



PROBABILISTIC APPROACH FOR LOCAL HIERARCHY CRITERIA OF EB-FRAMES

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SUMMARY: *The objective of this work is the study and the evaluation of the overstrength of short links in Eccentrically Braced Frames (EBFs) to be considered in the application of local hierarchy criteria. Starting from the results of experimental tests devoted to short links, linear multivariate regression is provided to evaluate the overstrength of links. The obtained mathematical relation accounts for the following geometrical and mechanical properties: the web slenderness of the link, the slenderness of the stiffening plates, the non-dimensional distance between the stiffening plates, the steel hardening, the non-dimensional length of the link and its ultimate rotation. Moreover, to account for the uncertainty of the regression model and the material variability, the first-order reliability method (FORM) is applied to the limit state function, which is used to describe the local hierarchy criterion for short links based on the rigid-plastic analysis. So, the authors defined an overstrength factor accounting for the geometrical and mechanical properties of links and including a safety factor to account for uncertainties.*

KEYWORDS: *Capacity Design, Eccentrically Braced Frames (EBFs), short link, stochastic approach, FORM Method*

1 Introduction

Eccentrically Braced Frames (EBFs) constitute a quite recent structural typology which gained prominence in the last decades thanks to the study of Popov and Kasai [Kasai and Han, 1997, Mirzai *et al.*, 2018, Bosco *et al.*, 2016, Farzampour *et al.*, 2019]. This structural type is well suited for tall buildings located in areas of high seismic intensity. In fact, under several seismic actions, EBFs constitute a good compromise between seismic-resistant MR-frames and concentrically braced frames, because they exhibit both adequate lateral stiffness, due to the high contribution coming from the diagonal braces, and ductile behaviour, due to the ability of the links, constituting the dissipative zones of such a structural typology, in developing wide and stable hysteresis loops [Roeder and Popov, 1978, Bruneau *et al.*, 2011]. EBFs can be classified in terms of link configuration which can be either horizontal (D-scheme, K-scheme and V-scheme) or vertical (inverted Y-scheme) (Figure 1).

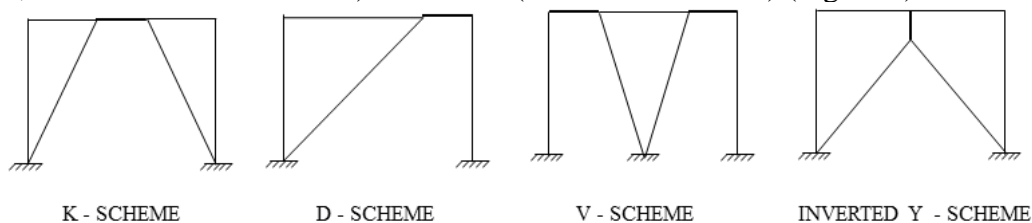


Figure 1 - Geometrical scheme of the eccentrically braced frames, proposed by EC8

In terms of length, links can be subdivided into [Eurocode 8]:

- Short links having $e \leq 1.6 M_p/V_p$;
- Intermediate links having $1.6 M_p/V_p < e \leq 3 M_p/V_p$;
- Long links having $e > 3 M_p/V_p$.

where e represents the non-dimensional length of links, M_p and V_p are the plastic moment and the plastic shear of the link cross-section, respectively. For the short links, the plastic behaviour can be assumed in pure shear; the long links are dominated by pure bending moment, while intermediate links are characterized by the moment-shear interaction [Montuori *et al.*,2014, Montuori *et al.*,2016, Nastri,2018, Nastri, 2020].

Links are designed according to the first principle of *Capacity Design* [Mazzolani and Piluso, 1996] conversely, the non-dissipative zones follow its second principle. Specifically, they are designed using the maximum internal actions transmitted by the dissipative members in their full yielded and strain hardened state. Therefore, the overstrength sources depend on the strain-hardening of material, which can assume different values in relation to the steel grade, the geometrical proprieties of link and the presence of stiffeners.

As the Eurocode 8 does not suggest any design criteria for local hierarchy in short, intermediate and long links, the only literature reference is given by Montuori *et al.*,2016 which introduced a design formulation based on the limit analysis. This design criterion, in case of short links, makes sense only for short links with K and inverted Y configuration (Figure 1) and allows preventing the yielding of non-dissipative zones (i.e. beams and columns) imposing that the sum of plastic moments of the diagonal members (M_d) and the beam (M_b) must be higher than the equivalent plastic moment of the link (M_{link}). However, the aforementioned condition is seen only under a deterministic light by neglecting the sources of randomness playing an important role both at local and global scale [Badalassi *et al.*, 2013]. Only an accurate evaluation of the role of the link overstrength can assure the respect of the local hierarchy, namely the involvement in plastic range of the link only.

In the last decades, many experimental campaigns and research works have been carried out to estimate the actual link overstrength and the parameters involved. Barecchia *et al.*,2006 provide an overstrength factor ranging between 2.00 and 2.50 for a link shear deformation equal to 0.10 rad. Okazaki *et al.*,2005 on the base of 23 experimental tests, proposed a coefficient ranging from 1.34 to 1.48 in case of short links and ranging from 1.12 to 1.28 in the case of longer links. Bozkurt *et al* carried out 31 experimental tests for only short links of steels grade S275, providing an overstrength factor, ranging from 1.70-3.20. Even though many experimental campaigns were performed a very large variability of the overstrength factor has been pointed out as also the test setup is the cause of high variability in the results.

In this paper, the results belonging to the available experimental tests [Okazaki *et al.*,2004, Okazaki *et al.*, 2005, Okazaki *et al.*, 2007, Okazaki *et al.*,2009, Bozkurt *et al.*,2017,Bozkurt *et al.*,2018, McDaniel *et al.*,2003, Maalek *et al.*,2012, Yin *et al.*,2018, Ji *et al.*,2016, Mansour *et al.*, 2011, Lian *et al.*,2016] are used to evaluate a first overstrength factor for only short links by multivariate regression defined as function of the main geometrical and mechanical characteristics of dissipative zone such as the slenderness of web and the stiffeners, the hardening of material, the non-dimensional length of the link, the ultimate rotation and the non-dimensional distance between two stiffening plates.

Furthermore, to account for the uncertainty of the regression model and the random material variability, a safety factor γ_{rd} is calibrated through the First Order Reliability Method (FORM) [Pinto *et al*, 2004, Piluso *et al*, 2019]. The limit state event is given by the formulation of the local hierarchy criterion belonging to the application of the kinematic theorem of plastic collapse in the frame rigid-plastic analysis proposed by Montuori *et al*. The proposed procedure could be used to calibrate the overstrength factors to be applied in the framework of the local hierarchy criterion also for long and intermediate links.

2 The overstrength factor of short links

The definition of an overstrength factor for short links cannot neglect the wide experimental campaigns carried out in the recent past especially on short links. In fact, as it is known, short links can show wide and stable hysteresis loops and assure an ultimate plastic rotation that is conventionally assumed equal to 0.08 rad. Therefore, a database of 97 experimental data was collected (Appendix A) with reference to different links, divided in terms of steel grade (S235, S275, S355). For each experimental test, the following parameters have been identified:

- Non-dimensional distance between two stiffeners a/t_w ;
- Web slenderness $\lambda_w = \frac{h_i}{t_w} \sqrt{\frac{f_y}{E}}$;
- Slenderness of the stiffening plates $\lambda_s = \frac{d_w}{t_s}$;
- Hardening of the material f_u/f_y ;
- Non-dimensional length of link $\bar{e} = \frac{e}{M_p/V_{y,EC8}}$;
- Ultimate rotation ϑ_u ;
- Overstrength of link $\gamma_{ov.sh} = V_{u.exp.}/V_{y,EC8}$.

where a is the distance between two consecutive stiffening plates, t_w is the web thickness, d_w is the link depth, E is the Young's modulus of the steel, f_y and f_u are the yield and ultimate strength of the measured material properties, respectively, and e is the length of the link. $V_{u.exp.}$ is the measured ultimate shear; while $V_{y,EC8}$ is the shear resistance of link according to Eurocode 80:

$$V_{y,EC8} = \frac{A_s f_y}{\sqrt{3}} = \frac{t_w (h - t_f) f_y}{\sqrt{3}} \quad (1)$$

In Figure 2 the scheme of link with the indication of the geometry is reported.

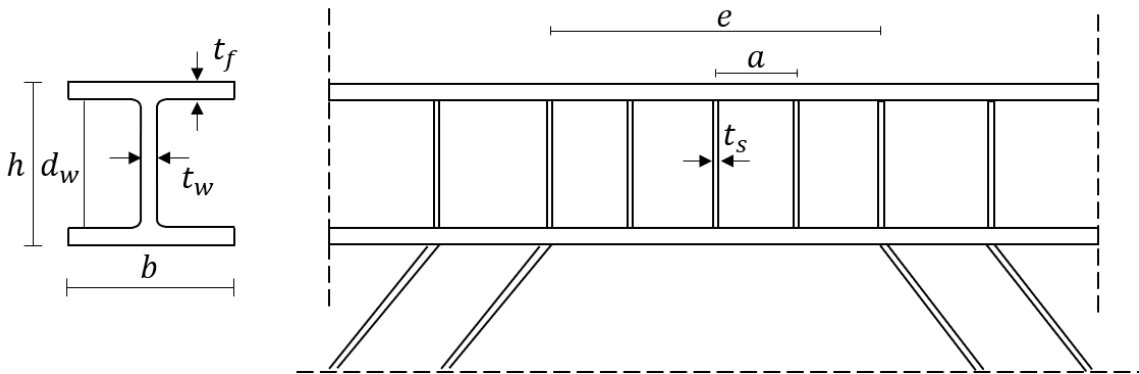


Figure 2 - Geometrical scheme of link in the EBFs

A link overstrength factor $\gamma_{ov.th}$ was evaluated considering all the geometrical and mechanical parameters previously described. In particular, the following multivariate regression is proposed

$$\gamma_{ov.th} = C_1 \left(\frac{a}{t_w} \right) + C_2(\lambda_w) + C_3(\lambda_s) + C_4 \left(\frac{f_u}{f_y} \right) + C_5 \theta_u + C_6 \bar{e} + C_7 \quad (2)$$

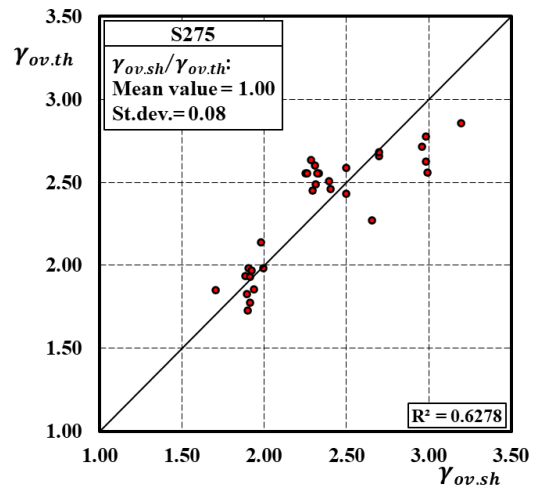
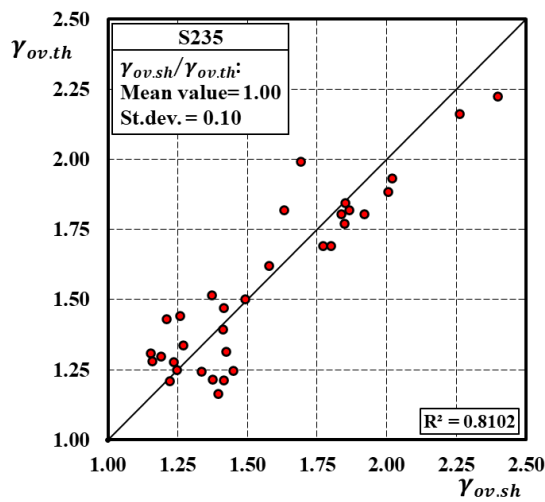
where C_i represent the coefficients whose values are reported in

Table 1 for the considered three steel grades.

For evaluating the accuracy of the last relationship, a comparison between the values of experimental overstrength $\gamma_{ov.sh}$, provided in Appendix A, and the values of $\gamma_{ov.th}$ obtained using Equation (2) is reported in Figure 3. For each steel grade, it is observed that the mean is equal to 1.00, while the standard deviation increases for increasing steel grades. In fact, for the short links made of steel S235, the standard deviation is 0.10 while it is 0.06 for links of steel S355. For S275 steel grade, the standard deviation assumes a middle and it is equal to 0.08. The high dispersion given by S235 is linked to its production. In fact, it is known that any waste material, belonging to other steel classes, are often classified as S235 making the inherent properties of such steel affected by a high dispersion.

Table 1 - The empirical coefficients of regression(2), adopted for different steel grades

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
S235	-0.011	-0.522	0.019	-1.274	-2.457	-0.152	4.554
S275	0.268	-6.679	0.480	0.463	-2.585	1.068	-7.120
S355	-0.052	3.061	-0.001	2.992	2.527	0.581	-6.246



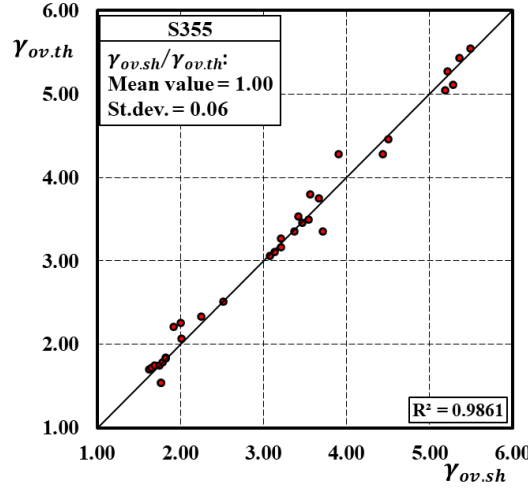


Figure 3 - Accuracy of the Eq. (5) for the steels of grade S235, S275 and S355

3 Probabilistic approach for the local hierarchy criterion

For ensuring the yielding of only link and consequently to assure the maximum dissipation of system, according to rigid-plastic analysis, the following conditions is suggested for K-scheme and Inverted Y scheme links:

$$M_d + M_b \geq M_{link} \quad (3)$$

where M_d and M_b are, respectively, the plastic moment of diagonal members and beam, while M_{link} represents is the equivalent moment of the link. By considering the overstrength factor $\gamma_{ov.th}$, the Equation (3) can be rearranged as:

$$W_{pl.d}f_{y.nom} + W_{pl.b}f_{y.nom} \geq \gamma_{ov.th} \left(V_{p.link} \frac{e}{2} \right) \quad (4)$$

where $W_{pl.d}$ and $W_{pl.b}$ are, the plastic modulus of diagonal members and beam, respectively, $f_{y.nom}$ is the nominal yield stress of steel, $V_{p.link}$ represents the ultimate shear resistance of link (Equation (1)) and e is the length of the link. It is computed using the nominal yield stress $f_{y.nom}$.

However, the relationship, presented in Equation (4), does not consider the randomness of the yield stress of the material and the uncertainty in the evaluation of overstrength factor. For this reason, a probabilistic procedure is applied in closed form to the deterministic version of Local Hierarchy Criterion, to provide a Safety Coefficient γ_{Rd} to be used as follows:

$$W_{pl.d}f_{y.nom} + W_{pl.b}f_{y.nom} \geq \gamma_{Rd}\gamma_{ov.th} \left(V_{p.link} \frac{e}{2} \right) \quad (5)$$

Starting from the deterministic relationship the limit-state condition G representing the separation between accepted and undesired event is defined. The reliability index of G is computed using a probabilistic method of the second level. Applying the cumulative distribution function to the previous index, it is computed the Probability of Failure (or

Probability of Success) of the limit-state condition, namely the probability of success of the design goal assuring only the link yielding.

3.1 Probabilistic assumptions

The assumptions of this procedure are referred to the definition of stochastic distribution of the random variables, concerning the variability of material and the randomness of the regression model. Specifically:

1. The steel material of members is the same.
2. The variability of the yield stress of the steel is represented by the Gaussian random variable Y , with mean μ_Y and standard deviation σ_Y :

$$Y \sim N(\mu_Y; \sigma_Y) \quad (6)$$

3. The uncertainty of model regression is indicated by another gaussian variable X with mean μ_X and standard deviation σ_X , expressed as:

$$X = \frac{\gamma_{ov.sh}}{\gamma_{ov.th}} \Rightarrow X \sim N(\mu_X; \sigma_X) \quad (7)$$

where $\gamma_{ov.sh}$ is the value of overstrength, provided by the experimental tests, while $\gamma_{ov.th}$ is the overstrength of link, computed through Equation (4).

4. There is no correlation between the random variables X and Y .

The last assumption can be verified proving that the correlation index presents a minimum value to be neglected. These assumptions allow using a reliability method to evaluate a reliability index for an assumed failure (or success) probability.

3.2 Application of first order reliability method (FORM)

Concerning a given limit-state condition, it is generally possible to define a function G , called the *limit-state function* of the stochastic variables:

$$\begin{aligned} G &> 0 \text{ (if the limit - state is not exceeded)} \\ G &= 0 \text{ (if the limit - state is reached)} \\ G &< 0 \text{ (if the limit - state is exceeded)} \end{aligned} \quad (8)$$

So, according to condition $G = 0$, i.e. when the limit-state is reached, Equation (3) can be arranged in the following form:

$$G(X, Y): M_d + M_b - M_{link} = 0 \quad (9)$$

Through the assumption 1, it is possible to introduce M_y , which represents the sum of the plastic moment of non-dissipative zones ($M_y = M_d + M_b$):

$$G(X, Y): M_y - M_{link} = 0 \quad (10)$$

Introducing the random variables X and Y and expressing the plastic moment through the geometrical properties of members, Equation (10) becomes:

$$G(X, Y): (W_{pl.d} + W_{pl.b})Y - (X \cdot Y) \left[\gamma_{ov.th} \frac{t_w(h - t_f)e}{2\sqrt{3}} \right] = 0 \quad (11)$$

To simplify the following steps, A and B are introduced to replace the multiplier factors of gaussian variables:

$$G(X, Y): A \cdot Y - B \cdot (X \cdot Y) = 0 \quad (12)$$

where:

$$A = W_{pl.d} + W_{pl.b} \quad B = \gamma_{ov.th} \left[\frac{t_w(h - t_f)e}{2\sqrt{3}} \right] \quad (13)$$

The probability of failure of function G is the condition of exceeding the limit-state. In this specific case, the condition is:

$$P_f = P_r\{G < 0\} = P_r\{B \cdot (X \cdot Y) - A \cdot Y > 0\} \quad (14)$$

For computing this probability, considering that the function G is the linear combination of the gaussian variables, the cumulative distribution can be applied to the reliability index β which represents the distance between the mean of function and the limit surface of probability which describes the boundary condition of the limit-state. In mathematical terms, this is equal to the ratio between the expected value and the standard deviation of the function G :

$$P_f = \Phi(-\beta) = \Phi\left(-\frac{E[G]}{\sqrt{VAR[G]}}\right) \quad (15)$$

where $\Phi(\cdot)$ is the cumulative distribution function (CDF), $E[\cdot]$ is an expected value linear operator, which provides the mean of G , while $VAR[\cdot]$ is a non-linear operator whose result is the variance of G :

$$P_s = 1 - P_f = 1 - \Phi(-\beta) \quad (16)$$

Considering the symmetry of limit-state function, the success probability, verifying that the limit-state is not exceeded, is complementary to the failure event:

$$P_s = 1 - \Phi(-\beta) = \Phi(\beta) = \Phi\left(\frac{E[G]}{\sqrt{VAR[G]}}\right) \quad (17)$$

Hence, the mean and the variance values of G must be provided. Using the linearity of the operator $E[\cdot]$, the mean value is equal to:

$$E[G] = A \cdot E[Y] - B \cdot E[X]E[Y] = A\mu_Y - B\mu_X\mu_Y \quad (18)$$

where μ_X and μ_Y are, respectively, the means of X and Y ; while the variance of G is:

$$\text{VAR}[G] = A^2 \cdot \text{VAR}[Y] + B^2 \cdot \text{VAR}[X \cdot Y] \quad (19)$$

Applying the Taylor series stopped to the second order, the variance of the product between two random variables is equal to:

$$\text{VAR}[X \cdot Y] = \mu_X^2 \sigma_Y^2 + \mu_Y^2 \sigma_X^2 + \sigma_X^2 \sigma_Y^2 \quad (20)$$

where σ_X and σ_Y represent the standard deviations of Gaussian variables. So, the Equation (19) can be expressed as follows:

$$\text{VAR}[G] = (A\sigma_Y)^2 + B^2(\mu_X^2 \sigma_Y^2 + \mu_Y^2 \sigma_X^2 + \sigma_X^2 \sigma_Y^2) \quad (21)$$

Introducing the coefficient of variation as the ratio between the deviation standard and the mean of the stochastic variable, the Equation (21) can be rearranged as:

$$\begin{aligned} \text{VAR}[G] &= (Ac_Y\mu_Y)^2 + B^2(\mu_X^2\mu_Y^2c_Y^2 + \mu_Y^2\mu_X^2c_X^2 + \mu_Y^2\mu_X^2c_Y^2c_X^2) = \\ &= (Ac_Y\mu_Y)^2 + B^2\mu_Y^2\mu_X^2(c_X^2 + c_Y^2 + c_Y^2c_X^2) \end{aligned} \quad (22)$$

where c_X and c_Y are, respectively, the variation coefficients of X and Y . Substituting the Equations (18) and (22) in Equation (17), the probability of success is equal to:

$$P_s = \Phi \left(\frac{A\mu_Y - B\mu_Y\mu_X}{\sqrt{(Ac_Y\mu_Y)^2 + B^2\mu_Y^2\mu_X^2(c_X^2 + c_Y^2 + c_Y^2c_X^2)}} \right) \quad (23)$$

Finally, fixed value for the success probability (or failure probability), it is possible to apply the reverse function of the cumulative distribution, obtaining u_Φ , which represents the normal standard variable associated to the value of P_s :

$$u_\Phi = \Phi^{-1}(P_s) = \frac{A\mu_Y - B\mu_Y\mu_X}{\sqrt{(Ac_Y\mu_Y)^2 + B^2\mu_Y^2\mu_X^2(c_X^2 + c_Y^2 + c_Y^2c_X^2)}} \quad (24)$$

3.2.1 Central safety coefficient “ γ_0 ”

In the reliability context, for a fixed probability of success, the central safety coefficient γ_0 is defined as the ratio between the means of the *Capacity* and the *Demand*. In the examined case, the *Capacity* is represented by the plastic moments of the non-dissipative zones (diagonal members and beam), while the *Demand* is characterized by the ultimate resistance of link, as follows:

$$\gamma_0 = \frac{E[M_y]}{E[M_{link}]} = \frac{E[M_d + M_b]}{E[M_{link}]} \quad (25)$$

Expressing M_y and M_{link} as a function of stochastic variables and using the linearity of the $E[\cdot]$, Equation (25) becomes:

$$\gamma_0 = \frac{E[A \cdot Y]}{E[B \cdot XY]} = \frac{A\mu_Y}{B\mu_Y\mu_X} = \frac{A}{B\mu_X} \quad (26)$$

Determining A as a function of γ_0 and substituting in the Equation (23), it is provided that:

$$u_\Phi = \frac{\gamma_0 B \mu_X \mu_Y - B \mu_X \mu_Y}{\sqrt{(\gamma_0 B c_Y \mu_X \mu_Y)^2 + B^2 \mu_X^2 \mu_Y^2 (c_X^2 + c_Y^2 + c_X^2 c_Y^2)}} \quad (27)$$

Then, it is possible to express the central coefficient as a function of u_Φ , c_X and c_Y :

$$\gamma_0 = \frac{1 + \sqrt{1 - (1 - u_\Phi^2 c_Y^2)[1 - u_\Phi^2 (c_X^2 + c_Y^2 + c_X^2 c_Y^2)]}}{1 - u_\Phi^2 c_Y^2} \quad (28)$$

Moreover, it is observed that this result is obtained under the assumption of no correlation between the random variables. However, it is possible to consider this condition, introducing the linear correlation index ρ_{XY} as:

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} \quad (29)$$

where σ_{XY} represents the covariance between the Gaussian variables, while σ_X and σ_Y are the standard deviations. Consequently, through some mathematical steps, the Equation (28) can be rearranged as:

$$\gamma_0 = \frac{1 + \sqrt{C^2 - (1 - u_\Phi^2 c_Y^2)[C^2 - u_\Phi^2 (c_X^2 + c_Y^2 + c_X^2 c_Y^2)]}}{1 - u_\Phi^2 c_Y^2} \quad (30)$$

where C is equal to:

$$C = 1 + \rho_{XY} c_X c_Y \quad (31)$$

3.2.2 Safety coefficient “ γ_{Rd} ”

With a few considerations, starting from the expression of central safety coefficient γ_0 , the safety coefficient γ_{Rd} can be obtained, by deriving the fractile values of the *Capacity* and *Demand*. In particular, by the introduction of the standard normal variable $z_{p,Y}$, referred to variable Y , the design bending moment $M_{y,Rd}$ is equal to:

$$M_{y,Rd} = E[M_y](1 + z_{p,Y} c_Y) \quad (32)$$

Regarding the *Demand*, the same procedure can be applied, but, preliminarily, it is necessary to introduce the coefficient of variation c_Z , referred to the new variable Z which is the product between the stochastic variables X and Y . Hence, from Equations (18) and (20), where the mean and variance of the product of X and Y are reported, c_Z can be expressed as :

$$c_Z = \frac{\sigma_Z}{\mu_Z} = \frac{\sqrt{\mu_X^2 \sigma_Y^2 + \mu_Y^2 \sigma_X^2 + \sigma_X^2 \sigma_Y^2}}{\mu_X \mu_Y} \quad (33)$$

and by considering σ_X and σ_Y with the coefficients of variation c_X and c_Y , it follows:

$$c_Z = \frac{\sigma_Z}{\mu_Z} = \sqrt{c_Y^2 + c_X^2 + c_X^2 c_Y^2} \quad (34)$$

Then, introducing the standard variable $z_{p,Z}$, the value of the design equivalent bending moment of the link is equal to:

$$M_{link.Rd} = E[M_{link}](1 + z_{p,Z}c_Z) \quad (35)$$

Finally, it is possible to define the safety factor γ_{Rd} as the ratio between the design values of the *Capacity* and the *Demand*:

$$\gamma_{Rd} = \frac{M_{y.Rd}}{M_{link.Rd}} = \frac{E[M_y](1 + z_{p,Y}c_Y)}{E[M_{link}](1 + z_{p,Z}c_Z)} = \gamma_0 \frac{(1 + z_{p,Y}c_Y)}{(1 + z_{p,Z}c_Z)} \quad (36)$$

So, computed the principle probabilistic index, after establishing a reliability target (u_Φ) and the values of the fractiles $z_{p,Y}$ and $z_{p,Z}$, the safety coefficient γ_{Rd} is, uniquely, defined and Equation (5) can be applied for verifying the local hierarchy criterion in the design of short links. In the following an example of this application is reported. In particular, considering the 95.00 % success probability, i.e. u_Φ equal to 1.645 and imposing 5% fractile for the standard variables ($z_{p,Y} = z_{p,Z} = -1.645$), the central and safety coefficients are computed for same steel grades considered in the evaluation of overstrength factor $\gamma_{ov.th}$. These results are reported in

Table 2.

Table 2 - Results of the FORM application for $u_\Phi = 1.645$ ($P_s = 95.00\%$)

	μ_Y	σ_Y	c_Y	μ_X	σ_X	c_X	c_Z	γ_0	γ_{Rd}
S235	274.67	40.52	0.15	1.00	0.11	0.10	0.18	1.46	1.56
S275	291.94	22.90	0.08	1.00	0.08	0.08	0.11	1.26	1.34
S355	366.15	15.20	0.04	1.00	0.06	0.06	0.07	1.14	1.20

It is important observing that the correlation index ρ_{XY} is evaluated for each steel grade (S235, S275, S355) and it is always less than 10%, therefore, it can be neglected.

Moreover, analyzing the values of the central coefficients γ_0 and, consequently, the safety coefficients γ_{Rd} , it is observed that they increase for increasing the yield strength of the steel grade.

Instead, in Figure 4 and Figure 5, the trends of the safety coefficients are reported by varying the reliability level (u_Φ). It is observed that when this value increases, the coefficient to be applied to the local criterion takes on higher values for decreasing the steel grade.

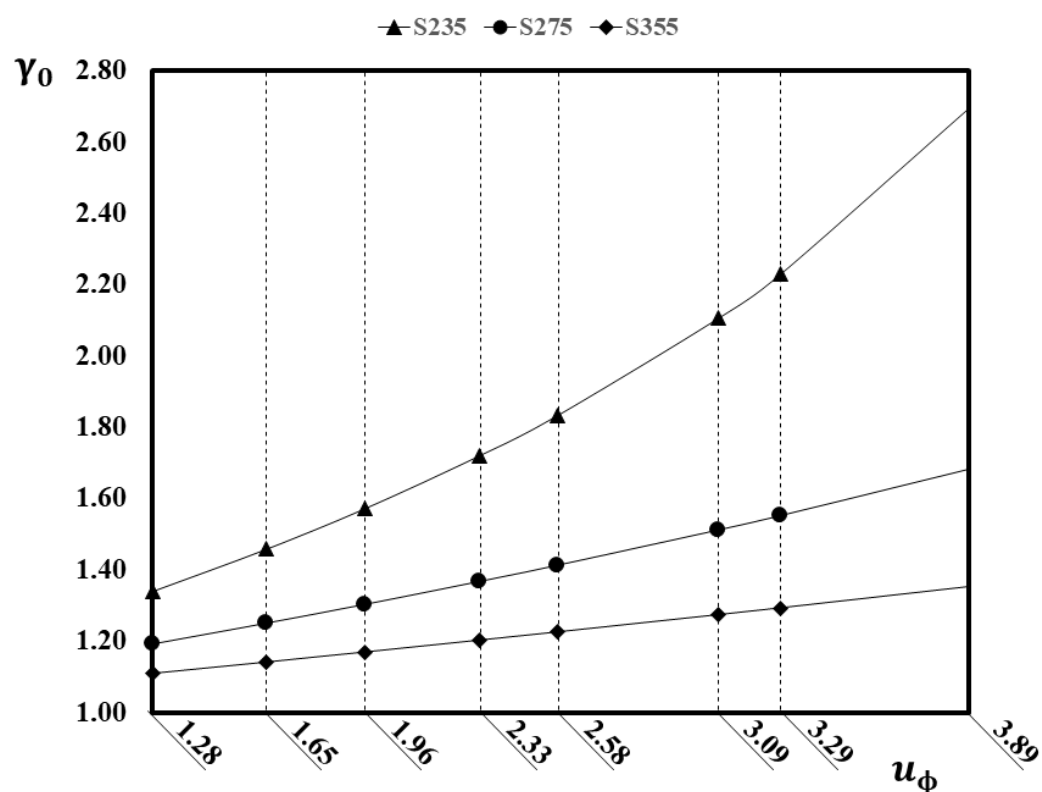


Figure 4 - The trend of the central safety factor γ_0 as a function of u_ϕ , for each steel grade.

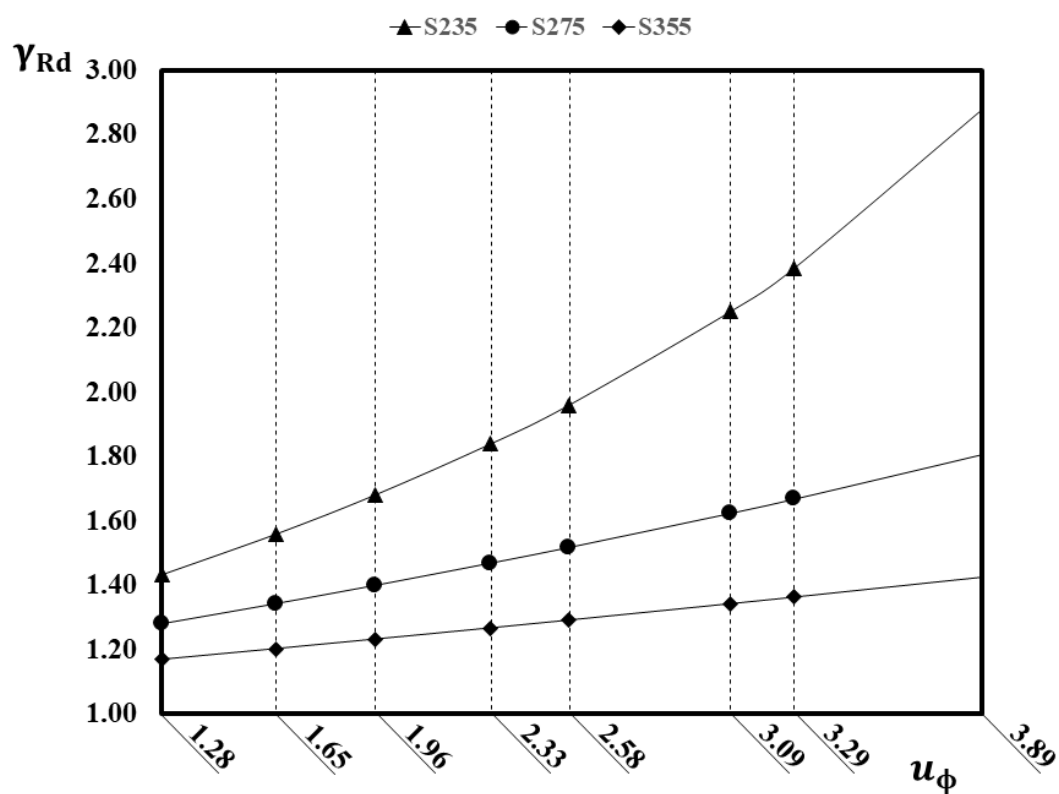


Figure 5 - The trend of the overstrength factor γ_{Rd} as a function of u_ϕ , for each steel grade

4 Application of the probabilistic approach in the local design criterion

Given the above, the local design criterion for Eccentrically Braced Frames (EBFs) can be applied according to the following steps:

1. definition of the vertical and seismic loads.
2. design of the link, according to the first principle of *Capacity Design*, i.e. as a function of the maximum internal actions provided by the load combinations.
3. determination of Overstrength source of the link by Equation(2). However, other models can be applied, as long as it is possible to characterize the uncertainty of the model to satisfy the assumption n. 3 on which the FORM is based.
4. assumption of a reliability Level (u_Φ), the safety coefficient γ_{Rd} is provided by Equation (36);
5. design the non-dissipative zones according to the following condition:

$$M_{b.Rd} + M_{d.Rd} \geq \gamma_{Rd} \gamma_{ov.th} M_{link.Rd}$$

The designer is not required to follow the probabilistic approach but only to compute the two overstrength factor $\gamma_{ov.th}$ by using Equation (2) and γ_{Rd} through Figure 4 and Figure 5 given the probability of success in achieving the design goal, namely the link yielding and the prevention of yielding of diagonal and beams. This practical design procedure is summarized in the flowchart in Figure 6.

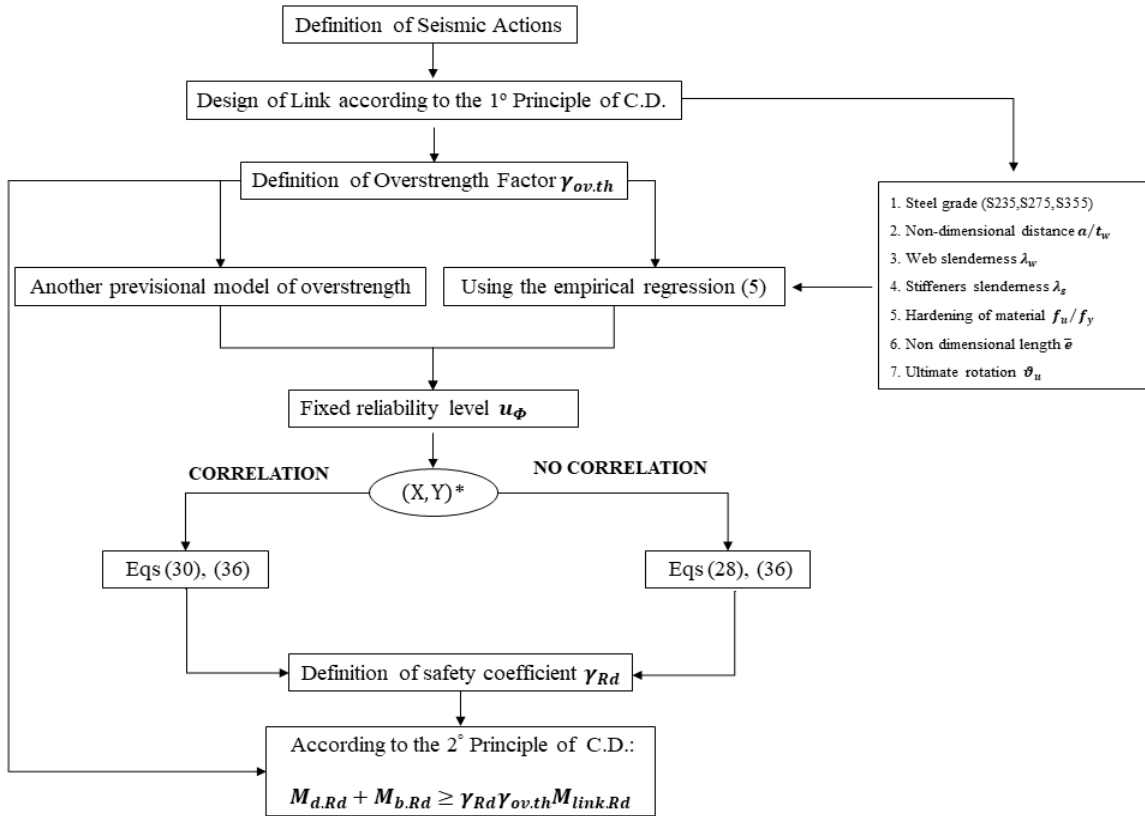


Figure 6 - Flowchart of the design of short links, according to the probabilistic version

5 Conclusions

In this paper, a design procedure to be applied for the local design of Eccentrically Braced Frames is proposed. According to the Capacity Design principles, non-dissipative zones must be designed through the maximum actions transmitted by the link in its ultimate condition. For this reason, the estimation of the overstrength sources of the link is of paramount importance.

As the Eurocode 8 does not provide any local design criterion for the EBFs the starting point of this design procedure is a local hierarchy criterion developed in the framework of the kinematic theorem of plastic collapse.

After collecting a large number of experimental data, reported in the technical-scientific literature, an empirical relationship is provided to predict the overstrength factor $\gamma_{ov.th}$ as function of the main following parameters as the non-dimensional distance between two stiffening, web slenderness, slenderness of the stiffening plates, hardening of the material, non-dimensional length of link and ultimate rotation.

Subsequently, to account for the material randomness and the uncertainty of the regression model used to compute the overstrength factor, a probabilistic procedure is applied to calibrate a safety coefficient γ_{Rd} for the deterministic version of local hierarchy criterion. The local hierarchy criterion can be regarded as a probabilistic event and the First Order Reliability Method (FORM) is used to compute the reliability index β , for a fixed probability of success in the design goal.

So, using both the coefficients $\gamma_{ov.th}$ and γ_{Rd} , the local hierarchy criterion is satisfied with a given target probability. The proposed procedure allows to design locally, the sections of diagonal and beam members, for this reason, this method must be associated with other design procedures in the framework of a global structural design. In light of future development, this probabilistic application could be extended to intermediate and long links. However, for these links, the available experimental data are not enough therefore opportunely calibrated FEM simulations can be performed to fill the lack of experimental results.

6 Appendix

In the following tables, the experimental data, presented in the scientific literature are reported:

Table A.1 - *Experimental Data for steels of grade S235*

Authors	No.	a/t_w	λ_w	λ_s	f_u/f_y	θ_u	\bar{e}	$\gamma_{ov,sh}$
Maalek et al, n.2	2	37.88	1.54	24.96	1.39	0.07	2.25	1.37
Maalek et al, n.3	3	37.88	1.53	24.96	1.40	0.08	2.25	1.49
Maalek et al, n.4	4	37.88	1.53	24.96	1.40	0.09	2.25	1.41
Yin et al, n.2	L2	33.33	1.41	23.20	1.63	0.08	2.67	1.42
Yin et al, n.3	L3	33.33	1.40	23.00	1.63	0.10	2.12	1.25
Yin et al, n.4	L4	25.00	1.39	22.80	1.63	0.15	1.95	1.45
Yin et al, n.5	L5	28.57	1.71	32.80	1.63	0.14	1.58	1.15
Yin et al, n.6	L6	28.57	1.71	32.80	1.63	0.09	1.58	1.21
Yin et al, n.7	L7	21.43	1.71	32.80	1.63	0.16	1.58	1.27
Yin et al, n.8	L8	21.43	1.71	32.80	1.63	0.15	1.35	1.41
McDaniel et al, n.1	TYPE 1	15.27	1.64	39.09	1.43	0.06	0.88	2.26
McDaniel et al, n.2	TYPE 3	11.36	1.64	39.09	1.43	0.07	0.66	2.40
Ji et al, n.1	L11C	22.00	1.25	38.00	1.45	0.14	2.23	1.85
Ji et al, n.2	L11D	22.00	1.25	38.00	1.45	0.08	2.23	1.69
Ji et al, n.3	L11	22.00	1.25	38.00	1.45	0.15	2.23	1.86
Ji et al, n.4	L12	22.00	1.25	38.00	1.45	0.17	2.23	1.85
Ji et al, n.5	L13	22.00	1.25	38.00	1.45	0.15	2.23	1.63
Ji et al, n.6	L21	22.00	1.25	38.00	1.45	0.15	1.49	2.02
Ji et al, n.7	L22	22.00	1.25	38.00	1.45	0.17	1.49	2.00
Ji et al, n.8	Q11	22.00	1.40	38.00	1.52	0.13	2.23	1.80
Ji et al, n.9	Q12	22.00	1.40	38.00	1.52	0.13	2.23	1.77
Ji et al, n.10	Q13	33.00	1.40	38.00	1.52	0.11	2.23	1.58
Ji et al, n.11	Q21	22.00	1.40	38.00	1.52	0.13	1.49	1.92
Ji et al, n.12	Q22	22.00	1.40	38.00	1.52	0.13	1.49	1.84
Dusicka et al, n.4	LL7-m	44.00	1.92	39.09	1.77	0.03	1.72	1.22
Okazaki et al, n.1	LL7-c1	28.90	1.92	38.00	1.77	0.06	1.72	1.19
Okazaki et al, n.2	LL7-c2	21.40	1.92	38.00	1.77	0.10	1.72	1.16
Okazaki et al, n.3	LL6-m	33.80	1.92	38.00	1.77	0.06	1.72	1.34
Okazaki et al, n.4	LL6-c1	29.72	1.92	38.00	1.77	0.06	1.72	1.24
Okazaki et al, n.5	LL6-c2	28.00	2.20	38.00	1.51	0.06	1.99	1.26
Okazaki et al, n.6	LL5-m	37.40	1.92	38.00	1.77	0.04	2.21	1.39
Okazaki et al, n.7	LL5-c1	44.70	2.45	38.00	1.39	0.03	2.88	1.38
Okazaki et al, n.8	LL5-c2	36.30	2.20	38.00	1.51	0.04	2.55	1.43

Table A.2 - *Experimental Data for steels of grade S275*

Authors	No.	a/t _w	λ_w	λ_s	f _u /f _y	θ_u	\bar{e}	$\gamma_{ov.sh}$
Bozkurt et al, n.1 (2017)	1	25.00	0.83	13.40	1.41	0.14	1.65	2.34
Bozkurt et al, n.2 (2017)	2	25.00	0.83	13.40	1.41	0.10	1.65	2.70
Bozkurt et al, n.3 (2017)	3	25.00	0.83	13.40	1.41	0.14	1.65	2.33
Bozkurt et al, n.4 (2017)	4	25.00	0.83	13.40	1.41	0.14	1.65	2.26
Bozkurt et al, n.5 (2017)	5	25.00	0.83	13.40	1.41	0.14	1.65	2.27
Bozkurt et al, n.6 (2017)	6	22.22	0.83	13.40	1.41	0.11	2.20	2.30
Bozkurt et al, n.7 (2017)	7	21.43	1.00	18.80	1.41	0.12	1.13	2.32
Bozkurt et al, n.8 (2017)	8	21.43	1.00	18.80	1.41	0.14	1.13	2.50
Bozkurt et al, n.1 (2018)	1	25.00	0.92	13.40	1.40	0.14	1.65	1.89
Bozkurt et al, n.2 (2018)	2	25.00	0.83	13.40	1.59	0.14	1.65	2.31
Bozkurt et al, n.3 (2018)	3	22.22	0.92	13.40	1.31	0.08	2.20	1.71
Bozkurt et al, n.4 (2018)	4	21.43	1.11	18.80	1.31	0.12	1.13	1.90
Bozkurt et al, n.5 (2018)	5	21.43	0.99	18.80	1.58	0.14	1.13	2.50
Bozkurt et al, n.6 (2018)	6	25.00	0.83	13.40	1.58	0.14	1.65	2.29
Bozkurt et al, n.7 (2018)	7	25.00	0.83	13.40	1.58	0.12	1.65	2.70
Bozkurt et al, n.8 (2018)	8	22.22	0.83	13.40	1.51	0.11	2.20	2.40
Bozkurt et al, n.9 (2018)	9	21.43	1.00	18.80	1.49	0.14	1.13	2.41
Azad et al, n.1	1	25.00	0.83	13.40	1.47	0.03	1.65	3.20
Azad et al, n.2	2	25.00	0.83	13.40	1.47	0.06	1.65	2.99
Azad et al, n.3	3	25.00	0.83	13.40	1.48	0.09	1.65	2.96
Azad et al, n.4	4	25.00	0.83	13.40	1.47	0.12	1.65	2.99
Azad et al, n.5	5	25.00	0.83	13.40	1.48	0.15	1.65	3.00
Azad et al, n.6	6	22.22	0.83	13.40	1.48	0.20	2.20	2.66
Azad et al, n.7	7	25.60	1.66	23.00	1.63	0.03	1.68	1.98
Azad et al, n.8	8	25.60	1.66	23.00	1.63	0.09	1.68	2.00
Azad et al, n.9	9	25.60	1.66	23.00	1.63	0.15	1.68	1.90
Azad et al, n.10	10	25.60	1.66	23.00	1.63	0.09	1.68	1.91
Azad et al, n.11	11	25.60	1.66	23.00	1.63	0.11	1.68	1.92
Azad et al, n.12	12	25.60	1.66	23.00	1.63	0.14	1.68	1.94
Azad et al, n.13	13	25.60	1.66	23.00	1.63	0.10	1.68	1.93
Azad et al, n.14	14	25.60	1.66	23.00	1.63	0.17	1.68	1.92

Table A.3 - *Experimental Data for steels of grade S355*

Authors	No.	a/t_w	λ_w	λ_s	f_u/f_y	θ_u	\bar{e}	$\gamma_{ov.sh}$
Mansour et al, n.1	UT-3A	19.05	1.28	32.04	1.29	0.10	1.31	1.77
Mansour et al, n.2	UT-3B	19.05	1.28	32.04	1.29	0.11	1.31	1.78
Mansour et al, n.3	EPM-11A	23.26	1.56	31.98	1.29	0.10	1.42	1.92
Mansour et al, n.4	EPM-11B	23.26	1.56	31.98	1.29	0.11	1.42	2.01
Mansour et al, n.5	UT-1A	27.78	1.62	27.96	1.29	0.13	3.29	3.38
Mansour et al, n.6	UT-1B	27.78	1.62	27.96	1.29	0.17	3.29	3.48
Mansour et al, n.7	UT-2A	34.48	2.10	29.06	1.29	0.07	1.52	3.22
Mansour et al, n.8	UT-2B	34.48	2.10	29.06	1.29	0.10	1.52	3.72
Mansour et al, n.9	EPM-12	30.00	2.01	24.02	1.29	0.11	2.23	3.68
Mansour et al, n.10	EPM-13	19.80	2.01	24.02	1.29	0.10	2.23	3.92
Mansour et al, n.11	EPM-14	19.80	2.01	24.02	1.29	0.10	2.23	4.45
Mansour et al, n.12	EPM-16	30.00	2.01	24.02	1.29	0.13	2.23	3.57
Mansour et al, n.13	EPM-15	30.00	2.01	24.02	1.29	0.09	1.94	3.43
Lian et al, n.1	1	19.44	1.50	20.50	1.39	0.09	1.07	2.26
Okazaki, n.1	1A	22.81	1.64	23.92	1.35	0.04	2.30	3.09
Okazaki, n.2	1B	22.81	1.64	23.92	1.35	0.06	2.30	3.15
Okazaki, n.3	1C	22.81	1.64	23.92	1.35	0.08	2.30	3.22
Okazaki, n.4	2	23.75	1.64	23.92	1.35	0.07	3.00	3.55
Okazaki, n.5	3	23.75	1.64	23.92	1.35	0.04	4.80	4.51
Okazaki, n.6	4A	19.73	1.31	22.53	1.37	0.06	1.24	1.63
Okazaki, n.7	4B	19.73	1.31	22.53	1.37	0.07	1.24	1.67
Okazaki, n.8	4C	19.73	1.31	22.53	1.37	0.08	1.24	1.70
Okazaki, n.9	5	21.08	1.31	22.53	1.37	0.07	1.97	2.03
Okazaki, n.10	6A	32.97	1.31	22.53	1.37	0.05	2.58	1.76
Okazaki, n.11	6B	32.97	1.31	22.53	1.37	0.06	2.58	1.80
Okazaki, n.12	7	32.97	1.31	22.53	1.37	0.04	3.93	2.52
Okazaki, n.13	8	17.73	2.21	38.20	1.46	0.08	2.21	5.36
Okazaki, n.14	9	27.07	2.21	38.20	1.46	0.05	2.90	5.23
Okazaki, n.17	12	18.25	2.31	22.50	1.38	0.09	1.41	5.19
Okazaki, n.18	4A-RLP	19.73	1.31	22.50	1.37	0.12	1.24	1.84
Okazaki, n.19	4C-RLP	19.73	1.31	42.82	1.37	0.12	1.24	1.83
Okazaki, n.20	8-RLP	17.73	2.21	22.53	1.46	0.12	2.21	5.50
Okazaki, n.23	12-RLP	18.25	2.31	22.53	1.38	0.12	1.41	5.29

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List of symbols

b Width of the link section;	$W_{pl.d}$ Plastic modulus of diagonal members;
h Total height of the link section;	$W_{pl.b}$ Plastic modulus of beam;
t_w Web thickness of link;	$f_{y.nom}$ Nominal yield stress of steel;
t_f Flange thickness of link;	$V_{u.nom}$ Ultimate shear of link;
a Distance between the consecutive stiffeners;	X, Y, Z Random variables;
d_w Height of link section;	$G()$ Limit-state function;
E Young's modulus of steel;	P_f Failure probability;
f_y Experimental yield strength of material;	P_s Success probability;
f_u Experimental ultimate strength of material;	β Reliability index;
A_s Shear area of link;	$\Phi()$ Function of cumulative distribution;
e Length of link;	$E[]$ Linear operator (expected value);
a/t_w Non-dimensional distance between the consecutive stiffeners;	$VAR[]$ Variance operator;
λ_w Web slenderness;	μ_X, μ_Y, μ_Z Means of variables X, Y, Z ;
λ_s slenderness of stiffening plates;	$\sigma_X, \sigma_Y, \sigma_Z$ Standard deviations of variables X, Y, Z ;
f_u/f_y Strain-hardening of material;	c_X, c_Y, c_Z Variation coefficients of variables X, Y, Z ;
\bar{e} Non-dimensional length of link;	u_Φ standard random variable;
ϑ_u Ultimate rotation of link;	γ_0 Central safety factor;
$V_{u.exp.}$ Ultimate shear of link, declared in experimental tests;	z_p Standard variable corresponding to 5% probability;
$V_{y.EC8}$ Shear resistance according to EC8;	γ_{Rd} Safety factor;
$\gamma_{ov.sh}$ Experimental overstrength of link;	σ_{XY} Covariance between the variables X, Y ;
M_d Plastic moment of diagonal members;	ρ_{XY} Linear correlation index between X, Y ;
M_b Plastic moment of diagonal beam;	$M_{d.Rd}$ Design plastic moment of diagonal member;
M_{link} Ultimate moment of link;	$M_{b.Rd}$ Design plastic moment of beam;
M_y Sum of the plastic moments M_b and M_d ;	$M_{y.Rd}$ Sum of the moments $M_{b.Rd}$ and $M_{d.Rd}$;
$\gamma_{ov.th}$ Theoretical overstrength of link;	$M_{link.Rd}$ Design plastic moment of link;

APPROCCIO PROBABILISTICO PER CRITERIO DI GERARCHIA LOCALE DI CONTROVENTI ECCENTRICI (EBFs)

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SUMMARY: Il presente lavoro ha come obiettivo lo studio e la valutazione della sovra-resistenza dei link corti per controventi eccentrici da considerare nell'ambito del criterio di gerarchia locale delle resistenze. In particolare, a partire da un database costruito su circa 238 prove sperimentali, sono stati estratti i soli risultati riferiti a link corti. Successivamente è stata effettuata una regressione multivariata per valutare la sovra-resistenza in funzione di una serie di parametri geometrici e meccanici, quali: la snellezza d'anima del link, la snellezza dei piatti di irrigidimento, la spaziatura adimensionale degli irrigidimenti, l'incrudimento del materiale, la lunghezza adimensionale e infine la rotazione ultima. Al fine di tener conto dell'incertezza del modello di regressione adottato e della aleatorietà delle proprietà meccaniche del materiale, un metodo affidabilistico del primo ordine (FORM) è stato applicato alla funzione di stato limite che descrive il criterio di gerarchia locale dei link corti, ottenuta mediante una analisi rigido-plastica. In tal modo, oltre a definire un coefficiente di sovra-resistenza, funzione delle proprietà meccaniche e geometriche dei link, è stato definito un coefficiente di sicurezza da applicare nel criterio di gerarchia locale al fine di tener conto delle incertezze in gioco.

KEYWORDS: *Principi del Capacity Design, Controventi Eccentrici (EBFs), Link corti, approccio stocastico, Metodo affidabilistico (FORM)*