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Construction and Empirical Research of Higher Mathematics Teaching Model Based on Flipped Classroom

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SUMMARY: *Advanced Mathematics is a very important foundational theory course in the undergraduate talent cultivation course system of universities. It possesses the ability to cultivate trainees' abstract thinking processes, logical reasoning abilities, and computational thinking abilities. This present thesis deals with the problem of teaching transformation in Higher Mathematics. This paper carries out deep discussion on the flipped classroom teaching method which is used for Advanced Mathematics course. This text introduces KBRL-Rec, which is an on-line examination question recommendation model that is established on the basis of reinforcement learning. On the foundation of the KBRL-Rec model, a precise teaching intervention framework for the flipped classroom of Advanced Mathematics is built, and an empirical exploration on the effectiveness of the teaching intervention is conducted. The research results show that when make comparison with the random recommendation method, the relevance degree of the exercises that recommended by the KBRL - Rec algorithm increases from 0.0084 to 0.0297. The undulation of difficulty degree descends from 0.2136 to 0.0325. Furthermore, the value of reward becomes more stable, hence the maximum value is comparatively more favorable. The utilization of the constructed generalized frame for accuracy education intervention to carry out accuracy education intervention has obtained better results and shows great application possibility. This present article provides a model for the teaching of Advanced Mathematics by making use of the flipped classroom teaching model, which therefore helps the realization of intelligent and dynamic accurate teaching intervention.*

KEYWORDS: *advanced mathematics; flipped classroom; enhanced learning; test question recommendation; precise instructional intervention*

1 Introduction

Advanced higher mathematics acts as an important foundational lesson for undergraduate learners in the domains of science and technology. This curriculum gets its difference from others through its all-round content, high degree of abstraction, strict logic organization, and wide practical utilization areas [1]. During a very long period, educational workers and learners have together coped with the difficult problem of promoting students' study enthusiasm for advanced mathematics, and letting them grasp this discipline by utilizing scientific and reasonable study methods[2, 3]. The tradition teaching method depends on making detailed explanations to formulas and theorems. This method has difficulty in causing students' learning passion and therefore is easy to arouse a feeling of fear towards hard assignments [4].Under the background of the information-leading epoch, great changes have been produced in many

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aspects of school education, for example education ideas, teaching objects, teaching aims, teaching ways, teaching processes, and media instruments. Therefore, hence, universities are step by step making attempts to set up the flipped classroom as one kind of teaching method for the carrying out of teaching [5, 6]. The turning-over classroom is an instruction method that gives students power to carry out self-guidance study through using modern technology and digit study platforms[7]. This method changes the traditional "teach first, after that learn" teaching pattern into a "learn first, after that teach" one. Before the class begins, students carry out directional pre-study on the basis of study tasks that are provided by the teacher. By means of self-directed learning, they have grasped foundational concepts, principles as well as skills, and have solved elementary problems[8-10]. Inside the classroom environment, the teacher designs a group of pointed questions according to the students' self-study results. These problems are designed for helping students solve the problems which they met when they did self-study. At the same time, they have the function of promoting students' deep-level understanding, and thus deal with the complicated nature of knowledge [11, 12]. After the class is finished, students carry out a group of exercises to make a deeper self-evaluation of their knowledge gaps and solve these problems. At the same time, the teacher by use of an online platform collects the feedback from students. This teaching method, which gets away from the traditional boundary division between teachers and students, thus effectively promotes students' participation in the study process. It also can arouse the studying passion of students, and thus can prominently improve the study effect in advanced mathematics[13-16].

High-level mathematics from long ago until now is a very important subject which has attracted much attention. Under the background of higher education, the flipped classroom method for advanced mathematics teaching possesses great importance in the course of teaching reform. Furthermore, the circle of academy has obtained prominent achievements in the digging research of this domain. Literature [17]The method system of the flipped classroom teaching model, it is given in a systemic way. Furthermore, the results of a practice project about the flipped classroom teaching pattern in advanced mathematics are given in this report. This report finds out students' acceptance degree of the flip classroom teaching pattern, as well as its strong points and weak points.Literature [18] pointed out the limitations of the traditional teaching model of advanced mathematics, proposed a flipped classroom teaching model, and implemented an adaptation of this teaching model in a course focusing on proof analysis to verify the effectiveness of the flipped classroom teaching model. Literature [19] assessed the impact of the application of the flipped classroom model in teaching advanced mathematics on students' test scores by using a quasi-experimental design and revealing a significant correlation between the flipped classroom model and students' test score performance through a comparative experiment. Literature [20] examined the extent to which the flipped classroom model impacts on students' performance in higher mathematics and pointed out that the flipped classroom model is effective in improving students' performance by conducting different types of student surveys. Literature [21] discussed the application of the flipped classroom model in the basic advanced mathematics course and systematically analyzed the course design under the flipped classroom, revealing that the flipped classroom transformed the traditional teaching mode and improved the effectiveness of mathematics teaching. Literature [22] explored the use of the flipped classroom teaching model in teaching advanced mathematics, and proposed possible improvements to be implemented for the next class of students by comparing the use of the model in different years.

Beside the advanced mathematics that has high difficulty, the flipped classroom teaching method has the important function in mathematics teaching of other education stages. Literature [23] emphasized the shortcomings of the traditional teaching model and indicated the applicability of the flipped classroom model to mathematics teaching, and the findings showed that the flipped classroom had a positive impact on students' mathematics achievement. Literature [24] discussed the impact of the flipped classroom model of mathematics teaching on students' achievement based on meta-analysis, and the results of the analysis pointed out that the impact of the flipped classroom model on students' achievement was significant. Literature [25], based on the literature review, emphasized that the flipped classroom model has a positive impact on mathematics education as a whole and pointed out that there are obvious limitations in the existing studies, such as significant similarities between studies, methodological uniformity, and lack of innovation. Literature [26] discussed the impact of the flipped classroom model of mathematics teaching on students' behavioral, affective, and cognitive engagement in mathematics courses based on the literature review, and the results verified the effectiveness of the flipped classroom model, but further research needs to be carried out in the areas of students' attendance and mathematics anxiety. Literature [27] explored the pedagogical transformation of a mathematics teacher in two mathematics classrooms based on the flipped classroom model, and the study describes the teaching and learning of the flipped mathematics classroom model, emphasizing that the model develops students' mathematical potential by focusing on students. Literature [28] compared two models of teaching mathematics in flipped and non-flipped classrooms and the results pointed out that students preferred the flipped classroom model because it improved their mastery of mathematics. Literature [29] reviewed the current literature on the flipped classroom model in mathematics with the aim of discussing the impact of the model on mathematics learning, and the results pointed out that the effect of the flipped classroom model on students' academic performance and cognitive aspects was unclear, and further research pointed out that effective flipped classrooms involve discussion, feedback, and peer collaboration. Literature [30] reviewed the impact of the implementation of flipped classrooms in mathematics teaching and emphasized that in most cases flipped classrooms help to improve students' mathematical knowledge and attitudes and also play an important role in terms of autonomy and self-regulation.

This research paper at the beginning completes the building of a reversed classroom teaching model for higher mathematics. The present paper is established upon the teaching reform idea of "laying firm base, putting emphasis on four key respects, and realizing two changes." Then, the KBRL-Rec recommendation model is constructed by introducing reinforcement learning theory into the exercise recommendation task, and the superiority of the model compared with the random recommendation is verified. Finally, based on the KBRL-Rec model, a generalized framework for accurate teaching intervention in the flipped classroom of higher mathematics is constructed, and it is used in teaching practice to test the usability of the framework.

2 Construction of a flipped classroom teaching model for higher mathematics

2.1 Teaching reform ideas

The present writing establishes a teaching structural system for high-level mathematics which is centered on the flipped classroom idea. It follows the education transformation tenet of "building up firm foundation, putting emphasis on four key points, and reaching two transformations.". That is to establish the fundamental position of the flipped teaching concept of "learner-centered, learning activity-oriented, and equal participation", and earnestly do a good job in the four links of course teaching design, teaching resource construction, classroom teaching activities, and teaching evaluation. Our aim is that we should shift the teaching philosophy from a "teacher-centric" method to a "student-centric" one. Furthermore, one should change the training objective from the production of "qualified course learners" to the cultivation of "whole-life learners".

2.2 Course instructional design

2.2.1 Pedagogical models

The method of flipped classroom causes a reverse of the procedures of knowledge spreading and knowledge absorbing. In traditional classrooms, the spreading of knowledge takes place in the class time, but for a flipped classroom, this process is moved to the time before class. In the same way, the activity of knowledge absorbing, which was originally done via after-class assignments, has now been shifted to in-class learning activities. The turned-over classroom teaching mode of the Advanced Mathematics course is showed in Figure 1. Before the class begins, students must use the network platform to look at the teaching software and finish the basic exercises according to the directed study plan that the teacher gives. After we enter the classroom, the interactive teaching activity is carried out between the teacher and the students. The interactive study in the turned-over classroom is realized by several modules. These contain group student speech shows, cooperating group work, teacher-student talks about hard questions, comments on the teaching effect, and students' own independent search to get study outcomes.

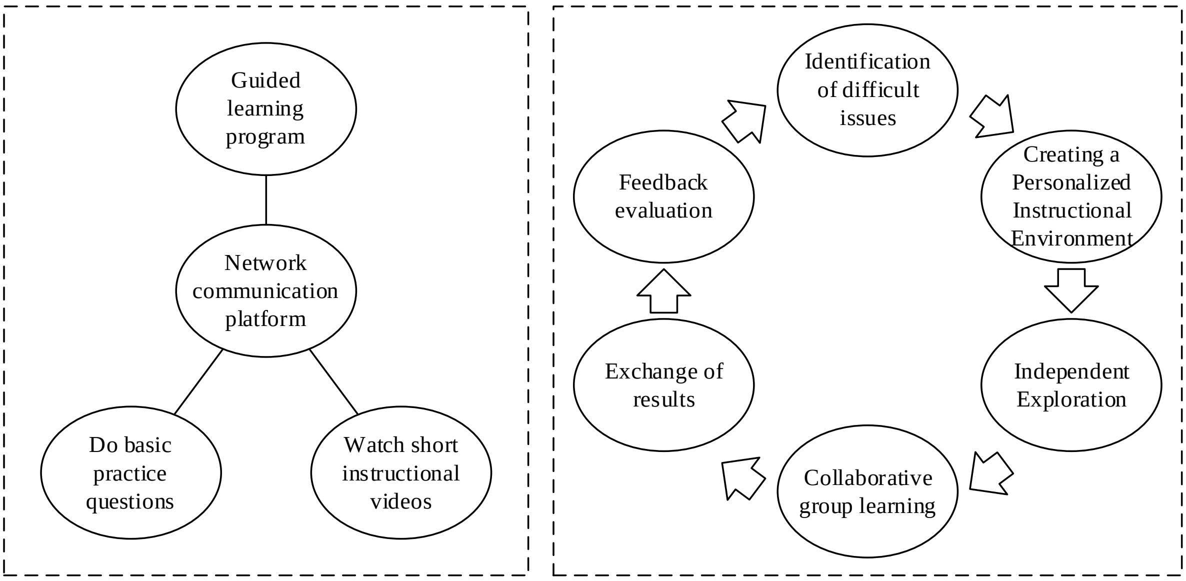


Figure 1: Teaching model of the "Advanced Mathematics" course

2.2.2 Instructional design

The total number of hours in the Advanced Mathematics program is 180, with 8 hours per week, 4 hours of pre-course study and 4 hours of classroom reinforcement. Teaching content covers function limits, continuity and discontinuity, differentiation, integration, infinite series, differential equations, and so on.

Stage 1: One week before the class, the teacher releases the study guide program, study software, basic exercises of Advanced Mathematics, etc., and the students will study independently and complete simple exercises and homework. After fully understanding their own learning tasks, students can flexibly arrange their own study time to achieve personalized learning.

Stage 2: In the first section of the lesson, students report in small groups, including basic knowledge and improved learning resources, and conduct interactive exchanges and discussions between groups. In the second section of the class, the teacher answers the difficult questions in the first section, and at the same time, the teacher gives additional explanations on some key contents and extended questions, and gives timely comments and counseling to each group.

Stage 3: After the class, the students complete and submit the homework of some comprehensive problems, and the teacher reviews the homework and gives feedback to the students.

2.3 Classroom teaching methods

In order to achieve the teaching effect of flipped classroom, the course of Advanced Mathematics adopts the teaching concept based on examples and problems, while combining the following three teaching methods to carry out teaching.

(1) PPT visual impact teaching, i.e., the teaching activity of adding animation or video display in the teaching PPT and supplementing it with relevant animation or video content questions.

(2) Classroom object teaching, that is, by showing the structure of the object or its movement and other forms of teaching activities based on the “question”.

(3) Group competition confrontation teaching, that is, through the teacher predetermined confrontation competition topic, let the students group free combination, to carry out knowledge independent learning teaching activities.

3 Reinforcement learning-based modeling of test question recommendations for online classrooms

In order to ensure the implementation effect of the flipped classroom model in advanced mathematics, this chapter constructs the test question recommendation model KBRL-Rec based on reinforcement learning theory.

3.1 Enhanced learning theory

Strengthened study is one important sub-category of machine learning calculation methods, which places itself together with supervised study and no-supervised study. When compared with supervised learning and unsupervised learning, reinforcement learning obtains knowledge through the interaction process which is between an intelligent body and its environment. Its goal is to find out the most effective method which can make the rewards of its own actions reach maximum.

3.1.1 Principles of Reinforcement Learning

Reinforcement learning lets an intelligent agent to carry out interaction with its environment under many different conditions and finally obtain the ability to choose actions which can make the rewards get optimized.The principle of reinforcement learning is shown in Fig. 2, where at moment t, the intelligent body obtains its own state s_t by observing the environment and itself, and also obtains its own reward R_t that it obtained at the previous moment ($t-1$). Next, the intelligent body makes an action a_t combining s_t and R_t according to the strategy set by reinforcement learning. The a_t passing through the environment will cause the intelligence to enter a new state s_{t+1}, while giving the intelligence a new reward R_{t+1}, and so on until the end of the round, denoted as moment T. The goal of reinforcement learning is to attain the maximum possible accumulative reward after the ending of one round of the learning process. That is namely, to maximize $R = \sum_{i=0}^{T} R_i$.

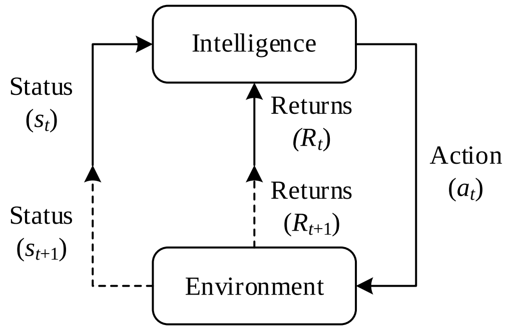


Figure 2: Principles of reinforcement learning

In conclusion, reinforcement learning is a process that an intelligent body carries out interaction with its environment and attempts to obtain as much rewarding signals as it is possible. It is mainly composed by the policy function, the reward mechanism, the value function and the environmental model. The policy function includes the collection of behaviors that the intelligent agent is able to carry out. At the same time, the environment model contains the collection of states which the intelligent agent possibly can take in the environment.

3.1.2 Markov decision process

In the process of reinforcement learning, the intelligent agent continuously carries out interaction with the environment. For example, starting the computation from the moment $t=0$, the intelligent body is in the state s_0, makes the action a_0, gets the reward R_1, and enters the state s_1. This cycle produces a sequence:

$$s_0, a_0, R_1, s_1, a_1, \ldots \tag{1}$$

This sequence is then called the sequential decision process.

In general view, the occurrence of a state at a specific time is connected with all earlier historical states, just as displayed on the right-hand side of the equality symbol in formula (2) Nevertheless, a Markov decision process usually chooses to ignore historical factors and puts forward that the state of the next time in a decision sequence is only connected with the state of the current moment, just as is shown on the left side of the equal sign in Equation (2).

$$P\left(s_{t+1} \mid s_t\right) = P\left(s_{t+1} \mid s_1, \ldots, s_t\right) \tag{2}$$

On this basis, define the probability that a specific state s moves to the next state s' after performing the action a as the when-state transfer probability:

$$P_{sas'} = P\left(s_{t+1} = s' \mid s_t = s, a_t = a\right) \tag{3}$$

For the set of all states and actions, $P_{sas'}$ constitutes a three-dimensional matrix, and assuming there are m actions as well as n states, this state transfer matrix P can be expressed as:

$$P = \begin{bmatrix} P_{1a1} & \cdots & P_{1an} \\ \vdots & \ddots & \vdots \\ P_{na1} & \cdots & P_{nan} \end{bmatrix} \tag{4}$$

where $P_{iaj} = \left[P_{i1j} \quad \ldots \quad P_{imj} \right], i, j \in [1, n]$.

According to Eq. (1), a return value $R_{sas'}$ is obtained for each sas' state transfer in the Markov decision process of reinforcement learning:

$$R_{sas'} = E\left\{ R_{r+1} \mid s_t = s, a_t = a, s_{t+1} = s' \right\} \tag{5}$$

3.1.3 Q-learning algorithm

The Q-learning arithmetic is one kind of reinforcement learning arithmetic, it carries out work without depending on one model which belongs to environment. This one belongs to the category of model-free algorithms, that is to say its operations do not get foundation from an already built environment model. Where $Q(s,a)$ is set as the expectation of the gain that an intelligent body in state s_t at a certain moment t can obtain after performing the action a_t, which is known as the state action value function. Model-free algorithms do not need to know the exact state transfer probability, the environment will return the corresponding return R by itself according to the action performed by the intelligent body. Based on this idea, the Q-learning algorithm constructs a Q-table to store each set of s and a values, and when selecting an action, it will select the action that maximizes the Q-value according to the current state as the next action:

$$a_t = \pi\left(s_t\right) = \underset{a}{\operatorname{argmax}}\, Q\left(s_t, a\right) \tag{6}$$

Q-learning, which is one algorithm that is used in reinforcement learning, also is classified into the category of Markov decision process. In the training stage, this Markov decision process can be solved by means of Bellman's equation. According to what the Bellman equation says:

$$Q(s,a) = E\left[G_t \mid a_t = a, a_t = s \right] \tag{7}$$

where $G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots$, represents the total discounted rewards from t onwards. Bringing in Eq. (7), we can finally obtain:

$$Q(s,a) = E\left[R_{t+1} + \gamma Q_\pi\left(s_{t+1}, a_{t+1}\right) \mid a_t = a, s_t = s \right] \tag{8}$$

Then, expanding the expectation, the optimal action value function is as in equation (9):

$$Q^*(s,a) = \max_{\pi} Q(s,a) \\ = \sum_{s'} P(s' \mid s,a)\left[R(s,a,s') + \gamma \max_{a'} Q(s',a')\right] \tag{9}$$

Based on the idea of time difference, the final formula for Q-value renewal is inferred in the following way:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\left[r + \gamma \max_{a'} Q(s',a') - Q(s,a)\right] \tag{10}$$

For the understanding of Eq. (10), $r + \gamma \max_{a'} Q(s',a')$ can be regarded as the algorithm takes the actual reward r obtained after performing the action a in the state s plus the maximum Q value after reaching s' after the maximum Q value as the realistic Q value, while the latter $Q(s,a)$ is the estimated Q value based on the original Q table. The error between the estimated and realistic values is what is taken into account in updating $Q(s,a)$. Where α is the learning rate, which represents how much of the current error will be updated to the Q table. The γ is the discount factor, which represents how far-sighted the Q-learning algorithm is, i.e., the closer the γ is to 1, the more pronounced the effect of the future reward gained on the current Q-value.

In the stage of action choosing, ε greed is used as a method in the process of decision making to get a balance between exploring and using.Assuming that ε is 0.1, it means that in 90% of the cases the system will follow the maximum value of the selected Q-table as the next action, while in the remaining 10% of the cases the action will be randomly selected as a way to cope with more different situations by trying other different behaviors.

3.2 Reinforcement learning based test question recommendation model KBRL-Rec

The recommendation model which is constructed on the basis of reinforcement learning has the capacity to imitate the real-time interactive procedure between an intelligent body and a user. It has the function of recommending the assigned content to the user. Through this method, it is able to obtain the true feedback from the user, which is therefore utilized by it for the updating of the recommendation strategy. When this model and this method are used to do recommendation work, the emphasis can be put on the long-term satisfaction degree of users and the promotion of their knowledge degree. Furthermore, the accuracy of the recommendation can be elevated, thus permitting the realization of the expected results.

3.2.1 Reinforcement Learning Based Recommendation Scenarios

The constituent parts inside the reinforcement learning model are divided into categories like this: Agent, Environment, State, Action, Reward, etc. The recommendation process simulated in this paper is as follows. The recommendation process simulated in this paper is as follows: during the learner's use of the platform, the Recommender Intelligence Body generates a certain number of recommendation sets $\mathbb{A}_T$ based on the user's initial state of knowledge S, where the recommendation set contains a number of exercises $\{q_1, q_2, \ldots, q_n\} \in \mathbb{A}_T$ and recommend to display to the user, after the user selects the recommended exercises, the recommending intelligence will continuously take new actions based on the user's knowledge state and the probability of answering the recommended exercise set correctly $\{p_1, p_2, \ldots, p_n\}$, i.e., generate new suitable learners' set of exercises, and the knowledge condition of the learner is subjected to unceasing renewals, and novel repayments are generated to be fed back to the recommendation intellectual system. According to the above narration, Figure 3 gives the flow exhibition of the recommendation model which depends on reinforcement learning.

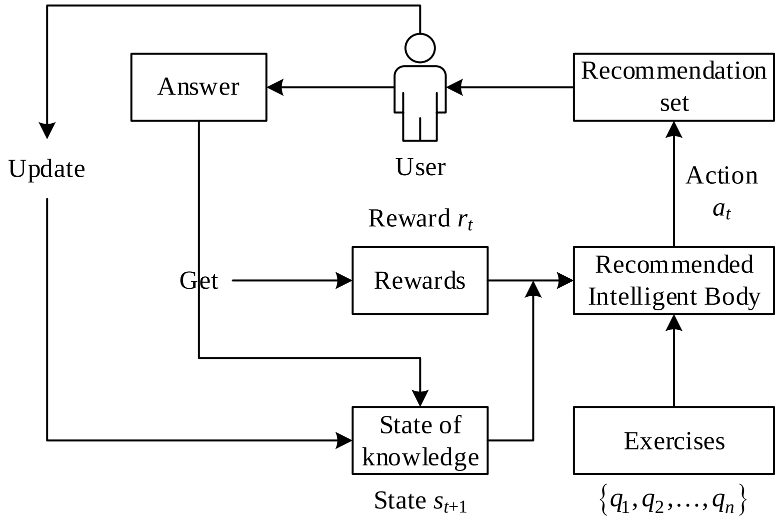


Figure 3: The process of the recommendation model based on reinforcement learning

3.2.2 Knowledge level representation based on deep knowledge tracking

In the present research paper, we utilize Deep Knowledge Tracing (DKT) for the building of a model of learners' knowledge status in a custom-made test recommendation system. The knowledge tracing process, also called the dynamic cognitive diagnosis process, involves dynamically acquiring student knowledge states and modeling the knowledge state matrix.

In this meeting, the main aim is to obtain the learner's knowledge level condition and the hidden expression of this knowledge condition, which is used to define the state S_t in the reinforcement learning recommendation scenario. Firstly, the DKT model is trained based on the user's history of answering sequences $\{x_1, x_2, \ldots, x_t\}$ and the history of answering results $\{y_1, y_2, \ldots, y_t\}$ as inputs.The outputs of the DKT y_t represents the probability of the current student answering the current question x_t correctly, i.e., the current student's mastery of the question, where the dimension of y_t is equal to the number of questions, i.e., $y_t \in R^{N\times i}$. In particular, $x_{t+1} = \{q_{t+1}, a_{t+1}\}$, where $a_{t+1} = y_t\left(q_{t+1}\right)$.

A model of the current student's personal knowledge state is derived at the end of training, and from this model the implicit representation of the student's knowledge state under the current time window t, h_t, is also the implicit matrix of the knowledge state. Therefore, the state of the recommender intelligence in the environment $S_t = f\left(h_t\right)$ can be obtained, where $f(\cdot)$ is the discretization processing function, and also $S_t, h_t \in R^{k\times i}, k$ is the number of knowledge points of the current subject of the learner in the process of learning on the platform . The learner knowledge implicit state h_t represents the overall knowledge point mastery of the student for the current learning subject, which is the current knowledge level state of the learner.

The process of modeling student knowledge state is shown in Figure 4.

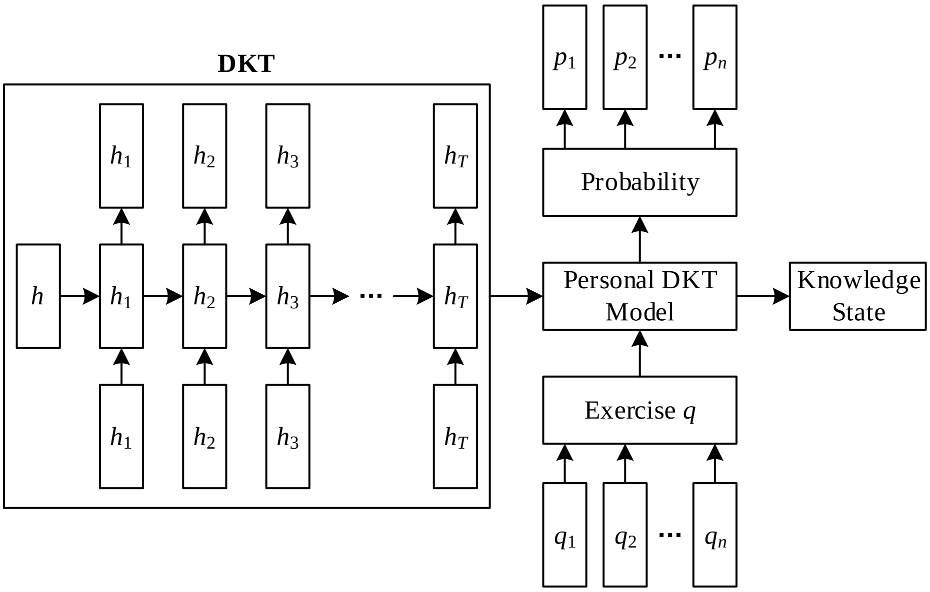


Figure 4: Modeling the students' knowledge status in the knowledge tracking part

3.2.3 Reward function design

In the present research paper, when we make the reward, we consider the knowledge proficiency level of the user. Furthermore, we extract the reward expression into the whole capability of the suggested exercise group for the present learner.

The representation of the reward function is shown in Equation (11):

$$r_t = \frac{1}{N}\sum_{i=1}^{N} f\left(q_i\right) \tag{11}$$

where N is the number of exercises in the current recommendation set $\{q_1, q_2, \ldots, q_n\} \in Q_n$, and q_i is the exercises in the recommendation set, and in particular, $f(\cdot)$ is denoted as the probability that the current user answered the question correctly, which process is determined by the user's s knowledge state matrix computed by interacting with the current exercise, represented as shown in Equation (14):

$$h_t = \tanh\left(W_{h_q} q_t + W_{hh} h_{t-i} + b_h\right) \tag{12}$$

$$y\left(q_i\right) = \sigma\left(W_{yh} h_t + b_y\right) \tag{13}$$

$$f\left(q_i\right) = probability\left(y\left(q_i\right)\right) = y\left(q_i\right) \tag{14}$$

where tanh is the tanh function, σ is the Sigmoid function, W_h, W_{yh} is the current user's mastery state of the current learning knowledge matrix, h_t is a matrix derived from the student's history of doing the problem or the data set of the training of the user's personal state of knowledge. , b_h is the bias of the hidden layer, and W_{hh} is the recursive weight matrix.

In particular, W_{yh} is the input weight matrix and b_y is the bias of the output layer.

At the same time, the sum total of rewards got from intelligence exploration is expressed as the sum of discounted rewards.

$$G_t = r_t + \sum_{n=1}^{N-n} \gamma^n r_t \tag{15}$$

where r_t is the current immediate reward for the intelligent body's exploration, followed by a cumulative sum of future reward accumulations, γ is the discount factor, and $\gamma \in [0,1]$.

3.2.4 Construction and Training of Recommendation Model KBRL-Rec

In the KBRL-Rec model which belongs to reinforcement learning, the intelligent body of the recommender makes policy selections through considering both the knowledge state of the learner and their recent response behaviors, which can be observed as a sequence of current actions and observations:

$$S_t = a_1\left(Q_{n_1}\right), s_1, a_2\left(Q_{n_2}\right), s_2, \ldots, a_{t-1}\left(Q_{n_{t-1}}\right)\Big), s_t \tag{16}$$

where Q_{n_i} is the currently generated action i.e. recommendation set, i.e. $\{q_1, q_2, \ldots, q_n\}$, i.e. the action of the recommending intelligences a_t, s_t is the current state of the learner's knowledge. Therefore, thus, the current procedure is formulated by us to be a Markov decision-making procedure. We put forward the hypothesis that the present stage of test problem proposing and studying will come to an end, hence the sequence is not continuous. This arrangement makes possible the employment of reinforcement learning methods to solve the current problem.

The situation of the environment, that is to say, the situation of the intellects, is described as the current knowledge situation of the current learner, denoted as s_t, t is the current time window, the knowledge state is obtained from the trained knowledge model, i.e., $s_t = f\left(h_t\right), h_t$ is the tacit knowledge state of the learner of the knowledge model, and $f(\cdot)$ is the discretization processing function.The condition of the environment experiences continuous changes together with the learner's study process and the procedure of replying to questions. This kind of advancement is obtained by means of regular, time-linked training exercises.

The set of actions is the entire teacher uploaded exercises in the current learning phase of the learner in the exercise repository $\mathbb{A}_{\mathrm{T}}$, and each action selected by the intelligent body a_t is the set of exercises to be recommended $\{q_1, q_2, \ldots, q_n\} \in Q_n$, while $Q_n \in \mathbb{A}_T$.

In terms of reward representation, this paper adopts Monte Carlo (MCMC) algorithm for training the intelligent body to explore.

Figure 5 gives a schematic description of the structure of the overall reinforcement learning recommendation model, KBRL - Rec.

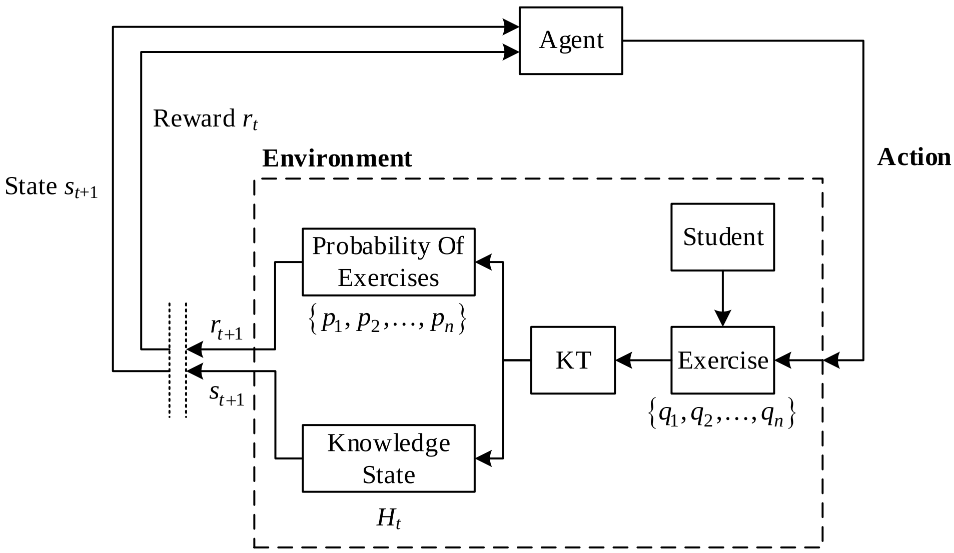


Figure 5: Structure of reinforcement learning recommendation model KBRL-Rec

After we have done the above-mentioned group of analyses, a test question suggestion algorithm which is based on reinforcement learning has been established. The detailed flow procedure of this algorithm is given in what follows:

Input: students' historical answer sequences, list of test questions $\mathbb{A}_T$.

Output: intelligent body action selection strategy $\pi(s)$.

(1) Initialization: current intelligent body environment, experience pool ψ , action selection strategy $\pi(s)$, intelligent body state s , action a , reward $r(s,a)$. where $\pi(s) \in \mathcal{A}(s), r(s,a)$ is empty and $\mathrm{s} \in \mathcal{S}, a \in \mathcal{A}(\mathrm{s})$.

(2) For training the obtained current user knowledge state model $Model_t$:

(3) Obtain the current user knowledge state h_t .

(4) Obtain the current state s_t of the intelligent body based on the user knowledge state h_t and discretize it.

(5) Obtain the set of exercises $\mathbb{A}_T$ according to the current learning stage of the current learner.

(6) Loop execution until the end of the round.

(7) Randomly take action a according to the user's current knowledge state according to the $\epsilon - Greedy$ randomized strategy approach, i.e., $\{q_1, q_2, \ldots, q_n\} \in Q_n \left(Q_n \in \mathbb{A}_T\right)$.

(8) Calculate the user's mastery of each question in the current action $\{p_1, p_2, p, \ldots, p_n\}$ separately according to equation (14).

(9) Calculate the reward payoff r for taking the action $a \in \mathcal{A}(s)$ in the current state $s \in S$ according to equation (11).

(10) Simulate the current learner to answer questions and input the answer records into the current DKT user knowledge state model $Model_t$ and obtain the current user knowledge state h_{t+1}.

(11) Train the learning using Q-learning algorithm, calculate the cumulative return G according to Eq. (15), and update the action selection policy $\pi(s)$.

(12) Store the explored local sequence data into the experience pool ψ.

(13) End the loop.

3.3 KBRL-Rec model performance testing

3.3.1 Data preparation

For verifying the effect of the KBRL-Rec model on the recommendation of higher mathematics test questions, this paper uses the industry public dataset skill builder data 2009-2010 provided by Assistment and the data mining competition dataset ADM-2017 by Assistment in 2017 for model testing.

(1) Knowledge state

Learning engagement describes the learner's learning effect over a period of time, and is calculated from the corresponding knowledge state after answering at each moment. Without applying the recommendation algorithm in this paper, the learning factor $\alpha = 0.7$, and the time period N=6, the change of engagement in a segment of the intercepted skill-builder dataset is shown in Fig. 6. We can observe that the participation degree has a steady decrease after Step 4. This downward tendency continues until Step 14, at this time the participation starts to get back at a slow speed. Following that, the degree of participation still changes in a not consistent way.

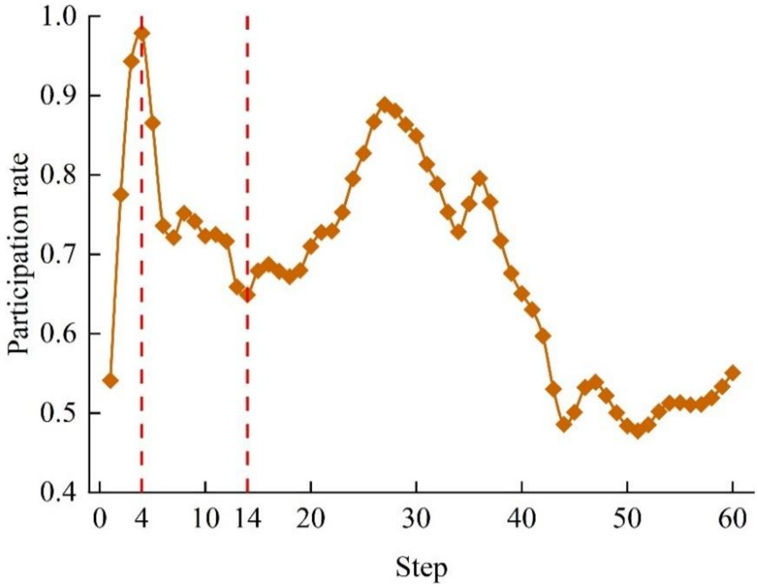


Figure 6: Changes in participation

(2) Comprehensive Difficulty of Knowledge Points

Study advancement means the change in the complication of the knowledge items that are being studied. When the change of complexity is excessively big, it thus can cause a drop of the learning result. For the purpose of the promotion of the learning experience that learners obtain, it is a necessary thing that the change of complexity must be as smooth without gap as it can be. Figure 7 shows the distributing of the combined complicated degree of the knowledge points which are in the skill-builder. According to what we can see, the total complication of the knowledge points is distributed over many different complication intervals. The major part of them are located in the scope of 0.3 - 0.8. This kind of distributing method can satisfy the demands that algorithm application puts forward, and therefore it can be used as the source data of experiment.

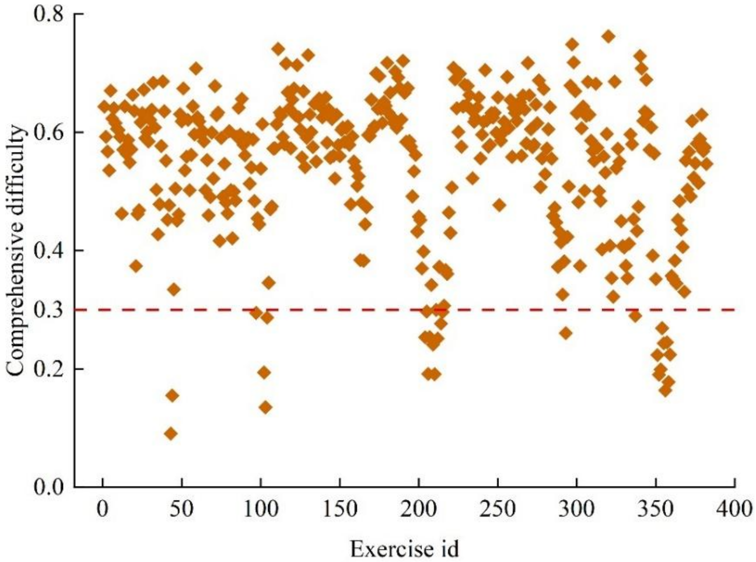


Figure 7: Distribution of comprehensive difficulty

(3) Knowledge point correlation

The knowledge points of a subject are not independent of each other, but have a forward and backward relationship, and this relationship between two knowledge points can be expressed by J_{ij}. Learning directionality refers to the use of forward and backward relationships between knowledge points to plan learning paths, thereby enhancing learning efficiency. The knowledge states output from the knowledge tracking model are processed to obtain the knowledge point relevance matrix $Y(ij)$, which in turn draws the domain knowledge relationship graph.

The knowledge status, comprehensive difficulty and knowledge relevance generated from the learning process dataset are the source data for the three learning objectives. After completing the data preparation, the following tests the model performance.

3.3.2 Performance testing process

(1) Validation of Decay Factor

Just as what is shown in (8), the attenuation factor is to measure the influence that the forward state exerts on the current state. This kind of influence becomes smaller and smaller when the interval of time becomes longer. Therefore, hence, if the Q-value of one present action holds the highest position, thus this action represents the optimal resolution for the whole system.

For verifying that the decay factor has effect when we select the most fitting action,an experiment is designed to compare the average reward values at $\gamma = 0$ and $\gamma = 0.8$. In this experiment, the KBRL-Rec model was used to simulate learners answering questions, and the learning factor $\alpha = 0.7$ was set according to the overall difficulty of the corresponding knowledge points of the exercises, and the length of the recommended sequences in each round was 12. The changes of the average reward value at different values of γ during the training period are shown in Fig. 8, the flat axle stands for the number of training episodes, and the upright axle expresses the average reward value for every single episode.

In the early stage, although the reward value is higher when $\gamma = 0.8$, the difference between the two is not obvious, and in the later stage, with the continuous optimization of the model, the advantage of $\gamma = 0.8$ is gradually obvious, and reaches more than 1.24 at the highest, and the part of the higher than $\gamma = 0$ has been maintained at more than 0.1. The experimental results show that global optimal recommendation ($\gamma = 0.8$) is more suitable for personalized recommendation than heuristic recommendation ($\gamma = 0$).

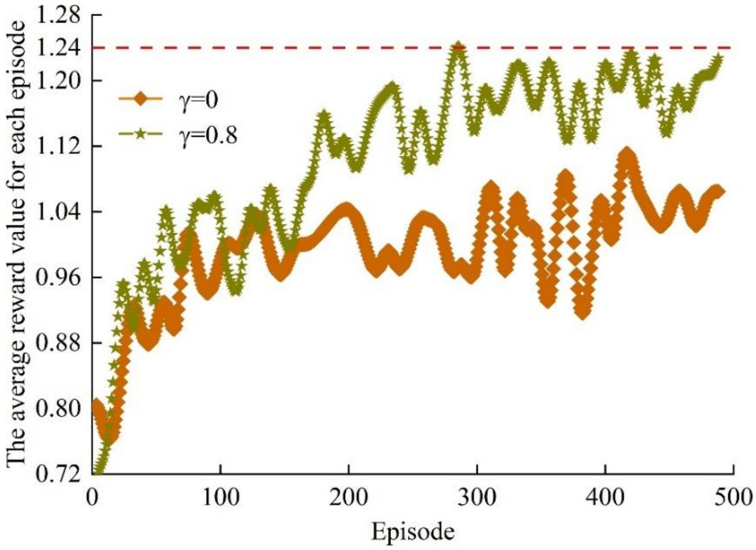


Figure 8: Changes in the average reward values for different γ values

(2) Overall model performance test

The change of Loss during the training process of the KBRL-Rec model recommended by the exercises built in this paper is shown in Fig. 9.The Loss value gradually decreases and converges to 0 during the training process, and the realization effect is in line with the expectation.

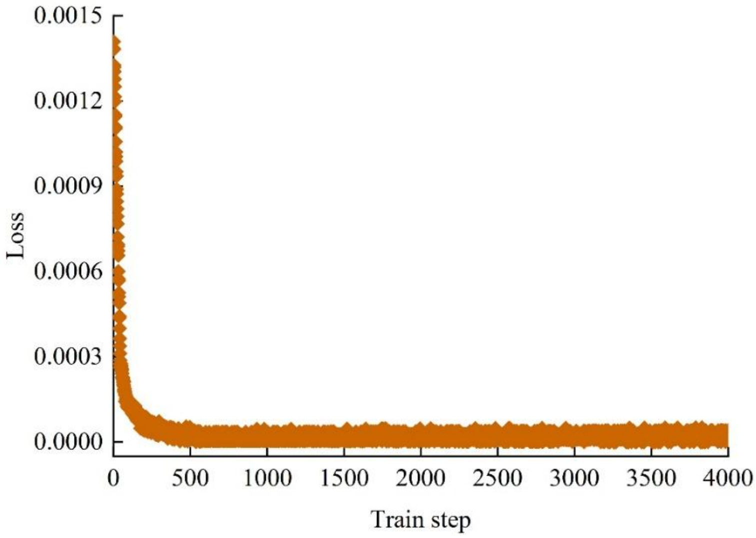


Figure 9: The variation of Loss during the model training process

Because the learning path is continuous and all recommended exercises affect the learning process, the path length may be a factor that affects the overall learning effect, and testing the appropriate length of the recommended sequence is part of the model optimization. After that, the relation between the length of the proposed exercise arrangement and the reward value has been checked and confirmed.

Figure 10 shows the change curves of the average reward value when the length L of the recommendation sequence is 10, 12, and 15. It can be seen that L=15 in most cases, its average reward value is smaller than L=12, its reward maximum value of 0.114 is lower than the 0.123 of L=12, its reward value fluctuation is larger, and the upward trend is relatively slower with L=12. The upward trend of L=10 is also not obvious, and the maximum value of 0.111 is much lower than the 0.123 of L=12, and at the same time, when L=10, the model training effect is not stable and the reward value fluctuates greatly. Therefore, based on the comprehensive consideration of model stability and functional reliability, L=12 is selected as the most appropriate recommended length for each episode of the model.

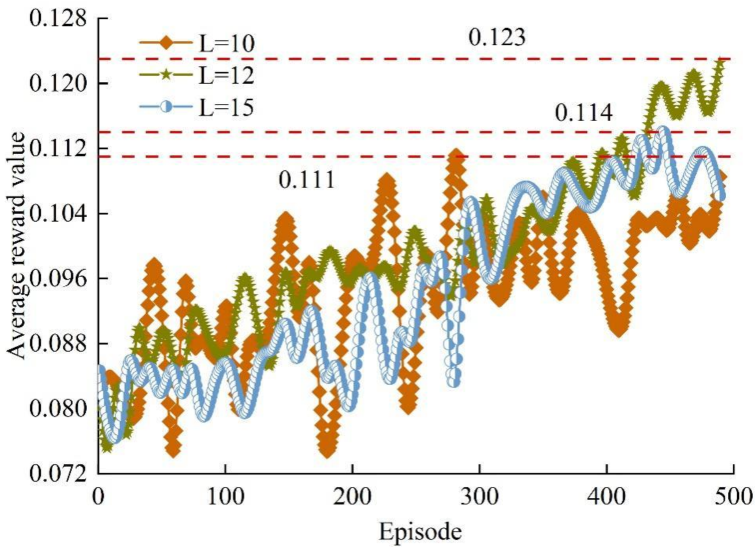


Figure 10: The average reward value changes when L=10, 12, 15

For the purpose of verifying the advantages of the KBRL-Rec algorithm, in this experiment, it has been compared with the average reward numerical value of the stochastic recommendation method. Both of them have the same parameters except for the recommendation mode, including the length of the recommendation sequence per episodes L=12, $\gamma = 0.8$, and the learning factor $\alpha = 0.7$.

Figure 11 gives the results obtained from the comparison that is done between the random recommendation mode and the KBRL - Rec algorithm. It is very obvious that in 500 training episodes, the average reward numerical value of the random recommendation method has been undergoing oscillation between 0.64 and 1.20. Furthermore, we can find that there does not exist any consistent rising development path. While KBRL-Rec algorithm's average reward value has been increasing from 0.64 to about 1.26 in 500 episodes of training, and the upward trend is obvious. In nearly all time points, the reward value which the personalized recommendation algorithm provides is higher than that which the random recommendation algorithm provides. Under the background of adaptive learning, the recommendation algorithm which this paper has developed obviously has better performance than the random recommendation on the ability of personalized recommendation.This indicates that with the continuous optimization of the model, the learning direction of the answer sequence generated by the recommendation algorithm overlaps with the overall learning goal of the algorithm.

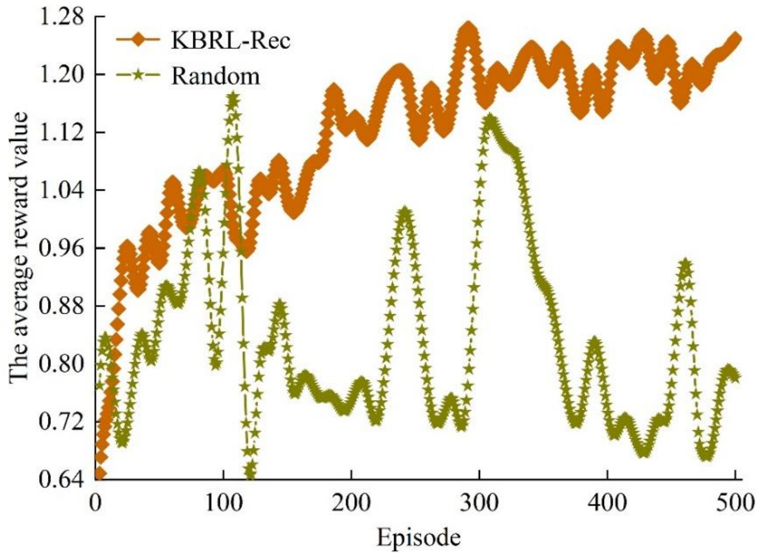


Figure 11: Random recommendation method and KBRL-Rec algorithm

For carrying out a more profound confirmation of the superiorities of the KBRL-Rec algorithm with respect to its capability to study directionality and progression in the recommendation process, comparative experiments are designed to compare the average relevance and average integrated difficulty fluctuation of the exercises recommended by KBRL-Rec algorithm and randomized recommendation approach. The experiment consists of a total of 60 Episodes, and the recommended sequence length of each Episode is L=12.

Table 1 gives the results of our contrast experiments. The average relevance degree of the KBRL-Rec algorithm is about 0.0297, much higher than 0.0084 of the random recommendation method, and the average comprehensive difficulty fluctuation is about 0.0325, much lower than 0.2136 of the random recommendation method, which suggests that in the process of personalized recommendation, the KBRL-Rec algorithm integrates both the considerations of learning directionality, and the recommended and forward exercises are more relevant to the corresponding knowledge points. This indicates that in the process of personalized recommendation, the KBRL-Rec algorithm integrates both learning directionality, recommending exercises with higher correlation with the corresponding knowledge points, and learning progressivity, reducing the difficulty fluctuation of adjacent exercises. Compared with randomized recommendation, the KBRL-Rec algorithm proposed in this paper has better stability and accuracy.

Table 1: Comparison of experimental results

	KBRL-Rec algorithm	Random recommendation method
Average relevance of recommended exercises	0.0297156427	0.0083540653
The average difficulty of the recommended exercises fluctuates	0.0324705612	0.2135675241

When all above-mentioned tests are got into consideration, it is thus clear that the total result obtained by the KBRL-Rec model, which puts reinforcement-learning interactive situations into the adaptive learning procedure, reaches the expected levels.

4 An empirical study of precise teaching interventions in the flipped classroom in higher mathematics

This chapter builds a generalized framework for precise instructional interventions based on KBRL-Rec, a model for recommending test questions for reinforcement learning, by means of empirical study, it has proven the appropriateness of the proposed overall framework for correct teaching interventions in the process of carrying out flipped classroom teaching in high-level mathematics.

4.1 Study population and data

4.1.1 Objects of study

In this study, 54 students in a class in the first year of college A were selected as the research object. In the higher mathematics instruction that is given to first-year college students, the class uses an intelligent online teaching and tutoring platform to support the learning process. The teacher has arranged flipped classroom lectures according to the teaching tasks and the plans of higher mathematics. After the completion of the lecture activities, the teacher has assigned the related practice questions. After that, students on the online platform carry out practice of solving problems.

Figure 12 shows the application frame of accurate teaching intervention which is built on the KBRL - Rec model that this paper develops. This process follows a series of definite steps: collection and handling of data, which is followed by the utilization of models and methods, then the making of accurate teaching intervention strategies, and finally the carrying out of accurate teaching intervention. Through carrying out interference teaching experiments upon advanced mathematics practice problems, the system solves the optimum decision-making function for practice interference. This goal is reached through utilizing the learning procedure data which is collected from students' finishing of exercise drills in the period of experiments. The result is an intelligent, active, and accurate exercise interference mechanism which helps students for promoting their mathematics study scores.

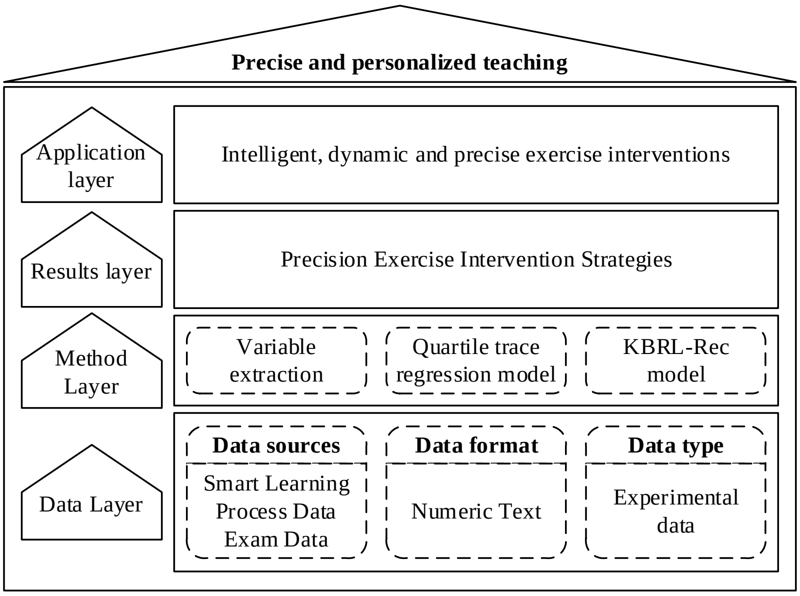


Figure 12: Application framework for precise teaching intervention

4.1.2 Data sources

The data that are used in this experience research are got from two main roads: students' study records from the intelligent network teaching and guidance platform and data from the advanced math class room. The study record files from this platform are the data of students' study process. These data include many different measurement items, such as the number, time length, and correct ratio of students' practice on different kinds of questions, including practice questions, location searches, and questions that are both easy to master and easy to make mistakes, among other targets. The time scope of these records covers the whole first-year university student academic year. To speak concretely, the study records include the data from students who already have finished 15 exercises on the network platform, and this amounts to a total of 810 pieces of records. On the other hand, the data of advanced mathematics classroom is constituted by the results of advanced mathematics tests and the corresponding rank of each grade. These exam outcomes come from three particular examinations: the entry test which is held one month after the academic year begins, the end term exam of the first semester, and the end term exam of the second semester.

4.2 Research Methodology and Process

In this study X is a vector consisting of variables extracted based on the students' learning process data, i.e., the total sum of questions, the whole correctness rate, the average time spent, the correctness rate of position-seeking queries, the correctness rate of questions which are easy to study and memorize, and the correctness rate of learning-related questions, A is the exercise intervention, and Y is the inverse-order grade rank of the students' performance in Advanced Mathematics. For illustrative purposes, this study uses semesters as a natural division, dividing the freshman year into two phases: the first school semester and the following school semester. The instructional interventions A_1 and A_2 for the two phases take the values of 0 or 1 to indicate whether or not the exercise intervention strategy was adopted in the corresponding phase, with 1 indicating practicing more than a quantitative number of study problems and 0 indicating practicing no more than a quantitative number of study problems.

Based on the data which was collected from the teaching experiment of exercise intervention, the accurate teaching intervention framework which is built on the KBRL - Rec model that this paper develops is used to solve the decision function of accurate exercise intervention. This hereby permits the formulation of a movement interference scheme and the execution of accurate movement interference.

4.3 Findings and analysis

4.3.1 Overall analysis of intervention effects

Figure 13 gives the contrast of the higher mathematics marks of the students inside the class. These achievement results come from the entrance examinations, and also from the end-of-semester examinations of the first and second semesters separately.

In the outcome of the entrance examination, the marks of students on the whole approached a normal distribution, with merely a small quantity of low marks dropping inside the 20 - 30 scope. When we talk about the final examination of the first semester, the whole distribution of students' score points was quite similar to that of the entrance examination. However, in the terminal examination of the second academic semester, there existed a quite obvious rise in the quantity of students who gained high scores, while the quantity of those students who possessed low scores had a big reduction. Through the analysis of the changes of students' score points among these three test papers, it is very clear that the whole study achievement of the students in this class has had an obvious promotion during the whole school year when the Precision Exercise Intervention was put into practice.

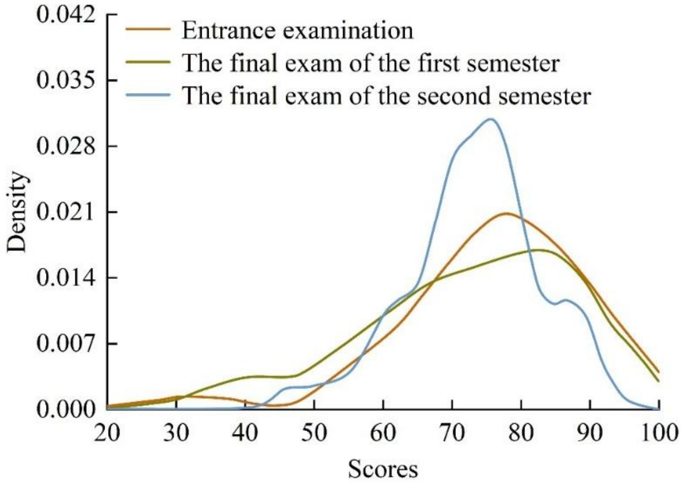


Figure 13: Comparison of advanced mathematics scores in the three exams

At the same time, because of the differences in question difficulty among different exams, this research therefore also investigates the rank of students' advanced mathematics grades within their class. This class in total has 1649 students. Figure 14 give a comparison of box-and-whisker diagrams for the grade orderings of the mathematics marks of 54 students in that class across three examinations: the entrance test, the first-semester end examination, and the second-semester end examination. It can clearly be seen that, the grade level orderings of the marks from the three tests increased one by one, and hence there is an obvious ascension in the grade level ordering of the marks from the second-semester end-of-term test.

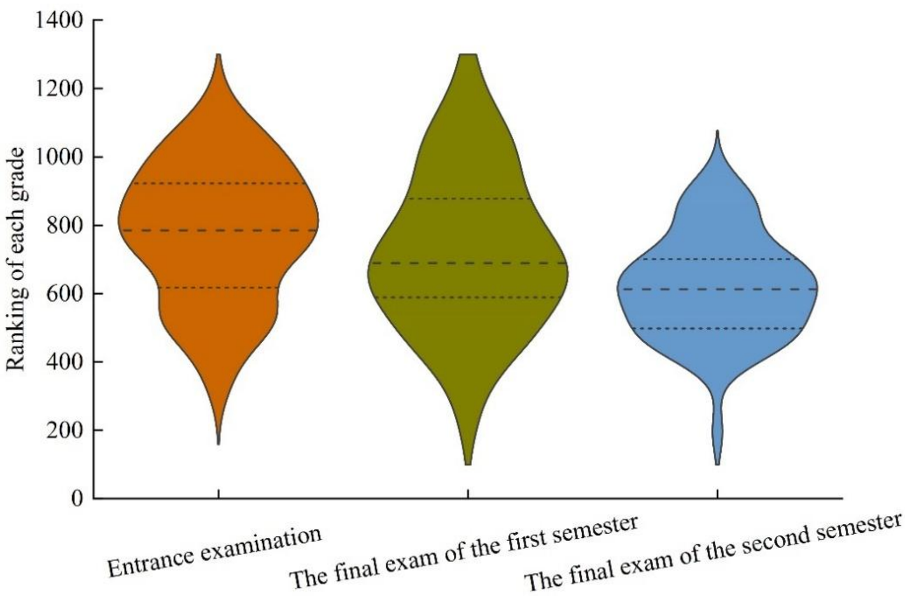


Figure 14: Comparison of grade rankings in the three advanced mathematics exams

We have implemented a paired-sample t-test upon the grade rankings of the three examinations of this class, hence we carry out the comparison between them one pair by one pair. The p-values that express the significance probabilities have been ascertained by us as 0.045, 0.004, and 0.213 respectively. Therefore, under the 0.05 significance level, the grade rankings of the entrance examination and the final examinations of the first and second semesters have obvious differences.

In conclusion, after a one-year directional movement interference, the students of this class have seen an obvious promotion in their whole study results.

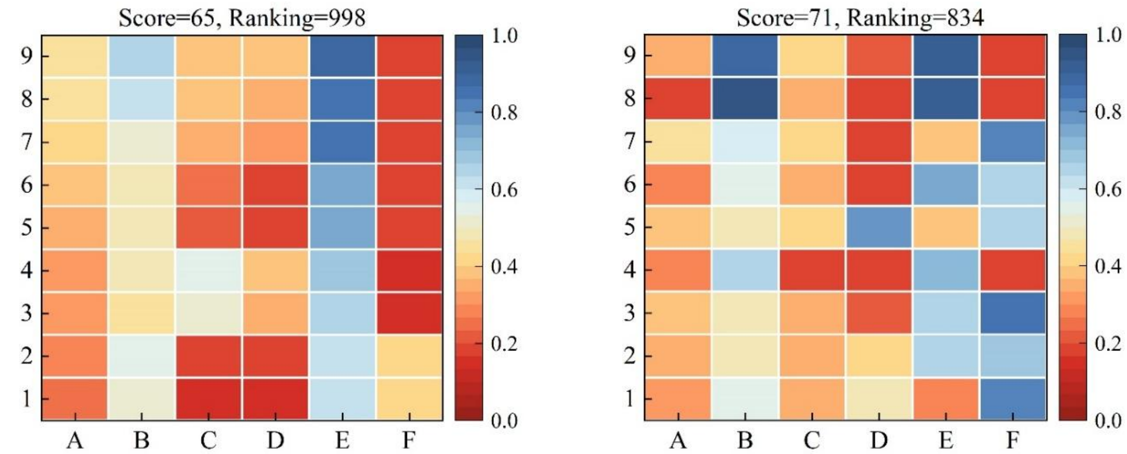
4.3.2 Intervention effectiveness case studies

For the purpose of carrying out a more penetrating analysis regarding the special influences that purpose-oriented teaching interventions under the flipped classroom pattern exert on students' learning situations and characteristics, this paper randomly selects some students from each of the four different exercise intervention strategies, and comparatively analyzes the changes in their learning images during the two phases.

(1) Exercise intervention strategy $A_1 = 1, A_2 = 1$

One individual student was selected by random method from the group. Figure 15 shows the study situation of the practices that this student finished in two semesters.In this aggregate, (a) and (b) separately represent the learning visual graphs for the first and the second study terms. Every single row is corresponding to one alone practice session, and every single column is representing one different variable. The six rows of numbers stand, following order, for the total question number, the total correct percentage, the mean used time, the correct rate for finding questions, the correct rate for questions that are simple to study but easy to make mistakes, and the correct rate for study questions. These variables are separately given names from A to F, and this naming method is kept unchanged from now on.

It is obvious that after the students carried out a large amount of practice for study questions in two semesters, the accuracy of locator questions, questions which are easy to learn but easy to make mistakes, general study questions, and the whole accuracy of the students' practice related to the topic all have shown a rising tendency. It is worth pointing out that the accuracy rate of the research questions has presented a quite big increase. This shows that the student's total ability to solve exercises has gotten better, especially in the aspect of finding out and correcting knowledge empty spots via practice. The student's advanced mathematics mark increased from 65 at the finish of the first semester to 71 at the finish of the second semester, hence it brought an improvement of 164 places in the grade rank. Carrying out an extra-large quantity of exercise problem training for two continuous semesters had an obvious influence on promoting the student's mathematics study achievement.

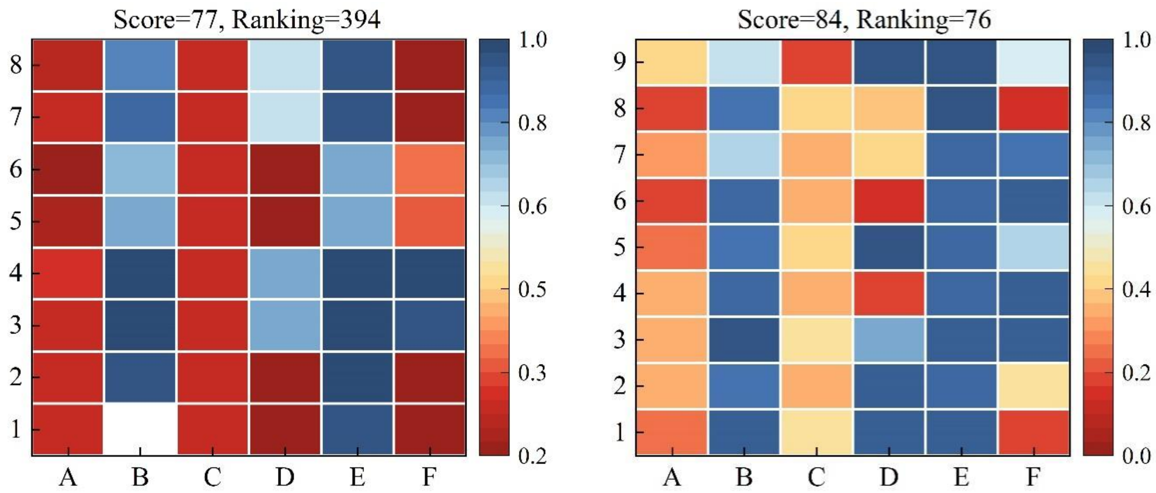


(a) The first semester (b) The second semester

Figure 15: Students learn image comparison of exercise intervention strategy A_1=1, A_2=1

(2) Exercise intervention strategy $A_1 = 0, A_2 = 1$

One single student was randomly selected from the group. Figure 16 gives the depiction of the learning profile of this student, who participated in exercise practice through the whole two semesters. In the first semester, it is very obvious that the student has obtained a high accuracy ratio and an overall high correct-answer ratio for questions which are easy to study but easy to make errors on. Nevertheless, the correct rate for position-finding questions and research questions was comparatively low. In the second semester, after a high-intensity, ultra-quantitative training on examination questions, the student presented a noticeable enhancement in the correct rate for position-finder and examination questions. Furthermore, the total correct answer proportion has been maintained at a relatively high level. The student's score has the ascent from 77 at the first semester's end to 84 at the second semester's end, and their grade rank has the progression of 318 positions. According to these observation results, we may tentatively make inference that this student possesses a relatively quick speed when he carries out the acquisition of new knowledge. Nevertheless, therefore, certain issues may exist in the students' grasping of knowledge points. The ultra-many practice on study questions can help the student to find out holes in their knowledge and promote the student's study of high-level mathematics concepts.

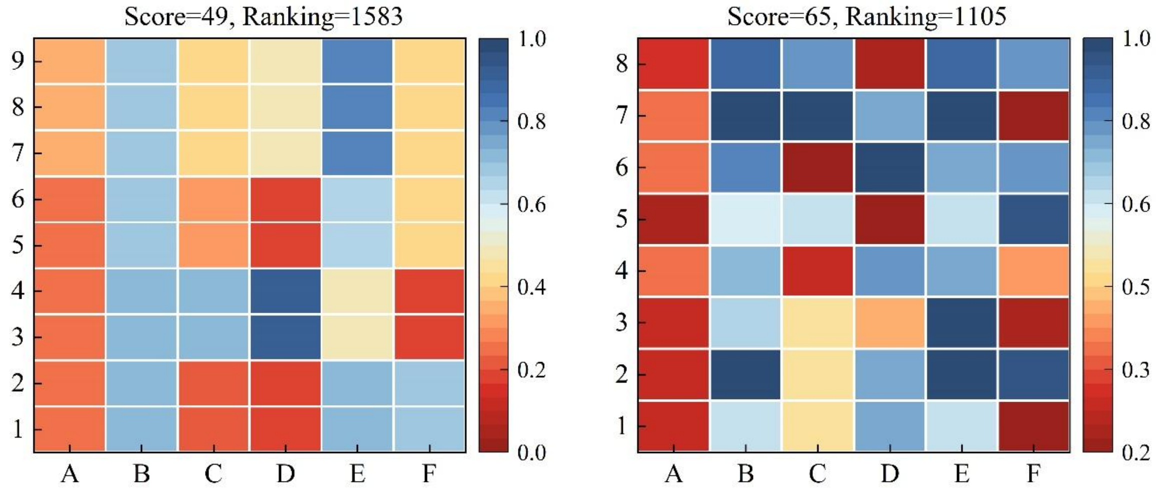


(a) The first semester (b) The second semester

Figure 16: Students learn image comparison of exercise intervention strategy $A_1=0, A_2=1$

(3) Exercise intervention strategy $A_1 = 1, A_2 = 0$

One student was randomly selected from this group, and the learning images of the two semesters in which the student completed the study problems are shown in Figure 17. It can be seen that the student's first semester of exercise practice had relatively fair correct rates for all items, indicating that practicing super-quantitative study questions had some effect. In the second semester, the student's average length of the second semester was relatively high, and the percentage of correctness of locator questions, easy-to-learn and easy-to-fail questions, study questions, as well as the total percentage of correctness increased compared to the first semester. At the same time, the student's grade went from not reaching the passing line at the end of the first semester (49 points) to reaching the passing line at the end of the second semester (65 points). This shows that there are weaknesses in the student's basic grasp of the knowledge points, but it does not mean that the student should be practicing more than a quantitative number of study questions, which should also be considered in the context of the average length of study.

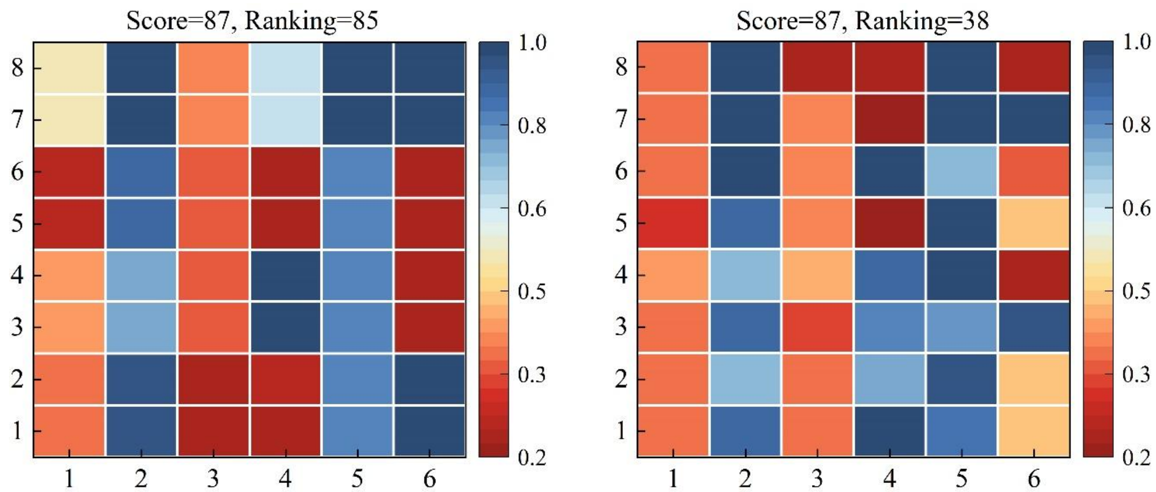


(a) The first semester (b) The second semester

Figure 17: Students learn image comparison of exercise intervention strategy A_1=1, A_2=0

(4) Exercise intervention strategy $A_1 = 0, A_2 = 0$

One student was selected by random, the descriptions of learning for the exercises that this student finished in two semesters are showed in Figure 18. It is very clear that, this student got a quite high correct rate for all practices in the whole two semesters, and the average study time length was comparatively short. Therefore, it was not needed to carry out the practice of more study questions after a certain quantity threshold for the reason of finding and correcting knowledge gaps. In terms of advanced math scores, the final exam math score was 87 in both semesters, but the grade rank improved by 47 places.



(a) The first semester (b) The second semester

Figure 18: Students learn image comparison of exercise intervention strategy A_1=0, A_2=0

Taken together, for these four randomly selected students, the implementation of suitable exercise interventions can increase the correct rate of exercise practice and improve exercise practice. It is very clear that carrying out pointed exercise interference measures according to particular situations can help students to promote their academic results and speed up their study of higher mathematics.

5 Conclusion

In this research treatise, we have constructed a flipped classroom teaching model for high-level mathematics. We have put forward a precise teaching intervention framework which is for the flipped classroom of advanced mathematics. This frame ground work is built on KBRL - Rec, which is a reinforcement learning exercise recommendation model. After that, we conducted an empirical research on the effect of this framework.

The KBRL-Rec algorithm has a significant upward trend in the average reward value in 500 episodes of training, from 0.64 to about 1.26. On the other hand, the average reward value of the random recommendation approach fluctuates between 0.64 and 1.20, and there is no sustained upward trend. It indicates that the personalized recommendability of the KBRL-Rec algorithm proposed in this paper is much better than that of the random recommendation method, and with the continuous optimization of the model, the learning direction of the answer sequences generated by the recommendation algorithm overlaps with the overall learning goal. Meanwhile, the average correlation of KBRL-Rec algorithm is about 0.0297, which is much higher than that of 0.0084 in the random recommendation approach, and the average integrated difficulty fluctuation is about 0.0325, which is much lower than that of 0.2136 in the random recommendation approach, which indicates that the stability and accuracy of the KBRL-Rec algorithm proposed in this paper are better compared with that of random recommendation.

One comprehensive instructional intervention framework that is based on the KBRL-Rec model can give intelligent and accurate instructional intervention selections in the teaching practice process. These selections have as the goal promoting students' academic study accomplishments. Furthermore, this frame can be put into practice in the education environments of flipped classrooms for advanced mathematics courses.

The flipped classroom teaching pattern of advanced mathematics and the accurate teaching interference frame which this research has set up have got good application results. However, the putting of them into teaching practice is still staying at the phase of small-scale experiment. Therefore, it is extremely necessary to enlarge case researches on the usage of accurate education in many different situations. In addition, the carrying out of comparison and verification work on the influences that precision teaching interventions bring within different education environments is a thing that is necessary. This will be helpful for the pushing forward of the flipped classroom teaching model and precision teaching interventions, and thus also for the wide use of these methods.

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