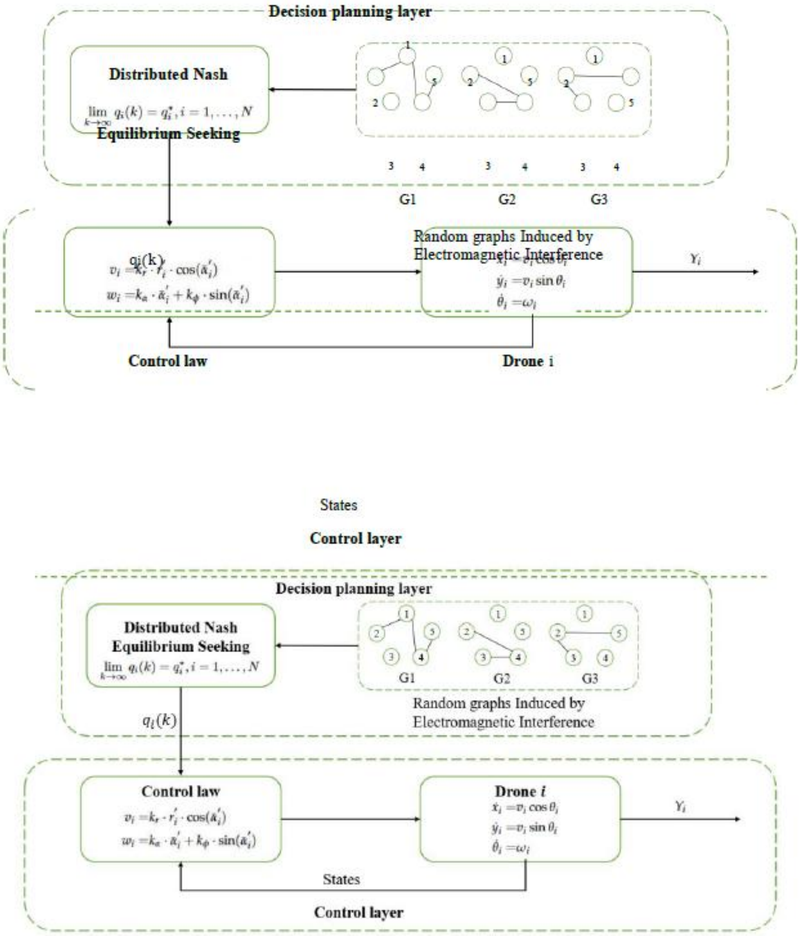

Figure 1: Schematic diagram of the distributed NE seeking algorithm for multi-drones system.

4.1 Design of Distributed NE Seeking Algorithm

The two-layer system of "decision-control" is adopted, and the decision layer is responsible for the communication of drones. This means that each drone shares information with its neighbors. The NE reference signal generation method is as follows:

$$q_i^i((k+1)T) = P_{Q_i}\Big[q_i^i(kT) - \eta_k\Big(\nabla_{q_i} J_i(q_i^i(kT), q_{-i}^i(kT)) + c\sum_{j=1}^{N} w_{ij}(kT)(q_i^i(kT) - q_i^j(kT))\Big)\Big]$$

$$q_{-i}^i((k+1)T) = q_{-i}^i(kT) - \eta_k c\sum_{j=1}^{N} w_{ij}(kT)(q_{-i}^i(kT) - q_{-i}^j(kT)). \tag{8}$$

where $q_i(kT) = [x_i^q(kT), y_i^q(kT)] \in \mathbb{R}^3$ is the auxiliary ref-erence signal obtained from distributed NE seeking, and T is the sampling period, which can be tuned. $q^i = (q_i^i, q_{-i}^i)$ represents the estimated value, c > 0 is a constant, the step-size η_k is a decaying positive sequence and satisfies: i) $\eta_0 \geq \eta_1 \geq \cdots$; ii) $\sum_{k=0}^{\infty} \eta_k = \infty$; and iii) $\sum_{k=0}^{\infty} (\eta_k)^2 < \infty$.

In the control layer, we design the controller v_i and $w_i, i = 1, \ldots, N$, such that the states of all drones will eventually con- verge to the Nash equilibrium point, i.e., (4). For the proof that follows, we first define the following general tracking errors.

Definition 2 Consider the drone *i* and the Nash equilib-rium $q_i^* = [x_i^*, y_i^*]$, the tracking errors are defined as follows:

$$e_{xi} = x_i^* - x_i, e_{yi} = y_i^* - y_i \tag{9}$$

To simplify the controller design, we transform the state of the drone from the Cartesian coordinate system to the polar coordinate system. Then, we define the radial error r_i and angular error α_i as follows:

$$r_i = \sqrt{(x_i^* - x_i)^2 + (y_i^* - y_i)^2} \tag{10}$$

$$\alpha_i = atan2(y_i^* - y_i, x_i^* - x_i) - \theta_i$$

where r_i represents the distance between the i-th drone and the target position, and α_i represents the angular differ- ence between the current orientation and the target direction. Thus, we are ready to present the active control problem of multi-drones system under electromagnetic interference.

Note that when the angle error α_i exceeds 90 degrees, the drone may face in the opposite direction of the target posi- tion. To avoid this, we define a new angle error $\bar{\alpha}_i$ to adjust the control law, ensuring that the drone always faces the tar- get location.

$$\bar{\alpha}_i = \begin{cases} \alpha_i, & |\alpha_i| \leq \frac{\pi}{2} \\ \alpha_i - \text{sign}(\alpha_i) \cdot \pi, & |\alpha_i| > \frac{\pi}{2} \end{cases} \tag{11}$$

In this way, α-i is constrained within $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Now, we propose the control law with adjusted angle $\bar{\alpha}$ as follows:

$$v_i = k_r \cdot r_i \cdot \cos(\bar{\alpha}_i)$$

$$w_i = k_\alpha \cdot \bar{\alpha}_i + k_\phi \cdot \sin(\bar{\alpha}_i) \tag{12}$$

where k_r, k_α and k_ϕ are parameters, which can be tuned. Note that we cannot directly obtain the position information of the equilibrium point, but rather the information of the tracking signal $\mathrm{q}_i(\mathrm{kT})$. Therefore, Equation (10) and (11) is modified as follows:

$$r_i^{'} = \sqrt{(x_i^q - x_i)^2 + (y_i^q - y_i)^2}$$

$$\alpha_i^{'} = atan2(y_i^q - y_i, x_i^q - x_i) - \theta_i \tag{13}$$

$$\bar{\alpha}_i^{'} = \begin{cases} \alpha_i, & |\alpha_i| \leq \dfrac{\pi}{2} \\ \alpha_i^{'} - \mathrm{sign}(\alpha_i^{'}) \cdot \pi, & |\alpha_i^{'}| > \dfrac{\pi}{2} \end{cases}$$

Thus, the control law we actually use is as follows:

$$v_i = k_r \cdot r_i^{'} \cdot \cos(\bar{\alpha}_i^{'})$$

$$w_i = k_\alpha \cdot \bar{\alpha}_i^{'} + k_\phi \cdot \sin(\bar{\alpha}_i^{'}) \tag{14}$$

Problem 1 For the game $\Lambda(\mathcal{V}, \mathcal{Q}_i, J_i)$ of the multi-drones system (1) under electromagnetic interference, a distributed controller is designed such that the output $Y_i(k)$ of the sys- tem converges to the NE point Y_i^* under any initial states $x_i(0), y_i(0), \theta_i(0)$. It is also equivalent to satisfying the fol- lowing equations.

$$P\{\lim_{t\to\infty} r_i = 0\} = 1$$

$$P\{\lim_{t\to\infty} \alpha_i = 0\} = 1, \quad i = 1, \ldots, N. \tag{15}$$

In conclusion, the specific process of the distributed NE seek- ing algorithm is given in Figure 2.

Algorithm 1 Distributed NE Seeking Algorithm Design
1: for k = 1, 2, ... **do**
2: for each drone $i \in \mathcal{V}$ **do Iteration**
3: Design the qi (k) over random graphs $q_i^i(k+1) = P_{\mathcal{Q}_i}\left[q_i^i(k) - \eta_k\left(\nabla_{q_i} J_i(q_i^i(k), q_{-i}^i(k)) + c\sum_{j=1}^N w_{ij}(k)(q_i^i(k) - q_i^j(k))\right)\right]$ $q_{-i}^i(k+1) = q_{-i}^i(k) - \eta_k c\sum_{j=1}^N w_{ij}(k)(q_{-i}^i(k) - q_{-i}^j(k))$
4: Design the tracking controller $v_i = k_r \cdot r_i \cdot \cos(\bar{\alpha}_i)$ $w_i = k_\alpha \cdot \bar{\alpha}_i + k_\phi \cdot \sin(\bar{\alpha}_i)$
5: Design the distributed NE seeking with multi-drones systems under electromagnetic interference
6: end for
7: end for

Figure 2: The distributed NE seeking algorithm.

4.2 Convergence Analysis

Let $\mathbf{q} = col(q^i)_{i\in\mathcal{V}} \in \mathbb{R}^{nN}$, and $q = col(q_i^i)_{i\in\mathcal{V}}$. The $\mathbf{q}(kT) \in \mathbb{R}^{nN}$ is the output at sampling time kT. The extended pseudo-gradient $\Gamma(\mathbf{q}) = col(\nabla_i J_i(q^i))_{i\in\mathcal{V}}$ and $N_{\mathcal{Q}}(q) = \prod_{i=1}^N N_{\mathcal{Q}_i}(q_i),$, with $N_{\mathcal{Q}_i}(q_i) = \{x \in \mathbb{R}^{n_i} \mid x^T(q-y) \le 0, \forall y \in \mathcal{Q}_i\}$.

Assumption 4 The extended pseudo-gradient Γ is as- sumed to be θ-Lipschitz continuous, i.e., there exists a con- stant $\theta > 0$ such that

$$\|\Gamma(\mathbf{q}) - \Gamma(\mathbf{p})\| \le \theta\|\mathbf{q}-\mathbf{p}\|, \quad \forall \mathbf{q},\mathbf{p} \in \mathbb{R}^{nN}. \tag{16}$$

Lemma 1 Under Assumptions 1-4, based on the dis- tributed stochastic forward-backward (DSBF) algorithm (8), the virtual first-order integrator system can converge to the game NE q^* of undirected random topology $\mathcal{G}_k$, that is,

$$\lim_{k\to\infty} q_i(kT) = q^*, i = 1,...,N. \tag{17}$$

Proof of Lemma 1 To facilitate the analysis, we intro- duce the following two operators:

$$\mathbb{A}_k := \mathcal{R}\Gamma(\mathbf{q}) + cL_k\mathbf{q},$$

$$\mathbb{B} := \mathcal{R}N_{\mathcal{Q}}(\mathbf{q}), \tag{18}$$

where $L_k = \mathcal{L}_k \otimes I_N, \mathcal{R} := diag(\mathcal{R}_1, \cdots, \mathcal{R}_N)$ and

$$\mathcal{R}_i = col(0_{n_1\times n_1}, \cdots, 0_{n_{i-1}\times n_{i-1}}, I_{n_i\times n_i}, 0_{n_{i+1}\times n_{i+1}}, \cdots, 0_{n_N\times n_N}) \tag{19}$$

It should be noted that algorithm (8) constitutes a prototypical stochastic forwardbackward method associated with the operators $\mathbb{A}_k$ and $\mathbb{B}$.

Let $\mathcal{F}^i(q^i(kT)) = col(0_{n_1},\cdots,0_{n_{i-1}},\partial\mathcal{I}_{\mathcal{Q}_i}(q_i^i(kT)),0_{n_{i+1}}\cdots,0_{n_N}) \in \mathbb{R}^n$, and $\mathcal{F}(\mathbf{q}(kT)) = col(\mathcal{F}^i(q^i(kT)))_{i\in\mathcal{V}} \in \mathbb{R}^{nN}$. Define $f(q^i(kT)) = q_i^i(kT) - \eta_k\big(\nabla_{q_i}J_i(q^i(kT))$

$$+c\sum_{j\in\mathcal{N}_i} w_{ij}(kT)(q_i^i(kT)-q_i^j(kT))\big). \tag{20}$$

Then, we have

$$q_i^i((k+1)T) = \arg\min_{q_i^i}\{\frac{1}{2}\| q - f(q^i(kT))\|^2 + \partial\mathcal{I}_{\mathcal{Q}_i}(q)\}$$

$$\in f(q^i(kT)) - \partial\mathcal{I}_{\mathcal{Q}_i}(q_{-i}^i((k+1)T)) \tag{21}$$

which implies that the update algorithm of (8) can be rewrit- ten as

$$\mathbf{q}((k+1)T) = \mathbf{q}(kT) - \eta_k\big(\Gamma(\mathbf{q}(kT)) + c\mathcal{R}L_k\mathbf{q}(kT) \;-\mathcal{F}(\mathbf{q}((k+1)T))\big) \tag{22}$$

Assume $\mathbf{q}^* = col((q^i)^*)_{i\in\mathcal{V}}$ is a fixed point in the algorithm (8). And according to the algorithm (8), we can have

$$0_{n_i} \in \nabla_{q_i}J_i((q_i^i)^*,(q_{-i}^i)^*) + N_{\mathcal{Q}_i}((q_i^i)^*), i\in\mathcal{V}, \tag{23}$$

and $(q_i^i)^* = (q_{-i}^j)^*$, which implies that $\mathbf{q}^* = \bar{q}\otimes I_N$ holds for some $\bar{q}\in\mathbb{R}^n$. Then, under Assumptions 1-2 and KKT condi-tion, $\bar{q} = q^*, x^*$ is an NE, thus $\mathbf{q}^* = 1_N\otimes q^*$, which is a unique fixed point of DSFB (8).

By KKT condition, we have

$$0_{nN} \in \mathcal{R}\Gamma(\mathbf{q}^*) + c\bar{L}\mathbf{q}^* + \mathcal{F}(\mathbf{q}^*). \tag{24}$$

Then, it follows from (22) that

$$\mathbf{q}((k+1)T) - \mathbf{q}^* - \eta_k\big(\mathcal{F}(\mathbf{q}((k+1)T)) - \mathcal{F}(\mathbf{q}^*)\big)$$

$$= \mathbf{q}(kT) - \mathbf{q}^* - \eta_k\big(\mathcal{R}\Gamma(\mathbf{q}(kT)) + cL_k\mathbf{q}(kT) - \mathcal{R}\Gamma(\mathbf{q}^*) - c\bar{L}\mathbf{q}^*\big),$$

which implies that

$$\left\|\mathbf{q}((k+1)T) - \mathbf{q}^*\right\|^2 \tag{25}$$

$$\le \eta_k^{\,2}\| \mathcal{R}\Gamma(\mathbf{q}(kT)) + cL_k\mathbf{q}(kT) - (\mathcal{R}\Gamma(\mathbf{q}^*) + c\bar{L}\mathbf{q}^*)\|^2$$

$$+\| \mathbf{q}(kT) - \mathbf{q}^*\|^2 - \eta_k^{\,2}\| \mathcal{F}(\mathbf{q}((k+1)T)) - \mathcal{F}(\mathbf{q}^*)\|^2$$

$$-2\eta_k\langle \mathcal{R}\Gamma(\mathbf{q}(kT)) + cL_k\mathbf{q}(kT) - (\mathcal{R}\Gamma(\mathbf{q}^*) + c\bar{L}\mathbf{q}^*), \mathbf{q}(kT) - \mathbf{q}^*\rangle$$

$$-2\eta_k \langle \mathcal{F}(\mathbf{q}((k+1)T)) - \mathcal{F}(\mathbf{q}^*), \mathbf{q}((k+1)T) - \mathbf{q}^* \rangle.$$

According to the study and analysis of [28, Lemma 2] and [29, Lemma 4], we have

$$<\Gamma(q) + cLq + F(q), q - q^* > \geq \bar{\mu} \| q - q^* \|^2 > 0, \tag{26}$$

where $\bar{\mu} := \lambda_{\min}\left(\begin{bmatrix} \frac{\mu}{n} & -\frac{\theta+\theta_0}{2\sqrt{n}} \\ -\frac{\theta+\theta_0}{2\sqrt{n}} & c\lambda_2(\mathcal{L}) - \theta \end{bmatrix}\right) > 0.$

Then, applying the inequality $(x+y+z)^2 \leq 3(x^2+y^2+z^2)$ for all $x, y, z \in \mathbb{R}^N$, we obtain the following bound for the first term on the right-hand side of (25):

$$\| \mathcal{R}\Gamma(\mathbf{q}(kT)) + cL_k\mathbf{q}(kT) - (\mathcal{R}\Gamma(\mathbf{q}^*) + c\bar{L}\mathbf{q}^*)\|^2$$

$$\leq 3\| \mathcal{R}\|^2 \| \Gamma(\mathbf{q}(kT)) - \Gamma(\mathbf{q}^*)\|^2 + 3c^2 \| L_k\|^2 \| \mathbf{q}(kT) - \mathbf{q}^*\|^2 + 3c^2 \| L_k - \bar{L}\|^2 \| \mathbf{q}^*\|^2$$

$$\leq (3\theta^2 + 12c^2)\| \mathbf{q}(kT) - \mathbf{q}^*\|^2 + 24c^2 \| \mathbf{q}^*\|^2,$$

where the second inequality holds since both of $\| L_k \|, \| \bar{L} \|$ are smaller than 2, and inequality (16) from assumption 3. By the definition of $\mathcal{R}$, we have $\| \mathcal{R}\|^2 = 1$.

By the definition of $\mathcal{F}(\cdot)$, it follows that the last term on the right-hand side of (25) is non-negative.

Combining the above results, we have

$$\left\| \mathbf{q}((k+1)T) - \mathbf{q}^* \right\|^2$$

$$\leq (1 + \eta_k{}^2(3\theta^2 + 12c^2))\| \mathbf{q}(kT) - \mathbf{q}^*\|^2 + 24\eta_k{}^2 c^2 \| \mathbf{q}^*\|^2$$

$$-2\eta_k \langle \mathcal{R}(\Gamma(\mathbf{q}(kT)) - \Gamma(\mathbf{q}^*)) + cL_k\mathbf{q}(kT) - c\bar{L}\mathbf{q}^*, \mathbf{q}(kT) - \mathbf{q}^* \rangle$$

$$-2\eta_k \langle \mathcal{F}(\mathbf{q}((k+1)T)) - \mathcal{F}(\mathbf{q}^*), \mathbf{q}((k+1)T) - \mathbf{q}^* \rangle.$$

Use Robbins-Siegmund quasi-martingale theorem [30] to prove Lemma 1, then, let

$$V(k) = \| \mathbf{q}(kT) - \mathbf{q}^*\|^2 + 2\eta_k \langle \mathcal{F}(\mathbf{q}(kT)) - \mathcal{F}(\mathbf{q}^*), \mathbf{q}(kT) - \mathbf{q}^* \rangle, \tag{27}$$

and we have

$$V(k+1) \leq (1 + \eta_k{}^2(3\theta^2 + 12c^2))V(k) + 24\eta_k{}^2 c^2 \| \mathbf{q}^*\|^2 - 2\eta_k \langle \mathcal{R}\Gamma(\mathbf{q}(kT)) + cL_k\mathbf{q}(kT)$$

$$+ \mathcal{F}(\mathbf{q}(kT)), \mathbf{q}(kT) - \mathbf{q}^* \rangle$$

$$-2\eta_k \langle \mathcal{R}\Gamma(\mathbf{q}^*) + c\bar{L}\mathbf{q}^* + \mathcal{F}(\mathbf{q}^*), \mathbf{q}(kT) - \mathbf{q}^* \rangle.$$

Next, we proceed to compute the following conditional ex- pectation:

$$E\{V(k+1)\,|\,\Pi_k\}$$

$$\leq (1+\eta_k{}^2(3\theta^2+12c^2))E\{V(k)\}+24\eta_k{}^2c^2\|\,\mathbf{q}^*\|^2$$

$$-2\eta_k\langle \mathcal{R}\Gamma(\mathbf{q}(kT))+c\bar{L}\mathbf{q}(kT)+\mathcal{F}(\mathbf{q}(kT)),\mathbf{q}(kT)-\mathbf{q}^*\rangle$$

where the filtration $\Pi_k, k\in\mathbb{N}$ is an increasing collection of sub- σ −fields satisfying $\Pi_0\subseteq\Pi_1\subseteq...\subseteq\Pi_k\subseteq...\subseteq\Pi$ with the probability space $(\Omega,\Pi,\mathcal{P})$.

Since $\sum_{k=0}^{\infty}\eta_k{}^2<\infty$, we obtain $\sum_{k=0}^{\infty}\eta_k{}^2(3\theta^2+12c^2)<\infty, \sum_{k=0}^{\infty}24\eta_k{}^2c^2\|\,\mathbf{q}^*\|^2<\infty$. Then, it follows from Robbins- Siegmund quasi-martingale theorem that

$$\sum_{k=0}^{\infty}\eta_k\langle \mathcal{R}\Gamma(\mathbf{q}(kT))+c\bar{L}\mathbf{q}(kT)+\mathcal{F}(\mathbf{q}(kT)),\mathbf{q}(kT)-\mathbf{q}^*\rangle<\infty \tag{28}$$

and $\lim_{k\to\infty}V(k)<\infty$ almost surely. By (26) and $\sum_{k=0}^{\infty}\alpha_k=\infty$, we have $\lim_{k\to\infty}\langle \mathcal{R}\Gamma(\mathbf{q}(kT))+c\bar{L}\mathbf{q}(kT)+\mathcal{F}(\mathbf{q}(kT)),\mathbf{q}(kT)-\mathbf{q}^*\rangle=0$. It follows from (26) that $\lim_{k\to\infty}\mathbf{q}(kT)=\mathbf{q}^*$.

This completes the proof.

Based on Lemma 1, we can design the two-stage controller v_i and w_i to track the NE reference signal $q_i(k)$ to solve the distributed NE seeking problem of multi-drones system under electromagnetic interference in the control layer.

Theorem 1 Under assumptions 1-4 and by employing distributed algorithm (8) and (12), the multi-drones system (1) under electromagnetic interference will asymptotically converge to the Nash equilibrium of the non-cooperative game (2), thereby solving Problem 1.

Proof of Theorem 1 According to (10) and (11), we have

$$\dot{r}_i=-v_i\cos(\bar{\alpha}_i),\quad \dot{\bar{\alpha}}_i=-w_i+\frac{v_i\sin(\bar{\alpha}_i)}{r_i}. \tag{29}$$

Substituting the actual control law (14), we obtain

$$\dot{r}_i=-k_r\cdot r_i'\cos(\bar{\alpha}_i)\cos(\bar{\alpha}_i') \tag{30}$$

$$\dot{\bar{\alpha}}_i=-k_\alpha\cdot\bar{\alpha}_i'-k_\phi\sin(\bar{\alpha}_i')+k_r\cdot\frac{r_i'}{r_i}\sin(\bar{\alpha}_i)\cos(\bar{\alpha}_i')$$

We choose the following Lyapunov function

$$V(r_i,\bar{\alpha}_i)=\frac{1}{2}r_i^2+b\cdot(1-\cos(\bar{\alpha}_i)) \tag{31}$$

where b is a positive constant. According to equation (30), the derivative of the Lyapunov function is

$$\dot{V}(r_i,\bar{\alpha}_i) = r_i \cdot r_i^{'} + b\sin(\bar{\alpha}_i)\cdot \dot{\bar{\alpha}}_i$$

$$= -k_r \cdot r_i^{'} \cos(\bar{\alpha}_i)\cos(\bar{\alpha}_i^{'}) \cdot r_i - b \cdot k_\alpha \sin(\bar{\alpha}_i)\cdot \bar{\alpha}_i^{'}$$

$$-b \cdot k_\phi \sin(\bar{\alpha}_i)\sin(\bar{\alpha}_i^{'}) + b \cdot k_r \cdot \frac{r_i^{'}}{r_i}\sin(\bar{\alpha}_i)^2 \cos(\bar{\alpha}_i^{'}) \tag{32}$$

Next, we will sequentially address the four terms on the right side of Equation (32).
The first term:

$$-k_r \cdot r_i^{'} \cos(\bar{\alpha}_i)\cos(\bar{\alpha}_i^{'}) \cdot r_i$$

$$= -k_r \cdot r_i^2 \cos(\bar{\alpha}_i)^2 - k_r \cdot r_i \cos(\bar{\alpha}_i)[r_i^{'} \cos(\bar{\alpha}_i^{'}) - r_i \cos(\bar{\alpha}_i)]$$

$$\le -\frac{1}{2} k_r \cdot r_i^2 \cos(\bar{\alpha}_i)^2 + \frac{1}{2} k_r [r_i^{'} \cos(\bar{\alpha}_i^{'}) - r_i \cos(\bar{\alpha}_i)]^2 \tag{33}$$

The second term: Since the inequality $|\sin(\bar{\alpha}_i)\bar{\alpha}_i| \le \frac{\pi}{2}(\sin(\bar{\alpha}_i))^2$ holds when $|\bar{\alpha}_i| \le \frac{\pi}{2}$, we have

$$b \cdot k_\alpha |\sin(\bar{\alpha}_i)\bar{\alpha}_i^{'}| \le b \cdot k_\alpha \frac{\pi}{2}\sin(\bar{\alpha}_i)^2 + b \cdot k_\alpha |\bar{\alpha}_i - \bar{\alpha}_i^{'}| \tag{34}$$

The third term: Since the same operation applies to the first term, we will omit some steps.

$$-b \cdot k_\phi \sin(\bar{\alpha}_i)\sin(\bar{\alpha}_i^{'}) \le -\frac{1}{2} b k_\phi \sin(\bar{\alpha}_i)^2 + \frac{bk_\phi}{2}(\sin(\bar{\alpha}_i) - \sin(\bar{\alpha}_i^{'}))^2 \tag{35}$$

The fourth term: Since $\cos(\bar{\alpha}_i^{'}) \le 1$when $|\bar{\alpha}_i^{'}| \le \frac{\pi}{2}$,we have

$$|b \cdot k_r \cdot \frac{r_i^{'}}{r_i}\sin(\bar{\alpha}_i)^2 \cos(\bar{\alpha}_i^{'})| \le b \cdot k_r \cdot \frac{r_i^{'}}{r_i}\sin(\bar{\alpha}_i)^2. \tag{36}$$

Finally, we obtain the following relationship for the deriva- tive of the Lyapunov function.

$$\dot{V}(r_i,\bar{\alpha}_i) \le -\frac{1}{2} k_r \cdot r_i^2 \cos(\bar{\alpha}_i) - b \cdot (\frac{1}{2} k_\phi - \frac{\pi}{2} k_\alpha - k_r \cdot \frac{r_i^{'}}{r_i})\sin(\bar{\alpha}_i)^2 + \psi(\xi) \tag{37}$$

where $\xi = [r_i, r_i^{'}, \bar{\alpha}_i, \bar{\alpha}_i^{'}]^T$ and

$$\psi(\xi) = \frac{1}{2} k_r [r_i^{'} \cos(\bar{\alpha}_i^{'}) - r_i \cos(\bar{\alpha}_i)]^2 + b \cdot k_\alpha |\bar{\alpha}_i - \bar{\alpha}_i^{'}| + \frac{bk_\phi}{2}(\sin(\bar{\alpha}_i) - \sin(\bar{\alpha}_i^{'}))^2 \tag{38}$$

It can be seen from the decision level analysis that $\lim_{k\to\infty}\mathbf{q}(k)=\mathbf{q}^*$, that is $\lim_{k\to\infty}\Delta_i(k)=\lim_{k\to\infty}(q_i(k)-q^*)=0$. Then, one has that $\lim_{t\to\infty}\psi(\xi)=0$, and we choose parameters to satisfy $\frac{1}{2}k_\phi-\frac{\pi}{2}k_\alpha-k_r\cdot\frac{r_i^{'}}{r_i}>0$ such that $E\{\dot{V}(r_i,\bar{\alpha}_i)\}<0$ when $r_i,\bar{\alpha}_i\neq 0$ .Thus, we have $\lim_{t\to\infty}E\{V(r_i,\bar{\alpha}_i)\}=0$.

This completes the proof.

Remark 4 The NE seeking problems are widely used in various fields, but more research focuses on single integra- tor systems, while this paper considers the mobility model of the drone. Meanwhile, we also need to consider the drones' ability to find Nash equilibrium solutions and effectively ad- dress complex decision-making under electromagnetic inter- ference.

5 Numerical Simulation

This section illustrates the effectiveness of the proposed distributed NE for multi-drones system over random graphs. Consider multi-drones consisting of seven drones in the form of a random graph shown in Fig. 3. Assuming four communi- cation topological graph, for each sampling, the probability of being visited is $p(\mathcal{G}_k=\mathcal{G}_1)=0.2$, $p(\mathcal{G}_k=\mathcal{G}_2)=0.3$, $p(\mathcal{G}_k=\mathcal{G}_3)=0.4$ and $p(\mathcal{G}_k=\mathcal{G}_4)=0.1$, respectively.

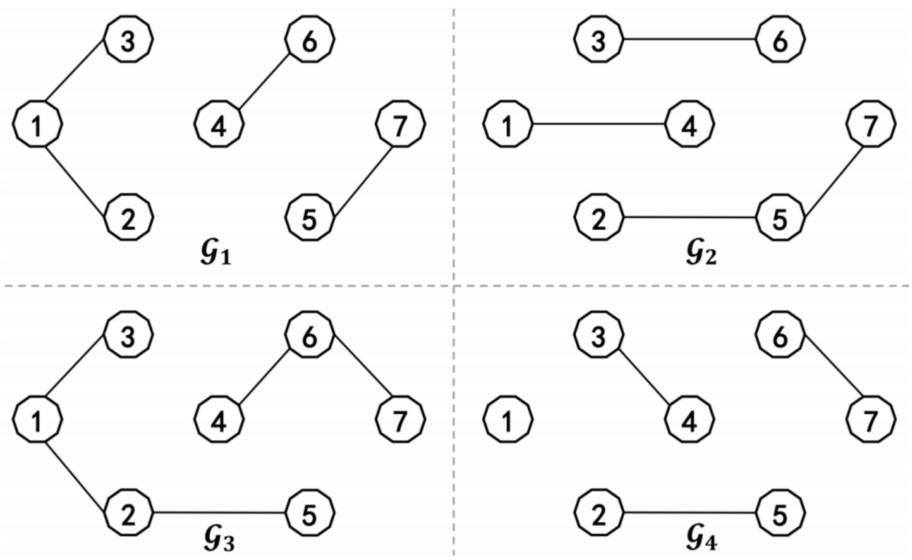

Figure 3: Random Graphs Gk of the drones

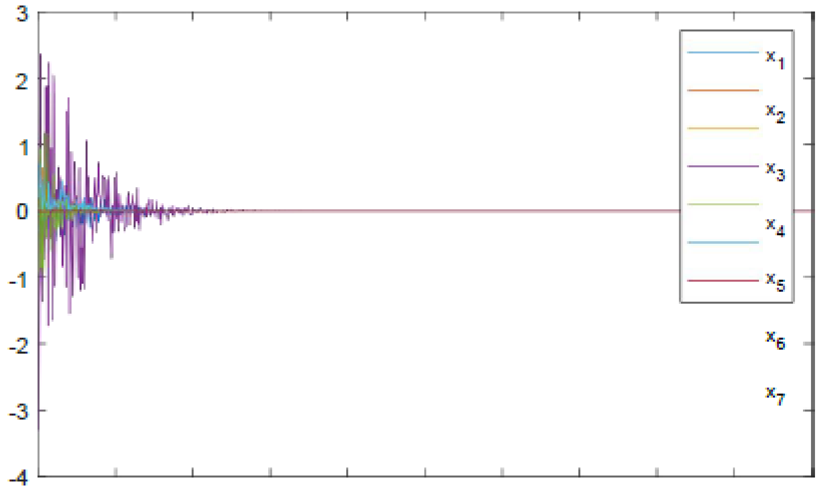

Figure 4: Error between the drones' output and the Nash equilibrium on the x-axis.

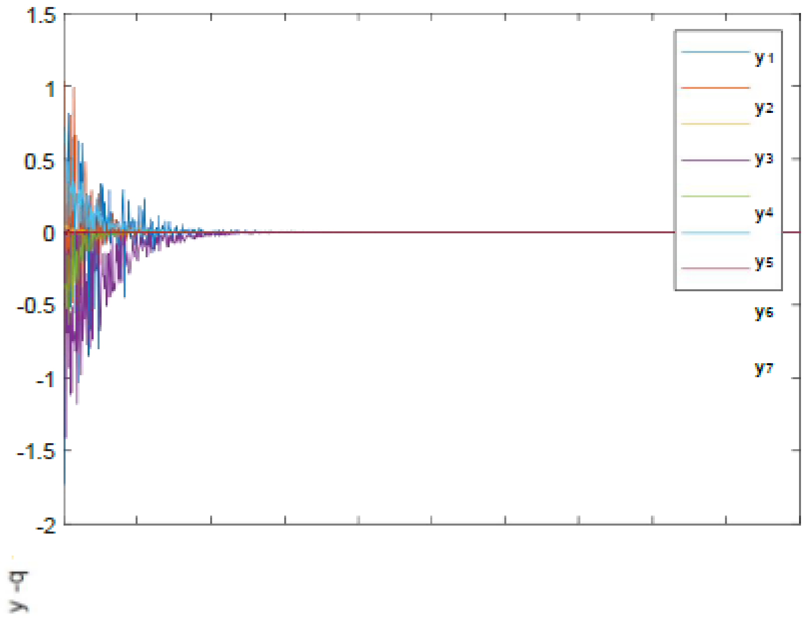

Figure 5: Error between the drones' output and the Nash equilibrium on the y-axis.

The weight in G*k* is chosen as:

$$w_{12}^1 = w_{13}^1 = \frac{1}{2}, w_{31}^1 = w_{21}^1 = w_{46}^1 = w_{64}^1 = w_{57}^1 = w_{75}^1 = 1,\ w_{ij}^1 = 0, otherwise; \tag{39}$$

$$w_{52}^2 = w_{57}^2 = \frac{1}{2}, w_{14}^2 = w_{25}^2 = w_{36}^2 = w_{41}^2 = w_{63}^2 = w_{75}^2 = 1,\ w_{ij}^2 = 0, otherwise$$

$$w_{12}^3 = w_{13}^3 = w_{21}^3 = w_{25}^3 = w_{64}^3 = w_{67}^3 = \frac{1}{2},\ w_{31}^3 = w_{46}^3 = w_{52}^3 = w_{76}^3 = 1, w_{ij}^3 = 0, otherwise;$$

$$w_{25}^4 = w_{34}^4 = w_{43}^4 = w_{52}^4 = w_{67}^4 = w_{76}^4 = 1,\ w_{ij}^4 = 0, otherwise;$$

Consider a game with seven drones and the multi-drone system (1). Each drone's output x_i and y_i are limited by the constraint set [-1.8, 0.8], and each drone is trying to mini- mize its own cost function, which is,

$$J_1(x)=2x_1^2+x_1(x_2+x_3+x_4+5)+3,$$
$$J_2(x)=x_2^2+x_2x_4-2,$$
$$J_3(x)=2x_3^2+x_3(x_1+x_6+x_7),$$
$$J_4(x)=\frac{3}{2}x_4^2+x_4(x_2+x_6+4)+5,$$
$$J_5(x)=\frac{5}{2}x_5^2+x_5(x_1+x_2+x_3+x_7+5),$$
$$J_6(x)=2x_6^2+x_6(x_3+x_4+x_5-1)+7,$$
$$J_7(x)=3x_7^2+x_7(x_1+x_2+x_3+x_4+x_5+3).$$

and

$$J_1(x)=2x_1^2+x_1(x_2+x_3+x_4+5)+3,$$
$$J_2(x)=x_2^2+x_2x_4-2,$$
$$J_3(x)=2x_3^2+x_3(x_1+x_6+x_7),$$
$$J_4(x)=\frac{3}{2}x_4^2+x_4(x_2+x_6+4)+5,$$
$$J_5(x)=\frac{5}{2}x_5^2+x_5(x_1+x_2+x_3+x_7+5),$$
$$J_6(x)=2x_6^2+x_6(x_3+x_4+x_5-1)+7,$$
$$J_7(x)=3x_7^2+x_7(x_1+x_2+x_3+x_4+x_5+3).$$

The initial values of the drone's state and the estimated vec- tor $q^i(0)$ are set to the zero vector. Select $c=1$ and $\eta=0.5/(k+1)^{0.6}$. By calculation, the NE of the reference signal is $q_{xi}^*=q_{yi}^*=[-1.0142, 0.8000, 0.0567, -1.8000, -0.9660, 0.8000, -0.0128]$..

Figures 4 and 5 illustrate that the errors between the *x*- and y-axis coordinates of the drones and the Nash equilibrium converge to zero, indicating that the output of the multi-drone system ultimately reaches the Nash equilibrium. In conclu- sion, the effectiveness of the distributed algorithms (8) and (12) is successfully verified.

6 Conclusion

This paper addresses the cooperative decision-making and control problem for a multi-drone system operating under electromagnetic interference. A two-layer decision-control architecture is proposed to tackle this challenge. In the de- cision layer, a distributed Nash equilibrium (NE) seeking algorithm is developed by integrating a stochastic forward- backward operator splitting method with a consensus proto- col, ensuring that the drone outputs stay within a predefined constraint set. In the control layer, a feedback tracking con- troller is designed to ensure that the drone outputs accurately track the NE trajectory. Numerical simulations are presented to validate the effectiveness and robustness of the proposed algorithm.

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