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SUMMARY: *The effect of the dimensions and locations of the openings and the influence of inclination on dynamic response of nine Persian historical brick masonry minarets constructed in the eleventh to fourteenth centuries is studied. The operational modal test was performed on a minaret constructed in the laboratory and the frequencies were obtained by the ARTeMIS software, and the ANSYS software was used for updating its finite element model. A range was determined for the values of the modulus of elasticity and bulk density to obtain acceptable ranges for the frequencies of the minarets. Then finite element model updating of the historical minarets was performed to determine their frequencies. Several dimensions and locations were assumed for the openings and two inclination angles were considered to study their effect on the frequencies. Minarets were then subjected to earthquakes in order to investigate the effect of openings and inclination on their seismic response.*

KEYWORDS: *brick masonry minaret, operational modal analysis, seismic response, updated finite element model, vibration test*

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1 Introduction

Operational modal analysis technique is frequently used in order to determine dynamic properties of historical masonry structures, as a non-destructive test. Results obtained from this technique can be used for updating the finite element model of the structure for further analysis. The finite element model updating based on operational modal analysis results has been widely used for historical masonry minarets.

In 2007, Gentile and Saisi [Gentile and Saisi, 2007] used ambient vibration testing for structural identification and damage assessment of a number of historical masonry towers. They identified the modal characteristics of the towers by ambient testing and then finite element model updating. In 2009, Bayraktar et al. [Bayraktar *et al.*, 2009] used ambient vibration test and operational modal analysis to identify the modal parameters of Hagia Sophia bell tower. Natural frequencies and mode shapes were determined by operational modal analysis and finite element model updating was used by changing the modulus of elasticity and boundary conditions. In 2011, Bayraktar et al. [Bayraktar *et al.*, 2011] used operational modal testing together with finite element model updating to study the seismic response of a historical masonry minaret. The ambient vibration test was performed on the minaret under natural excitations such as wind loading and human movement. The modulus of elasticiy and boundation conditions were changed for finite element model updating. In 2016, Hacıefendioğlu et al. [Hacıefendioğlu *et al.*, 2016] used operational modal analysis to determine the modal parameters of historical masonry minarets. Natural frequencies and mode shapes were obtained from operational modal analysis of the minarets under ambient conditions and finite element model updating was used to calibrate the model. In 2017, Bongiovanni et al. [Bongiovanni *et al.*, 2017] conducted experimental vibration tests on a historical masonry tower. Results were compared with previous experimental tests. Dynamic parameters of the tower obtained from vibration tests were used for finite element model updating of the tower. In 2018, Erdil et al. [Erdil *et al.*, 2018] investigated the effects of structural parameters on seismic behaviour of a historical masonry minaret damaged in the earthquake. They conducted vibration tests and determined the dynamic parameters of the minaret by the operational modal analysis. Then they created a finite element model of the minaret and updated the model by introducing the cracks according to the results of operational modal analysis. In 2019, Nohutcu [Nohutcu, 2019] investigated the seismic failure pattern of a historical masonry minaret under earthquakes. He used finite element model updating based on operational modal analysis and ambient vibration test and identified the vulnerable parts of the minaret against earthquakes.

In 2020, Alpaslan et al. [Alpaslan *et al.*, 2020] used operational modal analysis and surface-based finite element model updating to study a historical masonry minaret. The operational modal analysis technique was used to determine the modal parameters of the minaret from ambient test. Then the finite element model was developed and updated by changing materials properties and boundary conditions to get modal parameters similar to the experimental results. In 2020, Hökelekli et al. [Hökelekli *et al.*, 2020] studied the damage pattern of a historical masonry minaret under different ground motions by updating finite element models based on results obtained from operational modal analysis test. The updated paremeters were the modulus of elasticity and boundary conditions. In 2021, Altiok and Demir [Altiok and Demir, 2021] investigated the collapse mechanism of a historical masonry minaret considering soil-structure interation. Vibration test and operational modal analysis were used to determine the dynamic characteristics of the minaret. The finite element model of the minaret was updated according to experimental results. In 2021, Calayır et al. [Calayır *et al.*, 2021] used operational modal analysis method and finite element model updating to study the dynamic characteristics of masonry minarets. They considered soil-structure interaction in the finite element model. The finite element model updating was carried out by comparing the frequencies and mode shapes with experimental results.

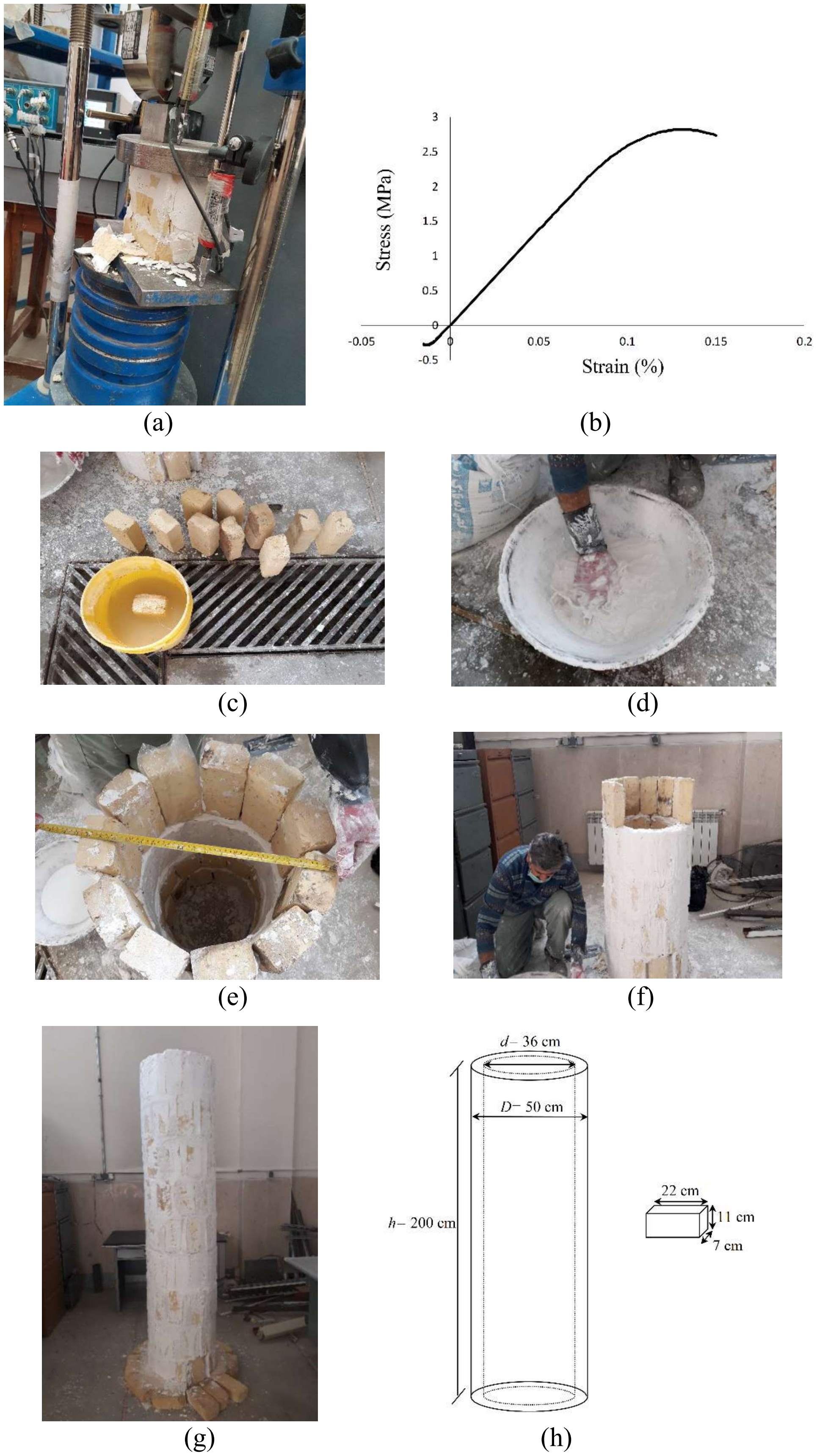
In 2022, Nastri and Todisco [Nastri and Todisco, 2022] studied three different failure critera for masonry structures. They conducted experimental tests and analysed their model with the finite element method. The concrete damaged plasticiy failure criterion showed that it is able to model the behaviour of the masonry structures in a promissing way. In 2023, Nastri et al. [Nastri *et al.*, 2023] investigated the capability of the concrete damage plasticity failure creiterion by using finite element modelling of typical tuff masonry constructions. Satisfacory resultes were obtained.

The minaret has always been one of the most important architectural elements in Persia. The history of minaret construction in Iran dates back to around two thousand years ago [Hejazi, 1997, Hejazi and Mehdizadeh Seradj, 2014, Hejazi *et al.*, 2015]. In order to identiy the modal parameters and investigate the seismic behaviour of a number of Persian historical brick masonry minarets by using the finite element model updating technique based on operational modal analysis results, this study is carried out. The study includes four stages. In the first stage, in order to perform vibration analysis, a brick masonry minaret was built in the laboratory. By installing sensors on this minaret, operational modal test was performed. In the second stage, the experimental frequency was determined using the ARTeMIS software [ARTeMIS Extractor Pro, 2014] and then the frequency was determined using the ANSYS software [Basic Analysis Guide for ANSYS 18, 2017], and the finite element model (FEM) was updated by changing the modulus of elasticity and bulk density. In the third stage, the modal behaviour of some minarets located in Isfahan, which date back to the eleventh to fourteenth centuries A.D., considering the dimensions and locations of the openings as well as the inclination of the minaret were studied. In the fourth stage, the seismic behaviour of the minarets and the effect of opening and inclination was investigated.

2 Operational modal tests

2.1 Brick masonry minaret constructed in the laboratory

A minaret was constructed in the laboratory for operational modal test and then finite element model updating. The minaret had a height h=200 cm, an outer diameter D=50 cm and an inner diameter d=36 cm. The materials used were clay bricks with dimensions of 70×110×220 mm^3 and gypsum mortar. Compressive and tensile tests were conducted on the assemblage of brick and gypsum mortar to determine the mechanical properties (Figures 1(a)-(b)). The modulus of elasticity was 987 MPa, and the bulk density was 1450 kg/m^3. The steps of constructing the minaret are shown in Figures 9(c)-(g). First, the bricks were placed in water for twenty-four hours to be completely wet. Then the gypsum mortar was made by mixing gypsum and water. At different steps of construction, the dimensions were controlled. At the last step, the overall dimensions were measured, and a suitable support was made around the bottom of the minaret by using bricks and gypsum mortar to connect the minaret to the solid floor of the laboratory.


Stress (MPa)
3
2.5
2
1.5
1
0.5
0
-0.5
-0.05
0
0.05
0.1
0.15
0.2
Strain (%)
d– 36 cm
D– 50 cm
h– 200 cm
22 cm
11 cm
7 cm

(a) (b)

(c) (d)

(e) (f)

(g) (h)

Figure 1 - *Mechanical test on the assemblage of brick and gypsum mortar and constructing the minaret in the laboratory: a) compressive test, b) stress-strain diagram, c) wetting the bricks, d) preparing gypsum mortar, e) controlling the diameter of the minaret, f) laying the fifth row of bricks, g) minaret after construction, h) dimensions of the minaret and brick*

2.2 Operational modal analysis and FEM updating of the minaret

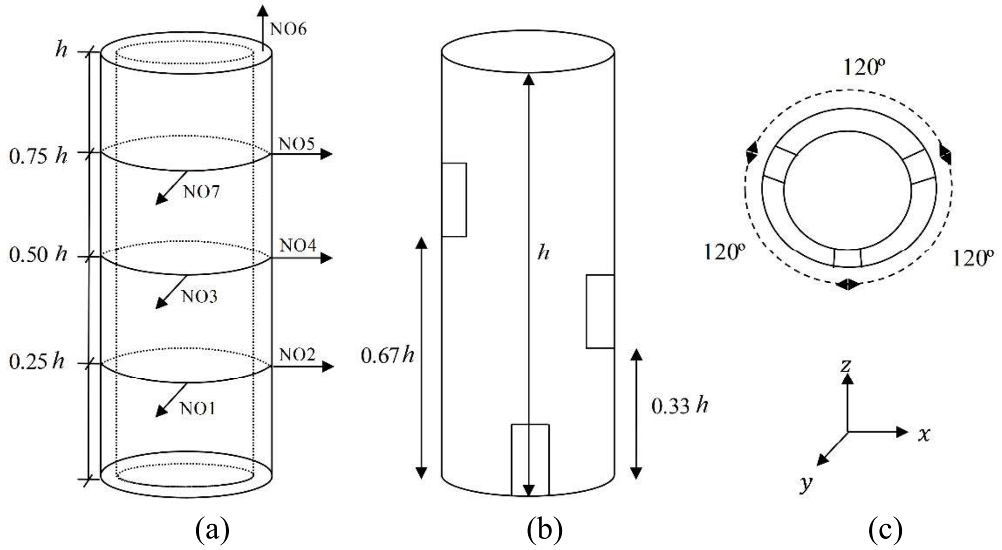
The operational modal analysis of the minaret constructed in the laboratory was performed in two cases, the minaret without opening and the minaret with three openings. For creating openings in the minaret, three bricks were removed from three different locations, i.e., one at the bottom, one at the one-third of the height ($0.33h$) and one at the two-thirds of the height ($0.67h$) each with a horizontal angle difference of 120º with respect to the others. In each case, seven piezoelectric accelerometer sensors with the sensitivity of 100mV/g were installed on the minaret. Three sensors were installed at one-fourth, half and three-fourths of the height in the x-direction, three sensors were installed at the same levels in the y-direction, and one sensor was installed at the top of the minaret in the z-direction in order to capture the response in three perpendicular directions (Figure 2).

The minaret was hit with a rubber hammer at suitable locations for excitation and the vibration was recorded (Figure 3). The data gathered from the accelerometers corresponding to specified points are shown in Figure 4(a). The experimental modal parameters of the minaret were obtained by the ARTeMIS software. The natural frequencies of the minaret were acquired by using FDD technique. The singular values of spectral density matrices of data set obtained from the FDD technique are shown in Figure 4(b). For finite element model updating of the models, the modal analysis in the ANSYS software started with changing the modulus of elasticity and then continued with changing the bulk density. In the finite element model, the SOLID 65 element with three translational degrees of freedom at each node was used. For boundary condition, all nodal displacements at the support of the minaret were constrained.

Before performing any type of finite element analysis, analyses should be performed to control the convergence of results and determine the number of acceptable elements with an acceptable error. To obtain this number of elements, by changing the dimension of the element and performing successive analyses on the minaret, the resulting errors were measured, and as soon as a negligible error was reached (less than 5%), the final number of elements was used for the analysis. In general, the Chehel-Dukhtaran minaret with 8792 elements, the Sin minaret with 10048 elements, the Bagh-i-Qush-Khana minaret with 11064 elements, the Gar minaret with 5827 elements, the Ziar minaret with 16414 elements, the Barsian minaret with 12517 elements, the Sariban minaret with 11237 elements, the Rahrvan minaret with 8851 elements and the Ali minaret converged with 14967 elements and analyses were performed with this number of elements. The results showed that the number of elements required for the convergence of each minaret can be obtained by dividing the height of each minaret in mm by 3.5. Each minaret was modelled including the outer shell, central column and spiral staircase.

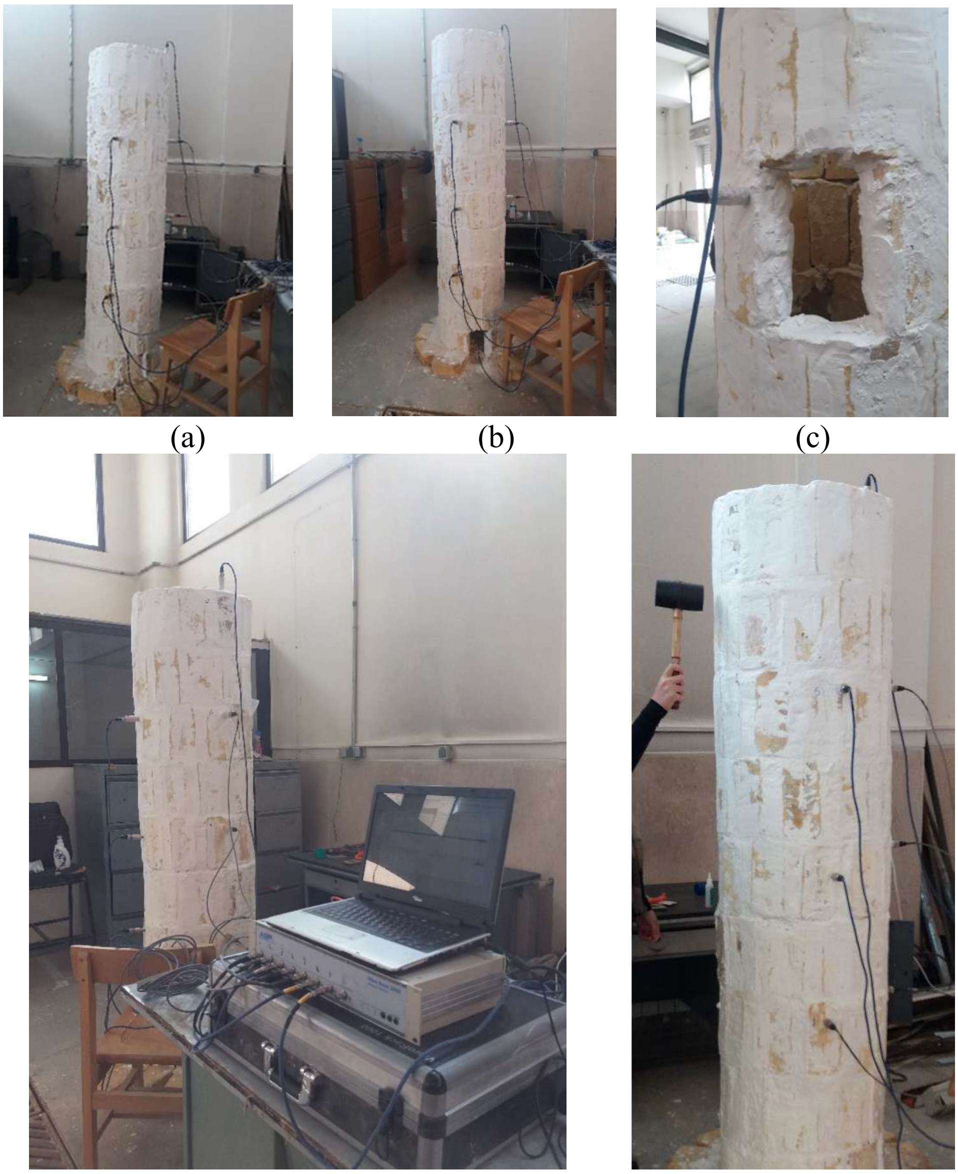
The finite element models of the minaret without opening and with opening were updated in 20 steps to change the modulus of elasticity and in 20 steps to change the bulk density as shown in Figure 5. To determine the effect of the modulus of elasticity, the bulk density was kept temporarily constant and equal to the value obtained from the mechanical test and the modulus of elasticity increased from zero until the resulting frequencies achieved the operational modal analysis values. A similar procedure was followed for the effect of the bulk density by increasing it from zero while keeping the modulus of elasticity temporarily constant. The values obtained for the modulus of elasticity and bulk density are respectively 1060 MPa and 1450 kg/m^3 after finite element model updating. Figure 6 shows a good agreement between finite element model updated frequencies and experimental frequencies.

Table 1 and Figure 7 show and compare the first seven frequencies of the minarets without and with opening obtained from operational modal analysis and finite element model updating. The second and fifth modes obtained from the finite element analysis cannot be determined from the operational modal analysis method. This is because the first and second natural frequencies, as well as the fourth and fifth frequencies of the laboratory minaret, are close to each other. In addition, due to the arrangement of the sensors, the third frequency of the minaret, which is the torsional frequency, cannot be determined. It is observed that the difference in the minaret without opening is less than 5%, it is less than 9% in the minaret with opening, and the results are satisfactory.



(a) (b) (c)

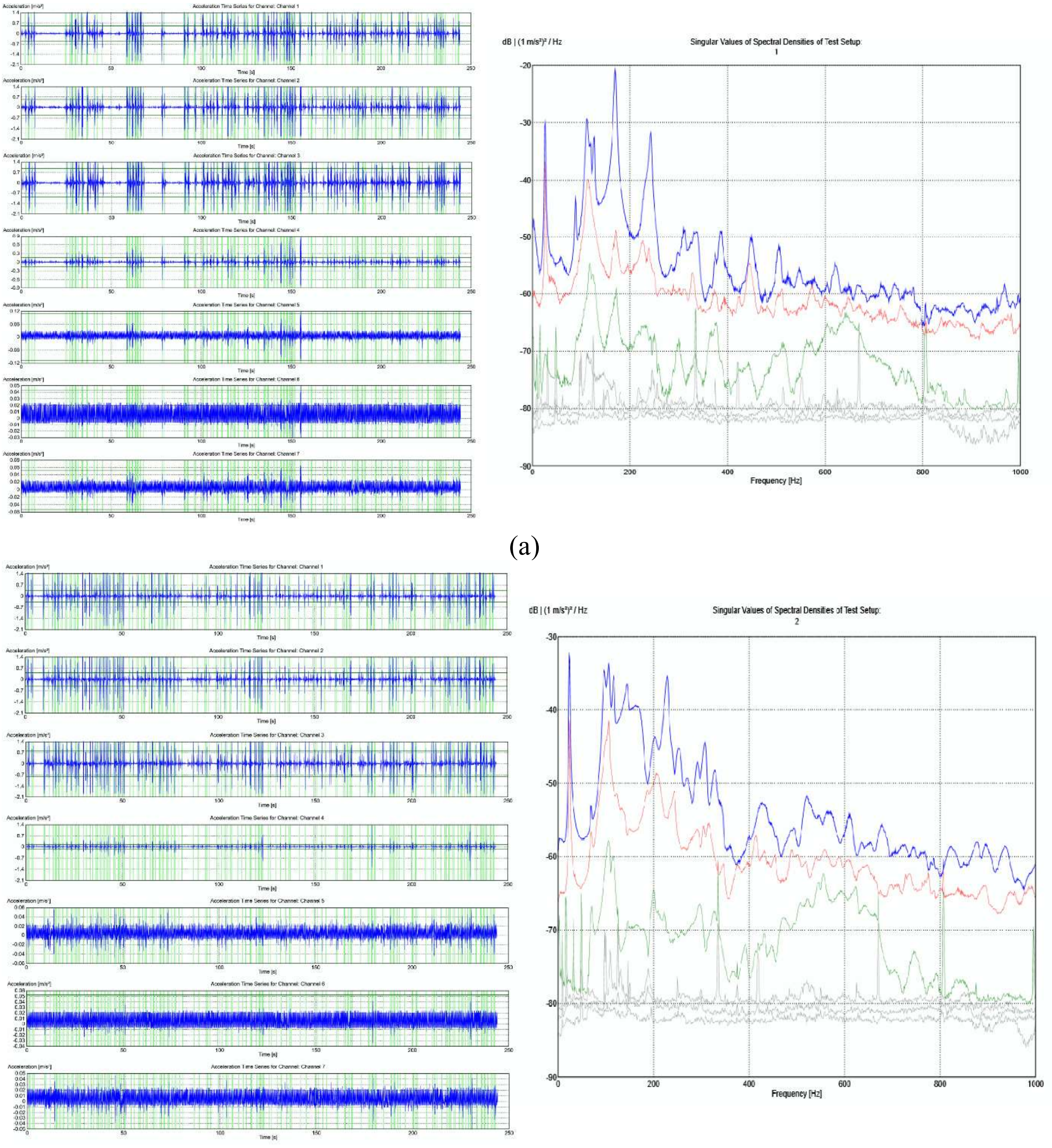
Figure 2 - *a) Locations of the sensors, b) vertical locations of the openings, c) angular plan for the locations of the openings*



(a) (b) (c)

(d) (e)

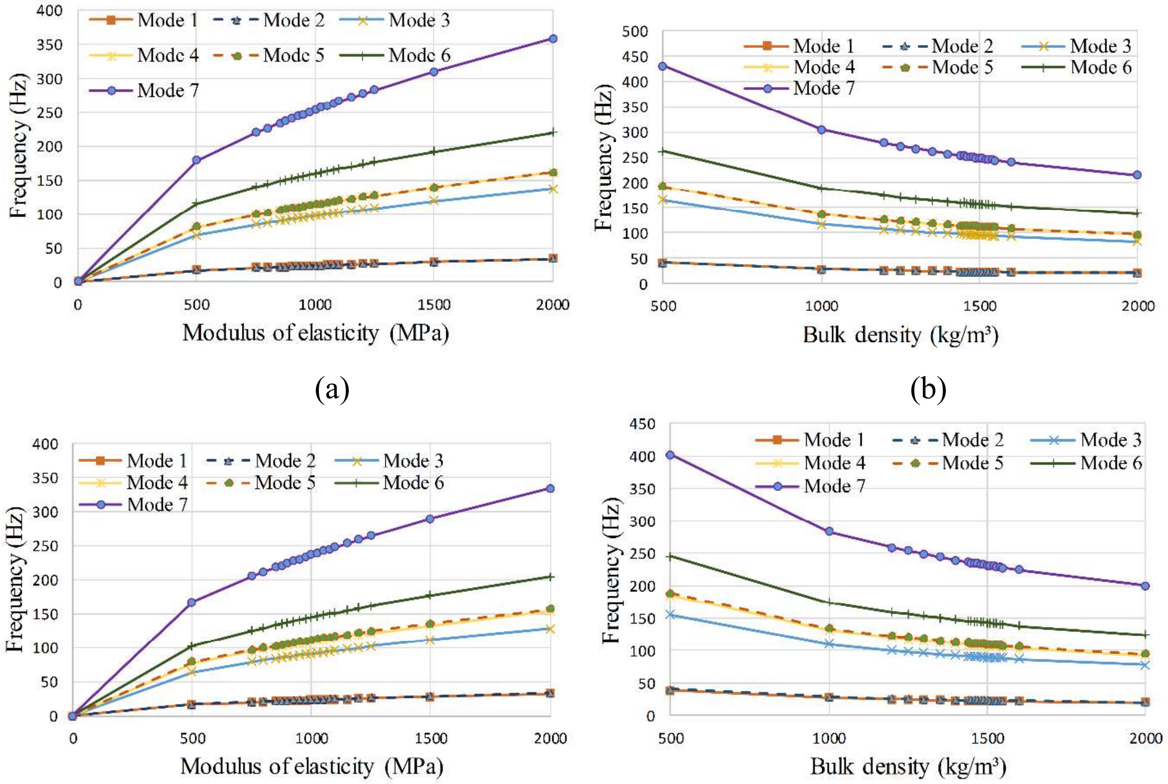
Figure 3 - *Operational modal test of the minaret in the laboratory: a) minaret without opening, b) minaret with three openings, c) sample of opening created in the minaret, d) sensors and data logger, e) excitation of the minaret with a rubber hammer*



(a)

(b)

Figure 4 - *Signal time series and average singular values of spectral density matrices: a) minaret without opening, b) minaret with opening*



(a) (b)

(c) (d)

Figure 5 - *Variation of frequency of the minaret in the laboratory against: a) modulus of elasticity, minaret without opening, b) bulk density, minaret without opening, c) modulus of elasticity, minaret with opening, b) bulk density, minaret with opening*

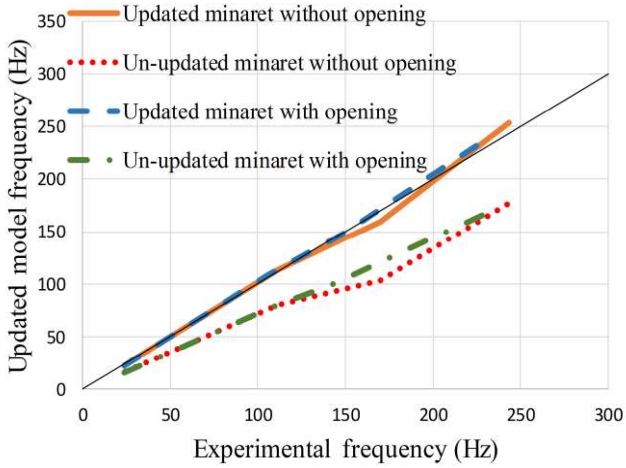


Figure 6 - *Updated and un-updated FEM frequencies against experimental frequencies*

Table 1 - *The first seven experimental and FEM updated frequencies of the minaret in the laboratory with the modulus of elasticity of 1060 MPa and bulk density of 1450 kg/m*^{3}

Frequency number	Minaret without opening			Minaret with opening		
	Experimental frequency (Hz)	FEM updated frequency (Hz)	Changes (%)	Experimental frequency (Hz)	FEM updated frequency (Hz)	Changes (%)
1	24.45	23.65	-3.24	23.438	22.57	-3.67
2	-	23.66	-	-	23.39	-
3	-	96.96	-	-	90.59	-
4	111.32	113.54	1.99	106.445	108.41	1.84
5	-	113.57	-	-	110.70	-
6	169.92	159.84	-5.93	144.531	144.33	-0.13
7	243.16	253.08	4.07	228.516	235.96	3.25

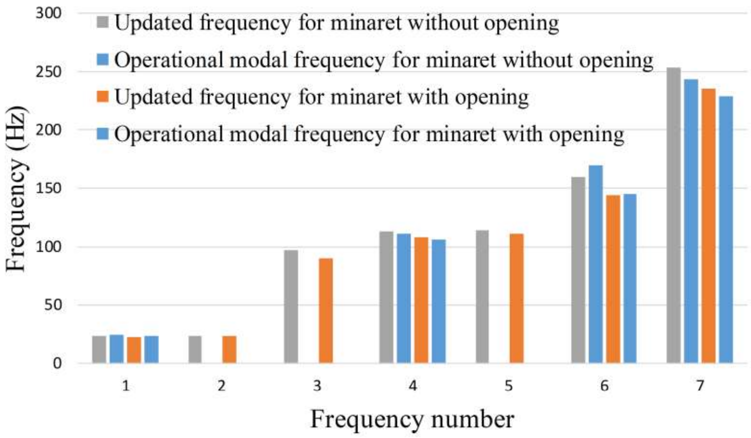
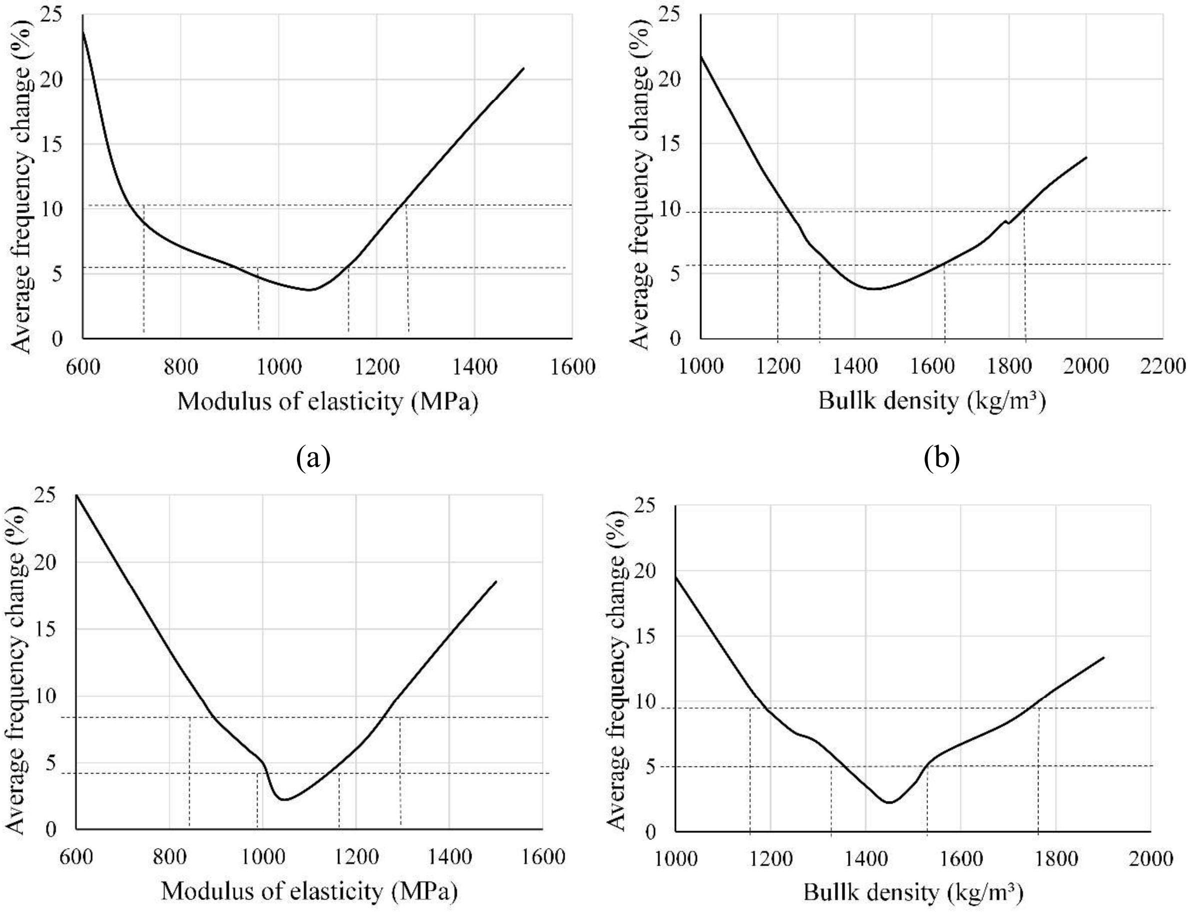


Figure 7 - *The first seven experimental and FEM updated frequencies of the minaret in the laboratory with the modulus of elasticity of 1060 MPa and bulk density of 1450 kg/m*^{3}

2.3 Range of acceptable variation for the modulus of elasticity and bulk density

In this section, the range of acceptable variation of the modulus of elasticity and bulk density is obtained so that if it is not possible to perform an operational modal test on a minaret, the values within the range can be used. The maximum acceptable variation considered for the frequency due to changes in modulus of elasticity and bulk density is 6% and variation above 10% is considered unacceptable.

Figure 8 shows that with a 12% variation in the modulus of elasticity and bulk density, the variation in the finite element frequency relative to the experimental frequency is about 6%. In addition, in case of variation of the modulus of elasticity and bulk density more than 22%, the variation of finite element frequency compared to the experimental frequency will be at least 10%. Based on these results, to determine the frequencies of minarets, it can be said that in minarets for which it is not possible to perform operational modal analysis, if the mechanical properties of their materials are determined through mechanical tests or any other reliable method and their support conditions are known, the frequencies can be determined using the finite element method. By changing the modulus of elasticity and bulk density in the range of ±12% in the finite element model, the frequency range of the minaret can be determined, and it can be expected to some extent that the actual frequency lies in this range.



(a) (b)

(d) (e)

Figure 8 - *Variation of average frequency of the minaret in the laboratory against: a) modulus of elasticity, minaret without opening, b) bulk density, minaret without opening, c) modulus of elasticity, minaret with opening, b) bulk density, minaret with opening*

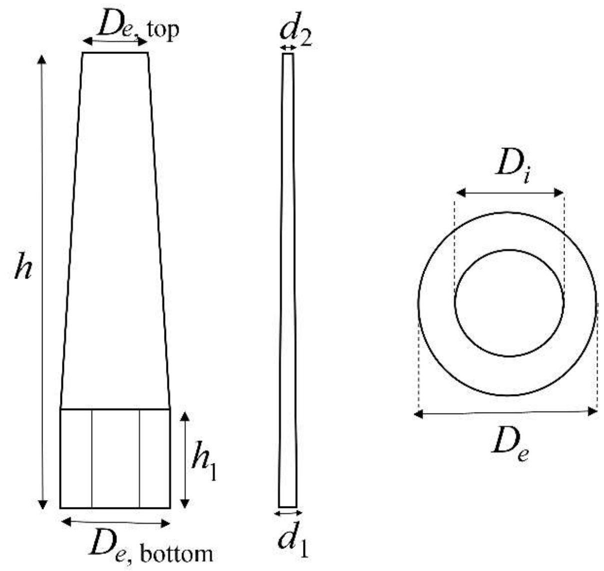
There are a number of historical brick masonry minarets from the eleventh to fourteenth centuries A.D. in Isfahan, Central Iran. They are the Barsian (1097 A.D.), Chehel-Dukhtaran (1107 A.D.), Ghar (1121 A.D.), Sin (1131 A.D.), Ali (eleventh to twelfth century A.D., Sariban (1155 A.D.), Ziar (eleventh to twelfth century A.D.), Rahravan (eleventh to twelfth century A.D.), and Bagh-i-Qush-Khana, fourteenth century A.D. minarets (Figure 9). Minarets consist of the outer shell, central column and spiral staircase. Their dimensions are shown in Figure 10 and Table 2. The height ranges from about 20 m to about 50 m. The outer diameter at the bottom is between 2.5 m and 6 m and it is from 2 m to 5 m at the top. The thickness of the outer shell ranges from 0.4 m to 1.8 m at the bottom and from 0.38 m to 1 m at the top.

In order to assess the aforementioned conclusion for real minarets, the mechanical properties of the brick masonry materials of the minarets in Isfahan determined from mechanical tests reported in [Hejazi *et al.*, 2016], i.e. the modulus of elasticity of 2730 MPa and bulk density of 1530 kg/m^3, are used to calculate the range of the frequencies for four historical minarets including the Chehel-Dukhtaran, Ghar, Sin and Ali minarets. The outer shell, central column and spiral staircase were considered in the finite element model.

Figure 11 shows the change of average frequency of these minarets against the modulus of elasticity and bulk density obtained from 32 analyses. Similar results to the experimental results are obtained. The results show that a 12% change in the modulus of elasticity and bulk density has an effect of about 6% on the frequency of historical minarets that is acceptable, and a 22% change in the modulus of elasticity and bulk density changes the frequency more than 10% that is unacceptable.

(a) (b) (c)

Figure 9- *a) Chehel-Dukhtaran minaret (1107 A.D.), b) Ali minaret (11th to 12th century A.D.), c) Sariban minaret (1155 A.D.)*

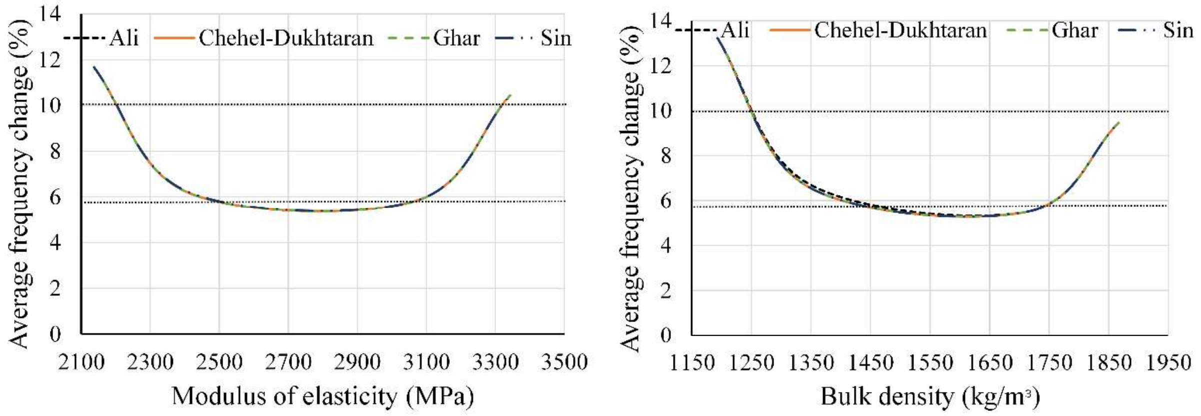


(a) (b) (c)

Figure 10 - *Dimensions of a minaret: a) outer view, b) central column, c) outer shell*

Table 2 - *Dimensions of historical minarets in Isfahan*

Minaret	Date (A.D.)	Total height (h) (m)	Base height (h_1) (m)	External diameter at the bottom ($D_{e,\text{bottom}}$) (m)	Internal diameter at the bottom ($D_{i,\text{bottom}}$) (m)	External diameter at the top ($D_{e,\text{top}}$) (m)	Internal diameter at the top ($D_{i,\text{top}}$) (m)	Column diameter at the bottom (d_1) (m)	Column diameter at the top (d_2) (m)
Barsian	1097	34.55	-	5.57	2.15	4.20	2.14	0.82	0.72
Chehel-Dukhtaran	1107	29	5	2.90	1.80	2.33	1.23	0.55	0.33
Ghar	1121	21	-	5.50	3.96	4.70	3.16	1.60	1.20
Sin	1131	33.70	6	2.65	1.852	2	1.20	0.80	0.60
Ali	11th-12th century	47.85	-	6	3.10	2	1.24	1.60	0.32
Sariban	1155	44.20	-	4.04	2.60	2.4	1	0.87	0.66
Ziar	11th-12th century	50	5.50	4.80	3.08	1.60	1	1.50	0.60
Rahravan	11th-12th century	29.50	-	3	2	2.40	1.20	0.80	0.40
Bagh-i-Qush-Khana	14th century	37.55	11	2.50	1.50	2	1.20	0.50	0.30



(a) (b)

Figure 11 - *Variation of average frequency for the Ali, Chehel-Dukhtaran, Ghar and Sin minarets against: a) modulus of elasticity, b) bulk density*

3 Modal analysis of historical minarets of Isfahan by the FEM

Modal analysis of the Barsian, Chehel-Dukhtaran, Ghar, Sin, Ali, Sariban, Ziar, Rahravan and Bagh-i-Qush-Khana minarets was performed using the updated mechanical materials obtained in the previous section. The outer shell, spiral staircase and central column of the minarets were modelled in the finite element analysis. The first twelve frequencies of the studied minarets are given in Table 3. The finite element model of the Ali minaret and its frequencies are shown in Figures 12 and 13, respectively. The order of the first frequency from the minimum to the maximum is 0.41 Hz of Bagh-i-Qush-Khana, 0.49 Hz of both Sin and Sariban, 0.6 Hz of Ziar, 0.71 Hz of Chehel-Dukhtaran, 0.74 Hz of Rahravan, 0.84 Hz of Ali, 1.1 Hz of Barsian, and 2.27 Hz of the Ghar minaret.

Table 3 - *The first twelve frequencies of the studied minarets*

Frequency number	Frequency (Hz)			Frequency number	Frequency (Hz)		
	Barsian	Ali	Sariban		Barsian	Ali	Sariban
1	1.10	0.84	0.49	7	12.73	9.86	8.23
2	1.10	0.84	0.49	8	12.76	10.38	8.70
3	5.28	2.82	2.31	9	21.05	11.14	10.85
4	5.28	2.84	2.34	10	22.21	11.21	10.96
5	8.63	6.32	5.86	11	22.23	17.02	17.03
6	11.06	6.35	5.90	12	29.28	17.12	17.17
Frequency number	Frequency (Hz)			Frequency number	Frequency (Hz)		
	Chehel-Dukhtaran	Rahravan	Ziar		Chehel-Dukhtaran	Rahravan	Ziar
1	0.71	0.74	0.60	7	10.48	10.36	8.36
2	0.72	0.74	0.60	8	11.78	11.98	8.40
3	3.98	3.98	2.04	9	19.18	18.93	8.59
4	4.01	4.01	2.05	10	19.27	19.00	9.40

5	9.07	9.41	4.65	11	23.87	24.47	12.99
6	10.41	10.31	4.68	12	29.67	29.26	13.08

Frequency number	Frequency (Hz)			Frequency number	Frequency (Hz)		
	Bagh-i-Qush-Khana	Sin	Ghar		Bagh-i-Qush-Khana	Sin	Ghar
1	0.41	0.49	2.27	7	7.44	8.20	26.78
2	0.41	0.49	2.29	8	9.58	10.34	26.99
3	2.18	2.64	11.14	9	10.77	13.07	31.44
4	2.19	2.66	11.58	10	10.82	13.16	44.14
5	5.73	6.97	11.68	11	17.08	20.43	44.32
6	5.76	7.01	15.14	12	17.16	20.63	44.62

(a) (b) (c)

Figure 12 - *FEM of the Ali minaret: a) outer shell, b) spiral staircase, c) central column*

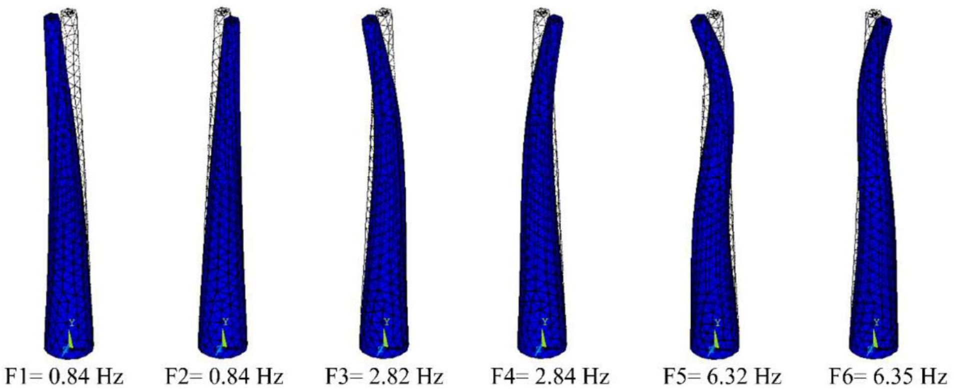


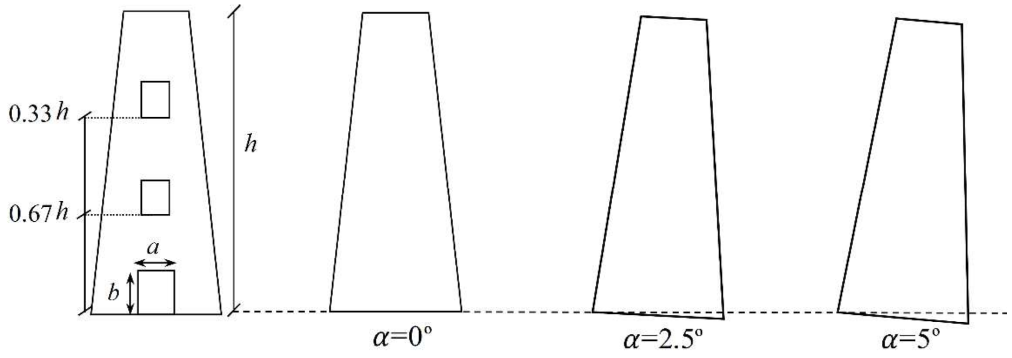
Figure 13 - *The first six modes of the Ali minaret*

4 Parametric study

After field studies and visiting several minarets, the dimensions and locations of the openings, which are usually used in Persian minarets, were determined according to Table 4 and Figures 14(a) and 15. The opening can be at the ground level, at one-third and at two-thirds of the height of the minaret. The horizontal and vertical dimensions of the opening are *a* and *b*, respectively (Figure 14(a)). The opening at the ground level has larger dimensions of *a*=1 m and *b*=2 m, and openings at one-third and two-thirds of the height have smaller dimensions of *a*=0.5 m and *b*=0.75 m. The location of the openings relative to each other can be different. Openings can be placed in a line or at angles of 120° or 180° to each other. There are fourteen cases for the openings as shown in Figure 14. For inclination, two inclination angles of 2.5° and 5° are considered (Figure 14(b)).

Table 4 - *Locations and dimensions of the openings (h=height of the minaret, a=horizontal dimension, b=vertical dimension)*

Location	*a* (m)	*b* (m)
0	1	1.5
0.33*h*	0.5	0.75
0.67*h*	0.5	0.75



(a) (b)

Figure 14 - a) *Dimensions and locations of the openings along the height: b) minaret without inclination and with inclination angles of 2.5° and 5°*

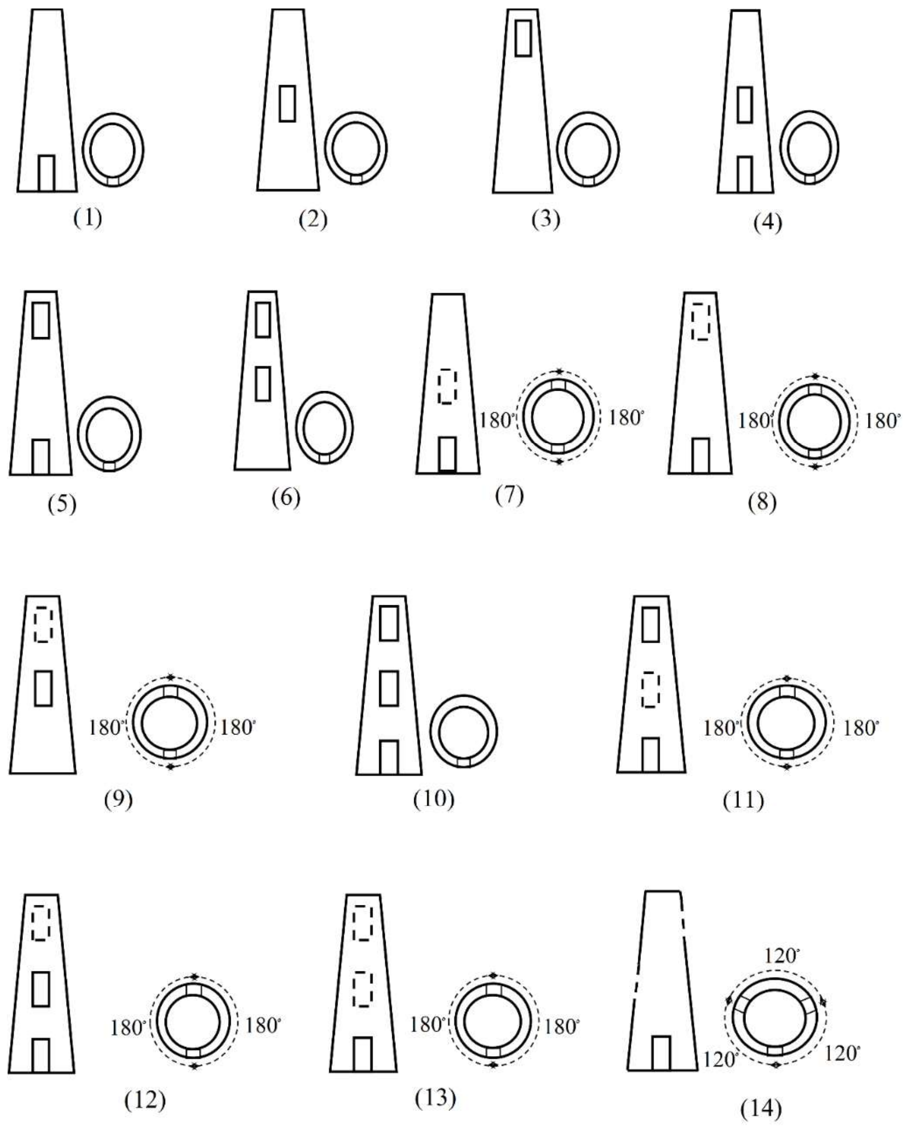


Figure 15 - *Fourteen cases of locations of openings*

4.1 Effect of the dimensions of the opening

Three groups of openings were considered for changing their dimensions. In the first group, the openings had a constant vertical dimension *b*= 1.5 m and different horizontal dimension *a*=0.5 m, 0.75 m and 1 m. In the second group, the vertical dimension was constant *b*=2 m and the horizontal dimension changed as *a*=0.5 m, 0.75 m and 1 m. In the third group, the horizontal dimension was constant *a*=0.5 m and the vertical dimension was considered *b*=1.5 m, 1.75 m and 2 m.

Analysis was performed for an opening located at the ground level with different opening dimensions. The Chehel-Dukhtaran, Ghar, Sin and Ali minarets were selected for study. The Ali minaret was chosen because of its high height and the Ghar minaret was selected for its low height and large diameter. A number of 36 models were analysed. Results for the change of first frequency due to different opening dimensions are shown in Table 5. As expected, larger dimensions of the opening have more effect on reducing the frequency. The maximum effect is a reduction 6.88% of the first frequency in the Chehel-Dukhtaran minaret with opening dimensions of *a*=1 m and *b*=2 m. Then a frequency reduction of 4.9% in the Sin minaret with *a*=1 m and *b*=2 m, a frequency reduction of 3.39% in the Ghar minaret with *a*=1 m and *b*=2 m, and finally a frequency reduction of 0.85% in the Ali minaret with *a*=0.75 m and *b*=2 m. In general, the effect of the dimensions of the openings on the first frequency is insignificant and is not related to the minaret height.

Table 5 - *Changes of first frequency of minarets with different opening dimensions located at the ground level with respect to minarets without opening*

Group	Opening dimensions (*a*×*b*)(m×m)	Minaret			
		Chehel-Dukhtaran	Ghar	Sin	Ali
	0.5×1.5	-2.08	-1.21	-2.04	-0.83
1	0.75×1.5	-2.93	-1.82	-2.46	-0.14
	1×1.5	-5.70	-2.59	-4.10	-0.64
	0.5×2	-2.67	-.158	2.45	-0.41
2	0.75×2	-3.62	-2.35	-3.27	-0.85
	1×2	-6.88	-3.39	-4.90	-0.67
	0.5×1.5	-2.08	-1.21	-2.04	-0.83
3	0.5×1.75	-2.43	-1.64	-2.25	-0.80
	0.5×2	-2.67	-.185	-2.45	-0.41

4.2 Effect of the location of the opening

In order to investigate the effect of the location of the opening on the frequencies of the minarets, the fourteen cases of Figure 15 were applied to the Chehel-Dukhtaran, Ghar, Sin and Ali minarets. Three groups are observed in these fourteen cases. The first group includes cases 1, 2 and 3 with one opening at different levels. The second group comprises cases 4-9 with two openings at different levels and different angles with respect to each other. The third group includes cases 10-14 with three openings at different levels and different angles on the plan. According to field studies the openings at the ground level have larger dimensions of *a*=1 m and *b*=2 m, and openings at one-third and two-thirds of the minaret height have smaller dimensions of *a*=0.5 m and *b*=0.75 m.

Table 6 shows the first frequency of the minarets for fourteen cases obtained from 56 models. In order to study the effect of the opening, the differences of the first frequency between the minarets with openings and the minarets without opening are presented in Table 7. In the first group with one opening, the maximum effect of the opening occurs in the first case in which the opening is at the ground level. In the second group with two openings, when one of the openings is at the ground level the effect of the openings is pronounced. In the third group with three openings, the effect is greater when the openings are at an angle of 120° to each other. In general, the difference of the frequencies is between 0.1% and 7% and therefore the effect of the presence and location of the opening is insignificant.

Table 6 - *The first frequency of minarets in fourteen opening location cases*

Opening location case number	Frequency (Hz)			
	Chehel-Dukhtaran	Ghar	Sin	Ali
1	0.68	2.19	0.47	0.83
2	0.71	2.26	0.49	0.83
3	0.71	2.26	0.49	0.84
4	068	2.23	0.46	0.83
5	0.68	2.22	0.47	0.83
6	0.71	2.26	0.49	0.83
7	0.68	2.22	0.46	0.83
8	0.68	2.21	0.47	0.83
9	071	2.26	0.49	0.83
10	0.68	2.22	0.46	0.83
11	0.68	2.22	0.46	0.83
12	0.68	2.22	0.46	0.83
13	0.86	2.21	0.46	0.83
14	0.66	2.22	0.45	0.82

Table 7- *The difference between the first frequency of the minarets with opening and the minarets without opening in fourteen opening location cases*

Opening location case number	Frequency difference (%)			
	Chehel-Dukhtaran	Ghar	Sin	Ali
1	-4.80	-3.40	-0.49	-0.94
2	-0.10	-0.10	-0.05	-0.87
3	-0.04	-0.22	-0.03	-0.52
4	-4.74	-1.77	-0.50	-0.87
5	-4.62	-2.19	-0.48	-1.12
6	-0.13	-0.22	-0.05	-0.87
7	-0.89	-1.91	-0.50	-1.39
8	-4.53	-2.28	-0.48	-1.55
9	-0.26	-3.12	-0.08	-0.99
10	-4.82	-2.21	-5.12	-1.27
11	-4.90	-2.25	-5.26	-0.99
12	-4.82	-2.08	-5.34	-1.30
13	-4.92	-2.33	-5.10	-1.59
14	-6.90	-2.24	-7.39	-2.80

4.3 Effect of the inclination angle

In some instances, due to soil settlement or other reasons, minarets slightly tilt and undergo permanent inclination. It is interesting to have insight into the effect of the inclination angle on modal parameters of the minarets. For this reason, two inclination angles of 2.5° and 5°, observed in some Persian minarets, are chosen. The inclinations were applied to the Chehel-Dukhtaran, Ghar, Sin and Ali minarets. Figure 16 compares the first frequencies of the inclined and non-inclined minarets and Table 8 presents their differences. In general, the inclination has little effect on the frequency of minarets, so that the frequency decreases slightly. The effect of inclination for inclination angles of 2.5º and 5º on the frequency of minarets is less than 3%, and therefore, negligible.

Table 8 - *The difference between the first frequency of inclined and non-inclined minarets*

Angle of inclination (º)	Frequency difference (%)			
	Chehel-Dukhtaran	Ghar	Sin	Ali
2.5	-0.01	0.29	-2.72	-0.64
5	-0.06	0.04	-2.73	-0.68

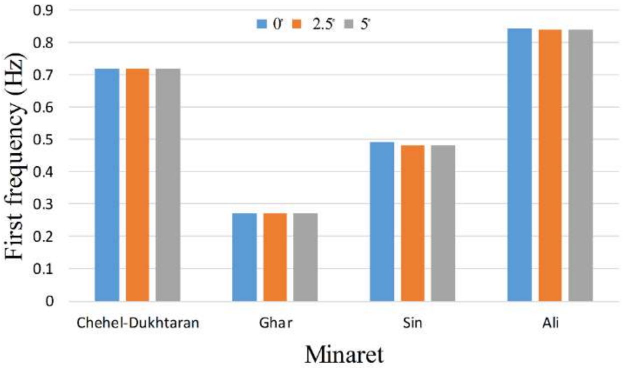


Figure 16 - *Comparison of the first frequencies of the inclined and non-inclined minarets*

5 Seismic analysis

In the previous sections, the effect of the dimensions and location of the opening and the inclination angle on the frequencies of historical minarets was discussed. In the following sections the effect of the same parameters on the seismic performance of the Chehel-Dukhtaran, Ghar, Sin and Ali minarets are studied. Because the information of the accelerogram data for the earthquakes occurred in Isfahan was not available, three acceleration time histories of the Coyote Lake, Parkfield and Whittier Narrows earthquakes were selected from the strong motion database collected by the University of California, Berkeley [Silva, 2020]. The selected earthquakes have geologic, tectonic, and seismologic and soil characteristics similar to Isfahan. Seismic analysis was performed for three cases including the minarets without opening and without inclination, minarets with fourteen cases of openings as shown in Figure 15, and minarets with inclination angles of 2.5º and 5º as illustrated in Figure 14(a). The time of the occurrence of the first crack was compared in the minarets.

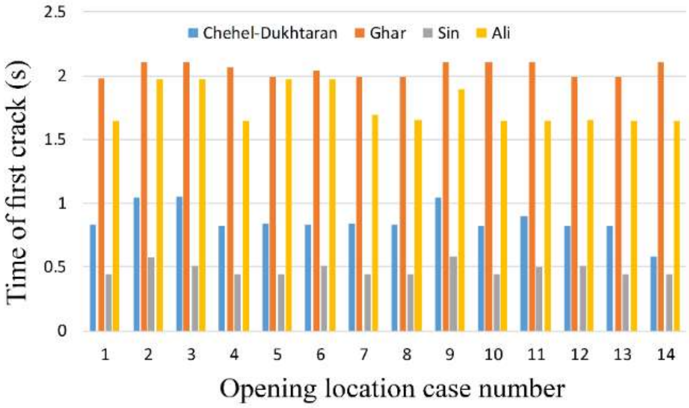
5.1 Effect of the location of the opening

A number of 189 analyses have been performed for the effect of the location of the opening under earthquakes. The time of occurrence of the first crack in the minarets without opening and without inclination under the Coyote Lake (C), Parkfield (P) and Whittier Narrows (W) earthquakes are presented in Table 9 with a maximum of 8.64 s for the Ghar minaret under the

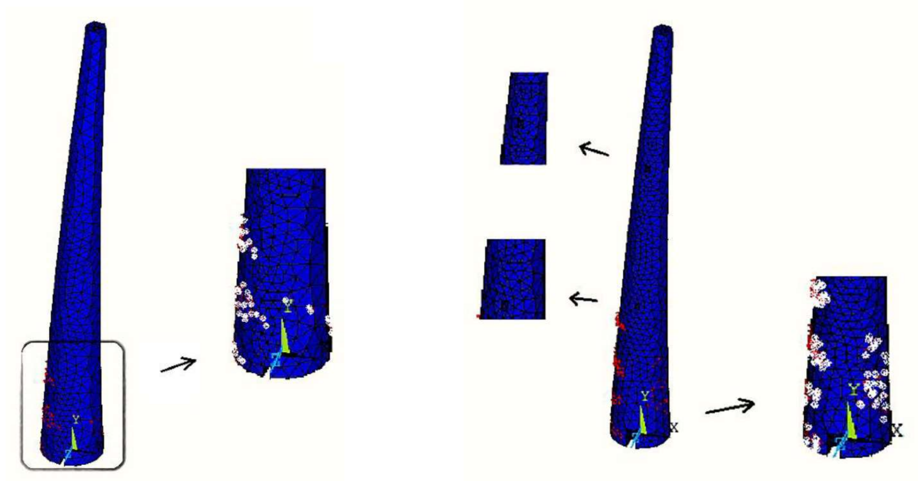
Whittier Narrows earthquake and a minimum of 0.59 s for the Sin minaret under the Coyote Lake earthquake. The results for fourteen opening location cases for the Coyote Lake earthquake, which has the highest effect on the minarets, are shown in Figure 17. It is observed that the Ghar and then Ali minarets have again the best performances. The times of the first crack occurrence in the minaret at different opening location cases under an earthquake are close to each other and practically the presence of the openings and their locations have an insignificant effect on the time of the first crack occurance. Cracks in the Ali minaret without opening and with opening location case 10, i.e. three openings along a vertical line, are shown in Figures 17(c)-(d). In both cases, cracks are increased near the bottom of the minaret and no crack is observed near the openings at the one-third and two-thirds of the minaret height. The opening at the ground level has caused more cracks near the base of the minaret compared to the minaret without opening.

Table 9 - *Time of occurrence of the first crack in minarets without opening and without inclination under the Coyote Lake (C), Parkfield (P) and Whittier Narrows (W) earthquakes*

Time of occurrence of the first crack (s)											
Chehel-Dukhtran			Ghar			Sin			Ali		
Earthquake			Earthquake			Earthquake			Earthquake		
C	P	W	C	P	W	C	P	W	C	P	W
1.09	4.93	5.18	2.03	7.31	8.64	0.59	1.99	2.40	2.01	6.83	7.17



(a)



(b) (c)

Figure 17 - *Historical minarets under the Coyote Lake earthquake in 14 opening location cases: a) time of occurrence of the first crack, b) cracks in Ali minaret without opening, c) cracks in Ali minaret in opening location case 10*

5.2 Effect of inclination

In the previous section, it was shown that the Coyoke Lake earthquake has a higher effect on the performance of the minarets than other two earthquakes; therefore, in this section the minarets with two inclination angles of 2.5° and 5° are studied under this earthquake. The earthquake is applied once in the plane of the inclination and once perpendicular to the plane of the inclination. Table 10 shows the times of occurrence of the first crack in inclined and non-inclined minarets and their difference obtained from 12 analyses. As expected, the time in inclined minarets is shorter than non-inclined minarets. By increasing the inclination angle, the time of the first crack occurrence decreases. When the earthquake is applied in the plane of inclination is more critical than when applied perpendicular to the plane of inclination.

When the earthquake is applied in the direction of the inclination, the reductions of the time of the first crack occurrence are 44% for the Chehel-Duckhtarn minaret at inclination angle of 2.5°, 23% and 24% for the Ghar minaret at inclination angles of 2.5° and 5°, respectively, and 11% and 46% for the Ali minaret at inclination angles of 2.5° and 5°, respectively. When the earthquake is perpendicular to the plane of inclination time reduction values are 39% for the Chehel-Duckhtarn minaret at inclination angle of 2.5°, 2% and 3% for the Ghar minaret at inclination angles of 2.5° and 5°, respectively, and 1% and 11% for the Ali minaret at inclination angles of 2.5° and 5°, respectively. The high effect of inclination on the tallest minaret, i.e. the Ali minaret, and the low effect of the inclination on the lowest minaret, i.e. the Ghar minaret, is observed.

Table 10 - *Time of occurrence of the first crack in inclined and non-inclined minarets and their differences*

	Chehel-Dukhtaran	Ghar	Ali

Angle of inclination (°)	Time of occurrence of the first crack (s)	Time difference with non-inclined minaret (%)	Time of occurrence of the first crack (s)	Time difference with non-inclined minaret (%)	Time of occurrence of the first crack (s)	Time difference with non-inclined minaret (%)
0	0.90	0	1.99	0	1.67	0
Earthquake in the plane of inclination						
2.5	0.50	-44	1.53	-23	1.48	-11
5	-	-	1.52	-24	0.89	-46
Earthquake perpendicular to the plane of inclination						
2.5	0.55	-39	1.95	-2	1.65	-1
5	-	-	1.93	-3	1.47	-11

6 Conclusion

In this paper, the modal and seismic analyses of a number of Persian historical brick masonry minarets were performed using finite element model updated with operational modal analysis. Parametric study was done by changing the dimensions and locations of the openings in the minarets. The effect of inclination angle was also investigated.

Obtained results indicate that in modal analysis, the presence of opening decreases the frequency of the minaret slightly compared to the minaret without opening. Larger dimensions of opening have more effect on decreasing the frequency. The lower location of the opening, the greater the effect on reducing the frequency. For minarets with one opening, when the opening is at the ground level it has the highest effect on decreasing the frequency of the minaret. For minarets with two openings, when one opening is at the ground level it has the highest effect on the frequency. The relative location of the opening, i.e. the angle between the openings on the plan be 0° or 180°, does not have a significant effect. For minarets with three openings, if the angles between the openings is 120° it causes the most reduction of frequency. The reduction of minaret frequency due to openings is less than 7%. By increasing the inclination, the frequency decreases slightly. The effect of inclination on frequency is less than 3%. In general, the effect of inclination on the frequency of minarets is less than the effect of the presence of opening, although the effect of both is insignificant.

It was showed that in cases where it is not possible to perform modal operational analysis to measure the frequency of minarets, if the mechanical properties of materials are determined from laboratory tests or other reliable methods, finite element analysis can then be used by changing the modulus of elasticity and bulk density by ±12% to obtain the range of frequency by an approximation of about 6%. If the variation of the modulus of elasticity and bulk density is within ±22%, the frequency approximation will be more than 10%.

In seismic analysis, the best and worst performances against the earthquakes is of the Ghar and Sin minarets, respectively. The presence and location of opening has little effect on seismic performance. The least vulnerability against the inclination belongs to the Sin and Chehel-Dukhtaran minarets and the most vulnerability is of the Ali and Ghar minarets. Larger inclination angles are more critical. Application of the earthquake in the plane of inclination causes more damage than when earthquakes are applied perpendicular to the plane of inclination.

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