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SEISMIC ASSESSMENT OF IRREGULAR EXISTING BUILDING: APPRAISAL OF THE INFLUENCE OF COMPRESSIVE STRENGTH VARIATION BY MEANS OF NONLINEAR CONVENTIONAL AND MULTIMODAL STATIC ANALYSIS

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SUMMARY: *The paper presents a research study concerning some critical issues related to the seismic modelling and analysis of typical existing RC buildings in Southern Italy. When speaking of existing buildings, first important issue is represented by the reliable appraisal of the parameters involved in the structural modelling, such as the concrete mechanical strength. It is evident that a correct assessment is crucial for obtaining an effective structural model in the elastic and inelastic field. Another key point is the choice of the proper method of analysis, since in some cases the standard methods proposed by European Codes are not effective in order to evaluate the structural behaviour of existing buildings. In order to investigate the effects of these aspects on the structural response, extensive comparisons have been performed on different structural models, obtained by properly varying the concrete strength value and by considering also a possible inhomogeneous distribution of the concrete class at different floors. Considering the specific framework of irregular buildings, analyses and consequent discussions have been performed by comparing conventional and multimodal pushover analyses and nonlinear incremental dynamic analyses.*

KEYWORDS: *in situ investigation, reinforced concrete, seismic analysis, irregular buildings, nonlinear analyses,*

1 Introduction

In the last few years, the issue of the structural safety of existing building has encountered a large interest within the scientific community, above all with regard to the investigation of modeling approaches and method of seismic analysis. In Italy, in particular, Census Data show that more than 70% of the building stock was built before 1974, year in which the first seismic code was issued, that is to say, many buildings were built in the absence of any anti-seismic design and, additionally, with a high probability that the quality of structural materials is inadequate (low materials' quality and strength, poor execution quality). Beside the presence of masonry buildings within historical centers, thence, there is a widespread presence of RC structures characterized by a potential high vulnerability to seismic actions.
Besides the well-known difficulties related to modeling and computational aspects (structural modeling, methods of analysis, capacity models,...), the vulnerability analyses and structural assessment of existing building has to face the crucial issue of the preliminary knowledge of actual features and conditions of the building. This is often a very difficult task, especially when the original documentation is poor or completely lacking.
European Building code [Eurocode 8, 2005], as well as Chapter 8 of Italian Building Code [Italian Ministerial decree 14/08/2008], provides a performance based approach with some

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general rules and guidelines which practitioners should follow. The focus is to the phase of the preliminary knowledge, by providing a precise sequence of steps, the first of which is represented by the historical analysis that includes the retrieval of original design drawings, information about foundations, ground conditions, dimensions and cross-sectional properties of building elements, properties of materials, constructive details. In addition, the documentary information gathered in the first step shall be completed by a direct survey aimed at the assessment of geometry, building details and by the execution of experimental in situ tests, such as destructive (D) and non-destructive (ND), aimed at the mechanical characterization of materials. On the basis of this “Path of Knowledge”, a “Knowledge Level” (KL) can be defined, depending on the quantity and quality of information found (Table 1), as indicated by Eurocode 8 - EN1998-3, part 1. The Knowledge Level is then related to a “Confidence Factor” (CF), which represents an additional “safety factor” to be used in the assessment phase.

Table 1 – *Knowledge level*

Denomination	KL	CF
Limited	1	1.35
Normal	2	1.20
Full	3	1.00

After completing the Knowledge Process, the final objective is to assess the Safety Level with respect to the different Load combinations provided by the Code for the relevant limit states, i.e. to dead and variable loads and seismic loads, through an adequate structural modeling approach. Nonlinear (NL) analyses (static or dynamic) are recognized to be the most complete and realistic approach for seismic analysis, especially for the assessment of existing buildings, as suggested by European and Italian codes. The possibility of considering the NL features of materials, sections and elements allows, in fact, of providing a realistic computational model and, eventually, to predict with more accuracy the safety level of structure and calibrate the global retrofitting strategies, avoiding the resort to generalized, invasive and excessively costly interventions.

The present paper is focused on two specific features related to the vulnerability assessment: the effectiveness of different structural methods of analysis and the sensitivity to the variability of mechanical parameters of in situ materials. In particular, an application to a real case study is presented, by performing comparisons between “standard” NL analysis (as proposed by European Code) and a NL analysis approach which takes into account the contribute of higher modes through a multimodal method, which is particularly effective for R.C. buildings irregular in plan. Furthermore, a sensitivity analysis of the structural response with respect to the variation of some mechanical parameters of in situ materials is performed and discussed.

2 THE ASSESSMENT OF THE SEISMIC RESPONSE OF R.C BUILDINGS

2.1 METHODS OF ANALYSIS: A BRIEF REVIEW

Within the context of vulnerability assessment, European Codes propose different methods for evaluating the seismic response of existing R.C. buildings, among which the more accurate one is, ideally, the Nonlinear Dynamic (NLD) analysis. The method provides the following steps: definition of a proper NL model of the structure, by introducing material and

geometric non linearity and accounting for stiffness variation; selection of a minimum of 3 group of accelerograms; application of the accelerograms to the computational model by integrating the equation of motion in the time domain; evaluation of the structural performance. Results are expected to be more accurate than other methods and are usually provided in the form of diagrams reporting the values of intensity measure (IM) and engineering demand parameters (EDP) of the maximum effects. Nevertheless, NLD analysis is rarely used in ordinary engineering applications, due to its complexity and relevant computational effort, besides to the difficulty of choosing representative design accelerograms, which are able to consistently reproduce the post-elastic and hysteretic behavior of the structure. In recent years, Nonlinear Static (NLS) analysis has gained increasing importance as an alternative NL method of analysis, especially for the vulnerability assessment of existing buildings. It is proved that usually the resulting response well represents the envelope of all the possible dynamic responses, and can so be used to replace a full NLD analysis. It can provide information about some important response characteristics that cannot be obtained from linear analyses and, at the same time, it is relatively easier than NLD analysis. Conventional NLS analysis consists in the application of a proper lateral load profile to the structure and then monotonically increasing it until the collapse mechanism is reached. On the basis, of this analysis method, Eurocode 8 proposes the “N2 Method”, which is a "Displacement Based" methodology [Fajfar, 2000] that allows evaluating the seismic response of a building. The name of the method derives from two distinctive features. The letter “N” indicates that the method is NL, whereas “2” refers to the use of two different computational models of the structure: a Multi-Degree of Freedom (MDoF) model, on which a pushover numerical analysis is performed, and an “equivalent” Single-Degree of Freedom (SDoF) model, which is derived by the previous one trough proper manipulations, and is used for the seismic assessment by the design response spectrum.
As previously mentioned, the 1st step of N2 method consists in the application of a horizontal static force profile on the structure that is monotonically increased until collapse is attained. Contextually, a control point, which usually coincides with the center of mass of the top floor, is monitored. The force profile employed is unimodal, according to the following relationship:

$$\{F\} = \lambda(t)[M]\{\Phi\} \tag{1}$$

where {F} is the vector of force of the load profile, [M] is the diagonal matrix of the storey masses; $\{\Phi\}$ is a proper displacement shape vector (normalized with respect to the displacement of the control point) and λ(t) is a scalar parameter that controls the evolution of the load history. The capacity curve obtained for the MDoF system is then scaled in order to represent the response of an SDoF system having an equivalent mass $m^* = \{\Phi\}^T[M] = \sum m_i\Phi_i$, by applying the modal participation factor:

$$\Gamma = \frac{\sum m_i \phi_i}{\sum m_i \Phi_i^2} \tag{2}$$

Finally, the SDoF curve is transformed into a simplified bilinear capacity curve characterized by a yield and an ultimate point. According to the Italian Seismic Code, this is made by imposing that the linear elastic branch intersects the curve at the point for which the force is equal to 60% of the maximum F_{bu}^*, whereas the perfectly elastic branch (yield force F_y^*) is defined on the basis of an equal area criterion, as shown in Figure 1. In this way, the capacity curve can be directly compared with the seismic demand through the elastic spectrum, Acceleration - Displacement Response Spectrum (ADRS).

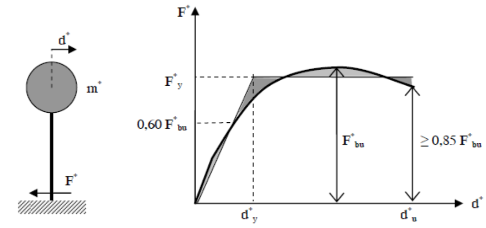

Figure 1 - *Bilinearization of the capacity curve of the equivalent SDoF system*

2.2 NON LINEAR STATIC ANALYSIS: MULTIMODAL AND ADAPTIVE METHODS

NLS procedures, despite their effectiveness, have two main limitations. Firstly, the assumption of an unimodal profile does not allow to take into account the effects of higher modes, which can significantly influence the structural response in some situations (e.g., tall buildings, presence of structural irregularities in mass or stiffness,…). In particular, the effect of the irregular distribution of mass and stiffness in the existing building provide an increase of the torsion effects on the structure. The direct consequence of this evidence is an increment of the seismic demand for the storeys element, especially for the ones on the building sides. This effect can increase the occurrence of brittle mechanisms in the structural elements. The torsional behavior of the existing buildings, in both elastic and inelastic field is highlighted in several works available in the scientific literature [De Stefano and Pintucchi, 2008, Anagnostopoulos *et al.*, 2015], where is shown the explanation of the main parameters, which influence the problem. Clearly, this effect provide different results, in terms of assessment and definition of structural performance and safety level, as shown in [Kreslin and Fajfar, 2012, Uva *et al.*, 2018]. A second drawback concerns the assumption of time invariant load-profiles, which neglects the changes of the dynamic properties of the structure (periods, shape of fundamental mode,…) due to the progressive plasticization of structural elements and consequent loss of stiffness.

For these reasons, in recent years, more refined approaches of NLS analysis have been developed, such as multimodal pushover analyses and adaptive pushover analyses. The first authors to assess the relevance of higher modes in NLS analysis [Paret *et al.*, 1996] defined a vulnerability index called “Modal Criticality Index” (MCI) for the identification of the critical modes of the building (The mode with the greater MCI was the critical one for the building). In particular, they presented a case study consisting in a building for which the structural elements are oversized at the lower floors and undersized at higher floors.

Subsequently, other authors [Sasaki *et al.*, 1998] presented a similar study in which they proposed to perform 3 different pushover analyses, with load profiles proportional to the first 3 main modes. The results of the research work confirmed that, in some cases, the actual damage mechanism is not associated to the first mode, according to the results obtained from NDL analyses. Chopra and Goel [Chopra and Goel, 2002], proposed a procedure known as “Modal Pushover Analysis” (MPA), which allows to study the post-elastic structural behavior as a combination (by means of a Complete Quadratic Combination- CQC or by a Square Root of the Sum of Squares - SRSS) of a number of pushover analyses selected according to the number of significant modes. Each pushover is performed under a load profile proportional to the specific modal shape. This method proved to be valid in the elastic range, where it provides results comparable to those of linear time history analysis, but have no theoretical foundation in the post-elastic range. In fact, when the plasticization effects became significant, there is a continuous change of the structural stiffness and thence of modal shapes and this caused a coupling of vibration modes, effect that cannot be neglected in a NL analysis. In a successive paper [Chopra and Goel, 2004], the same authors applied the aforementioned procedure for buildings non symmetric in plan. Numerical analyses showed that the method was well-applicable both to spatial "torsionally-stiff" systems and "torsionally-flexible" systems, whereas the method lost its accuracy for "torsionally-similar-stiff" systems (i.e. systems for which torsional stiffness and flexible stiffness are comparable) because the vibration modes become strongly coupled. In [Goel and Chopra, 2005], MPA was applied by the authors to tall building, performing numerical analyses on an extensive set of case studies. As a general result, they observed that higher modes are important because of the formation of mechanisms in the higher floors. In a few cases, instead, a different phenomenon occurred, called "reversal", in which the formation of plastic hinges causes an inversion of the direction of the top floor displacement. This effect did not allow the use of MPA. The same researcher [Chopra *et al.*, 2004] proposed a modified version of MPA, in which the inelastic response of buildings was derived from the combination of the inelastic response of the 1st mode plus the elastic contributions of higher modes. The main advantage of this method is to provide results comparable to MPA results, carrying out fewer pushover analyses and substantially decreasing the computational effort. Following the MPA idea [Moghadam, 2002], it was formulated a procedure similar to MPA, proposing to use in the elastic range a simplified combination of modes, through coefficients obtained by the modal participation factors of the considered modes,. Other authors [Hernadez-Montes *et al.*, 2004] showed that in some cases, such as in which occurs the case of "reversal", the capacity curve depicted by the usual control point (top-floor displacement vs. base shear) is not a good representation of the capacity of structure. In fact, one of the problems of MPA is that, in the inelastic range, the growth of the displacement of top floor is not proportional to other floors' ones as instead happens in the elastic range. In order to prevent problems related to the choice of the control point, the authors proposed an energetic procedure for the determination of the capacity curve. Subsequently it was proposed a procedure based on two load profiles obtained by sum and difference of the first two modes [Kunnath, 2004]. The results showed that, in the first case, the structural response, expressed in terms of interstorey drifts, was overestimated for lower floors, while in the second case there was an overestimation for higher floors. Through a subsequent campaign of numerical analyses for regular R.C. frame buildings, he proposed a procedure called "Method of Modal Combination" (MMC) that combines the aforementioned load profiles by modification factor "α_i", which depends by seismic demands and elastic period of mode considered. In order to appraise the contribution of higher modes, it was proposed to perform a multimodal analysis through a single load profile based on a combination of the modes, and defined new participation factors for each vibration mode, obtained by elastic response of the structure subjected to seismic action [Almeida and Barros, 2005]. Through a modal analysis, it was possible to define a new modal participation factor called "α", used in the development of load profile, proceeds by modal parameters of each vibration mode considered. They carried out pushover analyses on three-dimensional models with different features (symmetrical structure, stiffness asymmetric structure, mass asymmetric structure) and the results were compared with NLD analysis and unimodal pushover results. Other recent research works should be mentioned about the multimodal pushover, such as the ones proposed by [Kaatsiz and Suculoglu, 2014, Ferraioli *et al.*, 2016, Colajanni *et al.*, 2017, Ferraioli, 2017]

In the scientific literature, many adaptive pushover methods have been developed, and the first authors to propose it [Bracci *et al.*, 1997], developed a procedure valid for medium-low-rise buildings in which the load profile was updated when the stiffness variation became significant. The procedure allowed the calculation of a set of seismic demand curves to be compared with the capacity curve of the system. Based on the position of the bundle of curves with respect to the yield and failure point of the capacity curve, it was possible to understand the behavior of the system to seismic action. Following, it was proposed a procedure that provided the execution of a number of pushover analyses equal to the number of significant vibration modes [Gupta and Kunnath, 2000]. The analysis was conducted in incremental form, in which the modal shapes associated to the tangent stiffness matrix were calculated at the beginning of each load step. After evaluating the response parameters for each mode, they were combined through modal combination rules. The authors applied the method to buildings with different height, varying the strength and stiffness features of the cases study. With a different approach, it was proposed a procedure based on a single pushover analysis [Elnashai, 2001], in which at each load step the load profile was determined by combining (by a SSR combination rule) the profiles corresponding to the each vibration mode considered associated to the tangent stiffness matrix. The method, which is an arbitrary extension of linear modal analysis, did not always provide accurate results to with respect to NLD analysis, and revealed to be often numerically instable. Based on the procedure of previous research, other authors proposed an algorithm that allowed the development of an adaptive pushover able to account for the progressive stiffness deterioration and for the effects caused by higher modes [Antoniou and Pinho, 2004a]. The methodology, called Force Adaptive Pushover (FAP), was developed on several stick-models, showing that the results obtained for some response parameters, such as interstorey drifts, were not comparable with those calculated through NLD analyses. In order to improve the method, the authors later developed an alternative procedure [Antoniou and Pinho, 2004b] called Displacement Adaptive Pushover (DAP), in which the innovation was represented by the use of a lateral displacement profile instead of a force profile. The procedure proves to be effective thanks to the continuous update of the displacement profile shape, but suffers of a main limitation, consisting in the application of SRSS or CQC combination rules, which are valid for elastic systems, but scarcely indicated to NL systems. Other adaptive methods can be found in the literature, such as in which the lateral load profile is updated at the occurrence of a defined number of plastic hinges, according to two different methods [Raquena and Ayala, 2000]. One of these was based on the calculation of an equivalent vibration mode obtained by the SRSS combination of significant modes.

2.3 MODELLING THE MECHANICAL PARAMETERS OF MATERIALS BY IN SITU CHARACTERIZATION

The structural assessment procedure of existing buildings requires the availability of detailed preliminary information such as the geometry of structural elements, the diameter and position of steel bars and the mechanical features of concrete and steel reinforcements. Generally, the mechanical characterization of in situ materials represents a difficult but at the

same time crucial phase of the knowledge process, since an incorrect execution of in situ investigations can significantly affect the results of the safety assessment and spoil the decisions about intervention strategies, possibly leading to an unfounded increase in costs.
With regard to the methods for the in situ assessment of mechanical parameters of concrete and steel bars in RC buildings, there are two possible approaches: D (or slightly D) in situ testing and ND in situ testing. Table 2 reports the main type of in-situ tests that can be performed, respectively, on concrete and steel. In Figure 2, an example of extraction of a concrete core from a R.C. structural element is shown.

Table 2 – *Concrete and Steel test*

CONCRETE tests		STEEL tests	
ND	D	ND	D
Ultrasonic test	Drilling core	Pacometric test	Tensile test
Rebound hammer test	Pull-out test		Bending test
Windsor Probe Test	Pull-off test		
Georadar scan			
Tomographic test			

Figure 2 – *In-situ drilling core*

With regard to D tests, concrete coring is surely the most complex operation, both in the extraction phase and in the manipulation/preparation of the specimens. The main problem is represented by the possible disturbance and damage suffered by the cores, which will not represent anymore the actual mechanical properties of in-situ concrete. Other problems may occur in the compressive tests performed on drilled cores, where the values measured can be influenced by various factors, such as the confinement effect, the aspect ratio of specimens, the presence of embedded reinforcements, and so on. Thence, the compressive strength values provided by laboratory tests should be properly corrected in order to obtain the correct design value. In the scientific literature, there are some research studies about this topic, in which the quantification of degradation and damage due to concrete core sampling is investigated. Uva [Uva *et al.*, 2013], proposed a procedure for assessing the reliability of in situ concrete tests and improving the estimate of the compressive strength. It is based on the determination of a coefficient called C_{DD}, which estimates the decrease of in-situ-concrete strength in drilled cores as a function of the concrete compaction degree, allowing to account for the alterations induced by extraction and preparation of samples on the concrete strength evaluation. After this work, it was proposed an improvement of the procedure by correlating the results of D compressive tests, ND tests (ultrasonic pulse velocity tests) and the data documenting the compressive strength tested during the construction phases [Porco *et al.*, 2014]. Despite the high level reached in the quality control and production of concrete mixture in the constructive phases (placing, curing and control of concrete), which should guarantee the achievement of the design mechanical parameters, it is very frequent to encounter a significant level of dispersion and uncertainty about the measured values of in-situ parameters. This latter problem can be summarized with the Relative Standard Deviation (RSD), which is a dimensionless coefficient that, according to the American Code [FEMA 356, 2000], should not be greater than 14%. The high level of dispersion measured by RSD is sometimes dependent by the narrowness of the strength sample. In fact, the minimum required number of D tests required by the same Eurocode 8 for defining in-situ mechanical parameters is quite low, mainly for safety reasons, since D tests can cause damage on the structure, although they are the most representative possible measures. Nevertheless, precious additional information can be provided by ND investigations, which are faster, easier, induce a very limited damage on the structure and can extensively performed on the whole structure. On the same focus, it was performed a wide investigation about the dispersion of data collected by in situ diagnostic of the existing concrete by processing and analyzing the results of D tests performed on seven R.C. school buildings in the province of Foggia, for which it was observed that the values of RSD were higher than the aforementioned FEMA threshold [Fiore *et al.*, 2013]. An extensive analysis was performed on one of the buildings investigated, by processing and correlating additional results provided by ND tests (ultrasonic and rebound hammer tests), which showed to be homogeneous and consistent allowing to improve the estimate of the Relative Standard Deviation coefficient RSD that resulted to be lower. It is worth mentioning, anyway, that it is always difficult to obtain a reliable estimate of the mechanical parameters of in-situ materials to be used in the modeling phase. In order to improve the accuracy and quality of the information, and to avoid an excessive penalization of materials strength by CFs, the largest possible number of tests (both ND and D) should be performed, but, of course, the immediate consequence is the disproportionate in the immediate release of costs or can be decrease strength of materials (analysis results strongly penalized).

3. A SENSITIVITY ANALYSIS ON CONCRETE COMPRESSIVE STRENGTH VARIATION

3.1 ANALYSIS OF EXISTING RC BUILDINGS: A CASE STUDY

The case study investigated is a R.C. school building located in Bovino (Province of Foggia, Puglia, Southern Italy) having its plant inscribed in a rectangle of dimension 24.90 m x 35.65 m, 3 floors above ground and a pitched roof with total height of 15.55 m (Figure 3). The structure, built in the 60s' in the absence of specific seismic codes, was designed considering only vertical loads and it is constituted by a R.C. frame with beams, columns and slabs. The case study is representative of the structural typology that constituted a school building stock in abovementioned geographic area. The dimensions of the structural elements are variable, according to the design loads, as summarized in Table 3, which also reports the sectional parameters in terms of geometry, longitudinal and transversal steel bars. In the above Table, the amount of longitudinal steel bars is indicated as the number n of bars having diameter Φ ($n\Phi$), whereas the amount of transversal steel bars is indicated in terms of the ratio Φ/s expressed in centimeters (s = spacing of stirrups, Φ = diameter of stirrups). It is also reported the amount of longitudinal steel bars in the middle (M) and in the End (E) sections of the beams. The foundations are constituted by plinths connected by beams, while the warping of elevation beams is in just one way.

Figure 3 – *Photo of school Building study*

Table 3 – *Dimension and geometry of structural elements*

Level	Element	B (mm)	H (mm)	Longitudinal Bars (nΦ)		Transversal Bars (Φ/s)
				X	Y	
Ground floor	Columns	400	600	4 Φ18	4 Φ16	Φ6/20
		500	600	8 Φ18	4 Φ18	Φ6/18
		550	700	8 Φ18	4 Φ18	Φ6/18
		500	650	8 Φ18	4 Φ18	Φ6/18
	Beams	800	400	E: 3Φ18 – M: 6Φ18		Φ8/15
		800	200	E: 4Φ16 – M: 4Φ16		Φ8/20
		400	600	E: 3Φ16 – M: 6Φ16		Φ8/15
First floor	Columns	400	600	4 Φ16	4 Φ16	Φ6/20
		400	400	4 Φ16	4 Φ16	Φ6/20
		400	650	4 Φ16	4 Φ16	Φ6/20
		500	650	8 Φ18	6 Φ18	Φ6/20
		350	400	4 Φ16	4 Φ16	Φ6/20
	Beams	800	200	E: 5Φ18 – M: 8Φ18		Φ8/20
		400	800	E: 6Φ18 – M: 6Φ18		Φ8/15
		400	600	E: 3Φ16 – M: 3Φ16		Φ8/20
Second floor	Columns	400	400	4 Φ16	4 Φ16	Φ6/20
		400	600	4 Φ16	6 Φ16	Φ6/20
		350	400	4 Φ12	4 Φ12	Φ6/20
		350	500	4 Φ16	4 Φ16	Φ6/20
		300	300	4Φ12	4 Φ12	Φ6/20
	Beams	350	800	E: 5Φ16 – M: 6Φ16		Φ8/20
		350	700	E: 3Φ18 – M: 3Φ18		Φ8/20
		350	500	E: 2Φ16 – M: 2Φ16		Φ8/20

With regard to the KL, the building has been the object of a detailed investigation performed within an Agreement between AdB Puglia and Polytechnic University of Bari that provided, firstly, the development of “Guidelines for the vulnerability assessment of existing buildings” to be used by the professionals in charge of the vulnerability analyses on school buildings in the Province of Foggia, and secondly the execution of investigations and vulnerability analyses on a sample of School Buildings, under the supervision of a Team of experts of Polytechnic University of Bari. The school building of Bovino belongs to this sample, and thence a complete process of knowledge was carried out, by performing surveys, investigations and in situ tests on structural elements and achieving a full knowledge level (KL3). In order to estimate the mechanical properties of steel, the investigation plan provided the execution of widespread pacometric tests (ND tests) and tensile and bending tests of reinforcement bars (D), at each floor, which revealed the presence of smooth bars of "Aq42" steel class, a mild steel with yield strength f_{yd}= 401 MPa. With regard to the assessment of concrete mechanical properties, the extraction of 9 cylindrical cores was performed, which have been then subjected to compression tests (in Table 4, results are shown in terms of R_c - cubic strength and F_c - cylindrical strength).

Table 4 – *Results of compression tests*

Core	d(mm)	H(mm)	F_c (MPa)	R_c (MPa)	γc (kg/mc)
C1	94.0	94.0	11.60	12.05	2131
C2	94.0	94.0	25.20	25.95	2290
C3	94.0	94.0	15.50	16.30	2253
C4	94.0	94.0	17.50	17.65	2242
C5	94.0	94.0	11.90	12.15	2281
C6	94.0	94.0	13.10	13.50	2154
C7	94.0	94.0	12.80	13.20	2096
C8	94.0	94.0	11.50	11.65	2134
C9	94.0	94.0	14.80	15.45	2263

The results of compression tests have revealed a relevant dispersion of the strength values and consequently the average value cannot be assumed as representative parameter in view of the structural modeling. In particular, the difference between the maximum and the minimum values found for the compressive strength is equal to 14.3 MPa, with a Relative Standard Deviation RSD equal to 0.29, whereas the threshold indicated by FEMA is 0.14. In order to complete the investigation plan about mechanical parameters, 28 ND tests were performed, which allowed to identify the presence of different homogeneous strength classes of in situ concrete, confirming uncertainties related to characterization of these parameters.

3.2 THE NUMERICAL MODEL

The structural modeling of the case study was carried out by using the finite element software SAP2000. Beams and columns are modeled as one-dimensional frame elements, assuming fixed- end restraints at the base of the columns. Dead loads are applied to beams according to the direction of the joists and the influence area. In particular, dead loads are identified as G_1 = 2.5 kN/m^2, G_2 = 1 kN/m^2 for all floors; Q_v = 3 kN/m^2 for the 1st and 2nd floor while Q_v = 0.5 kN/m^2 for the top floor, according to the value provided by Italian codes for a service floor. Thanks to the completeness of the available documentation, it was possible to accurately reproduce the actual geometric configuration of structural elements. The pitched roof is modeled by considering the mass consistently distributed on the supporting beams (a preliminary analysis has shown that there is no significant variation by assuming a complete model with the actual inclination of joists). The NL behavior of the building has been modeled by a lumped plasticity approach, considering Takeda behavior for cyclic loads and neglecting P-Δ effect (because it doesn't varying analysis results and it doesn't significant in case study) and concrete cracking with reduction of E_{cm} (because, in NL analysis, this effect may be considered when the plastic hinges overcome the yield limit). Regarding to the plastic hinges, they have been defined through the evaluation of the chord rotation demand and the corresponding capacity, for each section investigated, such as provided by the formula provided by EC8. In particular, the plastic hinges are placed at structural elements' ends. It is worth remembering that the inelastic mechanisms considered for beams are only those due to simple bending, by defining a non-linear moment-rotation relationships. For the columns, combined axial stress-bending stress is considered for defining the non-linear moment-rotation relationships depending on the variation of the axial stress. The seismic action is defined by means of the elastic response spectrum calculated at the construction site and assuming a design working life V_N= 50 years and an Importance Class III (to which a Coefficient of importance c_u=1.5 is associated) in order to determine the reference Return Period for the seismic action for the limit state of life safety (SLV). According to the seismic hazard map of Italy, the peak ground acceleration (PGA) on A-type ground is equal to 0.2491/g, where g is the gravity acceleration. Based on the characteristics of the constructive elements of the floors, the hypothesis of infinite extensional stiffness has been assumed, by introducing an internal rigid constraint at each floor. Elastic modulus (E_{cm}) of concrete is calculated according to Italian code formulation (depending by f_{cm}); Tangential modulus of concrete (G) is calculated according to elastic formulation which links it with E_{cm} and Poisson's ratio (ν = 0.2). In this work, the possible activation of brittle shear mechanisms in the structural elements has been neglected, even if a very low amount of transversal reinforcements has been detected during the investigation phases. This is a quite common situation for Italian RC framed buildings designed in the absence of anti-seismic standards. It is thence evident that brittle shear mechanisms can develop since the very first steps of the analysis, undermining the objectives of the research study. It has been supposed that shear mechanisms will be anyway preliminarily prevented by means of proper intervention strategies. In order to investigate the possible differences in modeling methods and to include the influences of secondary structural elements (such as influence of slabs and joists) an additional numerical model has been implemented, by introducing horizontal elastic elements in the first two floors, where the presence of double joists or concrete bands has been detected, as showed in Figure 4. In the third floor a different type of slab system has been observed, consisting in a prefabricated slap called "SAP", typical of Italian 70's existing RC building, which cannot offer additional contribution as primary seismic elements. In each of the two models, steel strength has been assumed according to in-situ measured parameters (f_{yd} = 401 Mpa), whereas a parametrical analysis has been performed with regard to concrete strength, by considering 3 possible values of f_{cm}: 10 MPa, 20 MPa, 30 MPa, on the basis of the range of strength values provided by D testing. This choice clarifies that the purpose of this work is not determine the representative value of in situ concrete strength for the existing structure, possible future aim of research, rather it is that of study a several engineering scenarios related to assessment of RC existing building, analyzing the structural response in a range of values of in situ concrete strength. Overall, 6 different numerical models have been analyzed, in order to assess the influence of material variations and secondary horizontal elements on the seismic response of the building and on the effectiveness of advanced NL methods of analysis. For the considered case study, it was seen that the explicit modeling of floor elements (double joists or large concrete bands - “complete model”) does not significantly affect the mass and the stiffness of the model and, besides, does not determine a substantial variation in the modal parameters of the system and in the consequent stress states. However, all subsequent analyses will be performed with reference to the complete models which are closer to the real structural configuration.

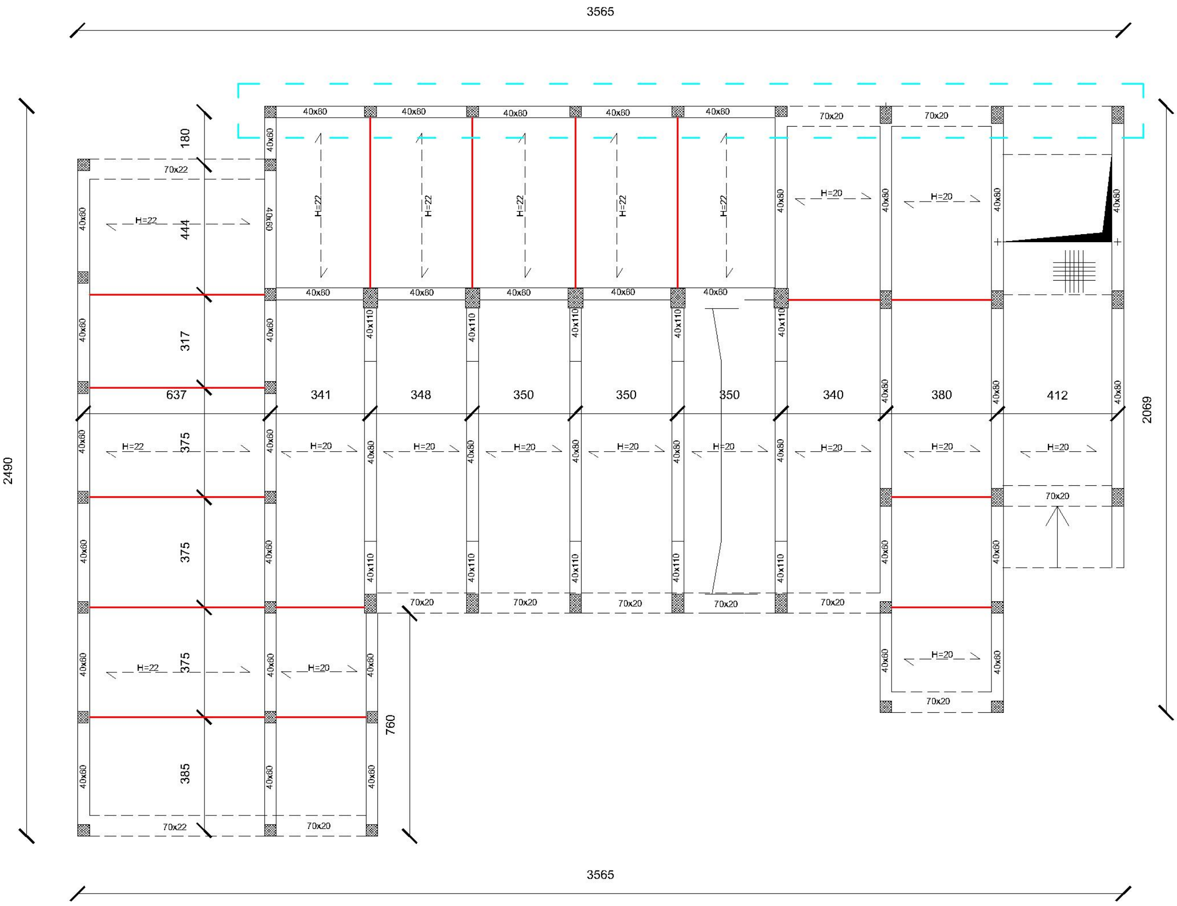

Figure *4 – Carpentry of building study. Red signs lead full bands and double joists*

3.3 EIGENVALUE AND LINEAR DYNAMIC ANALYSES

For the three numerical models described in the previous section, preliminary eigenvalue analyses have been performed, deriving the natural vibration modes. Figure 5 reports the first 2 modes for the “complete model”, for a concrete strength value f_{cm} = 10 MPa. This model will be herein after assumed as the reference one for all comparisons, because this model have concrete strength value nearest to average of results obtained by compressive tests. In Table 5, periods (T) and participating masses (M[%]) are shown for the first 6 vibration modes of the reference model, whereas in Table 6 the percentage variations of T and M[%] in the two main directions are shown for the models for which f_{cm}=20 MPa and 30 MPa. Subsequently, a linear dynamic analysis with response spectrum has been also performed, by assuming a behavior factor q=1.5, which is the minimum value for R.C. existing building, as indicated in European code. Results in terms of stress distribution are compared among the different models. Table 7 shows the comparison, in terms of maximum percentage variation, of the bending moments, shear stress and normal stress among the reference model and the models with f_{cm}= 20 MPa and 30 MPa. Figure 6 shows the differences of geometric dimension in a vertical section of buildings, showing the frame indicated in Figure 4 by dashed box. The above tables show that the stress variation, varying the f_{cm}, is up to about 15% in the elastic field.

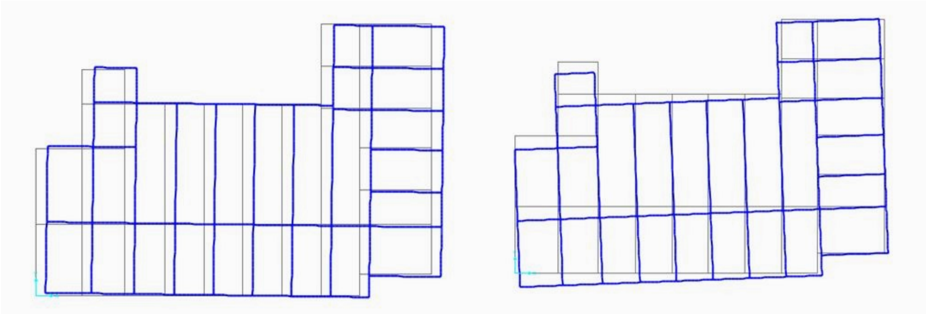

Figure 5 – *Vibration modes in main directions (X and Y)*

Table 5 – *Periods and participating masses of reference model*

Mode	T (s)	$M[\%]_x$ (%)	$M[\%]_y$ (%)	$M[\%]_z$ (%)
1	0.876	87.123	0.031	0.000000173
2	0.573	1.940	33.326	0.0000304
3	0.542	0.629	59.037	0.0000356
4	0.284	8.643	0.073	0.0000172
5	0.216	0.145	3.415	0.000150
6	0.179	0.00985	3.134	0.000298

Table 6 – Variations of periods and participating masses among reference model and models having fcm 20 Mpa and fcm 30 Mpa

	f_{cm} 20 MPa			f_{cm} 30 MPa		
Mode	ΔT(%)	$\Delta M[\%]_x$ (%)	$\Delta M[\%]_y$ (%)	ΔT(%)	$\Delta M[\%]_x$ (%)	$\Delta M[\%]_y$ (%)
1	-9.86	-0.01	0.00	-15.17	-0.02	0.00
2	-9.86	0.05	0.07	-15.18	0.10	0.11
3	-9.87	0.00	-0.05	-15.19	0.00	-0.07

Table 7 – Variations of stresses states among reference model and models having fcm 20 Mpa and fcm 30 Mpa

	f_{cm} 20 MPa			f_{cm} 30 MPa		
Rreference model	ΔN_{max}(%)	ΔT_{max}(%)	ΔM_{max}(%)	ΔN_{max}(%)	ΔT_{max}(%)	ΔM_{max}(%)
	7.11	1.62	11.63	10.36	2.60	16.99

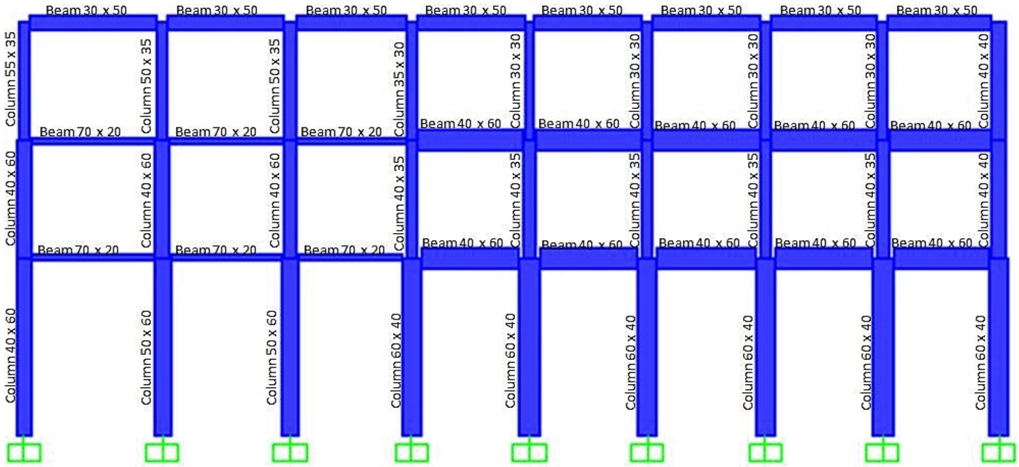

Figure 6 – *Geometric dimension of representative frame of building*

3.4 VULNERABILITY ANALYSIS BY NONLINEAR STATIC ANALYSIS: COMPARISONS BETWEEN CONVENTIONAL AND MULTIMODAL APPROACHES

In order to provide a more reliable assessment of the structural capacity, NLS analyses have been performed on all models, for the two main directions X and Y and taking into account a gravity loads before push structure with horizontal loads. Nevertheless, based on the eigenvalue analyses, it is important highlights that in the X direction, the participating mass of the fundamental mode is always greater than 85% for all models, whereas in the Y direction it is comprised in the range 60-65%, which is a level not sufficient for using a unimodal profile. Therefore, the focus is the subsequent evaluations is the Y direction and the subsequent analysis have been done just in this one. In particular, for each model, NLS analyses have been carried out by using the following load profiles: unimodal (ULP), inverse triangular; constant and finally, a multimodal load profile has been implemented (MLP).
The procedure used to define MLP is based on weighted contribution of significant modal shapes, considering new participation factors calculated using the response of the structure obtained by a linear modal analysis. The main advantages of the procedure adopted is due to the multimodal analysis is performed by using just one analysis with a unique load profile, which takes into account the higher modes. This is a benefit in terms of computational effort, especially from the practitioners' point of view. The adopted procedure is the one proposed by Caliò [Caliò *et al.*, 2010], and provides the following steps:

1. The modal shapes $\{\Phi\}_n$ are evaluated from the linear modal analysis. For each mode, the modal participation factors are computed as:

$$\Gamma_n = \frac{\{\Phi\}_n^T[m]}{\{\Phi\}_n^T[m]\{\Phi\}_n}\{e\} \tag{3}$$

being [m] the mass matrix of structure and {e} the influence vector associated to the input direction of motion;

2. From the elastic response spectrum, the modal displacements $\{D\}_n$ are determined.
3. The maximum nodal displacements are derived from the spectral values:

$$\{u\}_n = \Gamma_n D_n \{\Phi\}_n \tag{4}$$

4. The maximum response $\{u\}_{max}$ in terms of displacement is evaluated through the CQC combination rule.
5. The Modal decomposition of $\{u\}_{max}$ into the modal contributions of the each vibration mode is performed and the corresponding participation factors are calculated:

$$z_n = \frac{\{\Phi\}_n^T [m]}{\{\Phi\}_n^T [m] \{\Phi\}_n} \{u\}_{max} \tag{5}$$

6. A NLS analysis is performed by using a MLP determined as the combination of the vibration modes of the structure according to the participation factors obtained at step 5 and scaled with respect to floor masses. This resulting curve represents the structural response of the system.

The above-described procedure has been applied to each of the three numerical models of the case study, in order to obtain a multimodal load profile to be applied along the Y direction, monitoring the variation of the modal parameters (eigenvalues, eigenvectors, periods, modal participation factors). After calculating the new participation factors z_n by eq. (5), only the modes having a participating mass higher than 1% have been taken into account, since the remaining modes have a negligible influence on the computation of the load profile. In Figure 7, a typical load profile calculated through the specified procedure is shown, where in abscissa there is a force normalized with respect to the height and in ordinate there is the height. This MLP is compared with a reverse triangular load profile.

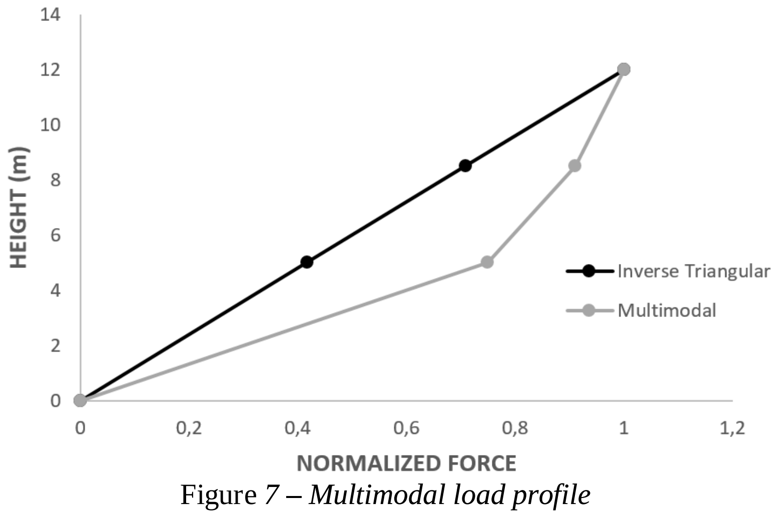

Figure *7 – Multimodal load profile*

Then, these load profiles have been implemented in the FE model in order to perform multimodal NLS analyses. For each of the 3 models, different analyses have been performed by varying the load profile shape and the mechanical parameters of materials, and the results obtained have been compared with the “reference” model, in order to assess the appraise the differences among the related capacity curves. A further comparison has been performed between the results of NS procedures and those provided by three Incremental Dynamic Analysis (IDA) [Vamvatsikos and Cornell, 2002] performed by applying 3 different artificial design-consistent accelerograms generated with SIMQKE software. In Figure 8, the accelerograms used is shown and clarifies that they are spectrum-compatible with the acceleration spectra of site of case study. The accelerograms have been appropriately scaled in amplitude up at several steps, in order to reproduce the trend of structural response, starting from low action up to strong acceleration that caused the structure collapse. To this scope, each accelerogram is composed by just one component, which has been applied (and after scaled) in the Y direction, in order to reproduce the structural behavior provided by the NLS analyses in the same direction.

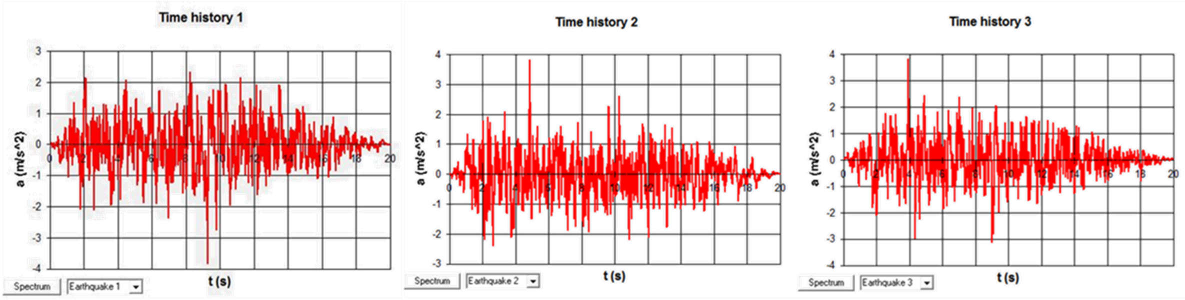

Figure 8 – *Accelerograms used in IDA, with duration equal to 20 sec and stationary part of duration equal to 10 sec.*

The results of the analyses obtained by increasing the accelerogram amplitude have been depicted in the same graph of the NLS analyses, in terms of Maximum displacement of the control point (center of mass of highest floor) versus Maximum base shear. Whit this regard, Figure 9 shows the comparison among NLS analyses performed with ULP and MLP for the 3 models (f_{cm}= 10 MPa, 20 MPa, 30 MPa) and IDA results, in order to perform a critical comparison. The trend of IDA results come near the NLS analyses results obtain using MLP in both elastic and inelastic field for all models and this confirmed the validity of MLP calculated to estimate the real structural response. The comparison is also summarized in Table 8 in terms of percentage differences of the base shear (ΔV_b) in the different cases, for two different values of roof drift (0.2% - elastic range; 0.4% - inelastic range). From the results, it is possible to appreciate the clear difference of the elastic stiffness with the variation of concrete from 10 MPa to 30 MPa. In addition, Table 8 shows that the uncertainty of the building response in the inelastic field related to the variation of in situ concrete strength (curves of different color), is greater than the one due to the approximation related to analysis method (curves and points of same color). This evidence, albeit limited to the case study examined and therefore not directly generalizable, clarifies the importance of the phase of in situ investigation about mechanical parameters in the vulnerability assessment of RC buildings, phase that is often disregarded with respect to those of structural modeling and analysis.

Table 8 – Percentage differences of base shear among different models, at defined values of Roof drift, due to the variation of f_{cm} in all structural elements of the building

	f_{cm} 10	f_{cm} 30	f_{cm} 10 – f_{cm} 30
Roof drift (%)	$\Delta V_{b(U-M)}$ (%)	$\Delta V_{b(U-M)}$ (%)	$\Delta V_{b(U)}$ (%)
0.2	9.62	20.58	30.76
0.4	15.38	16.67	29.23

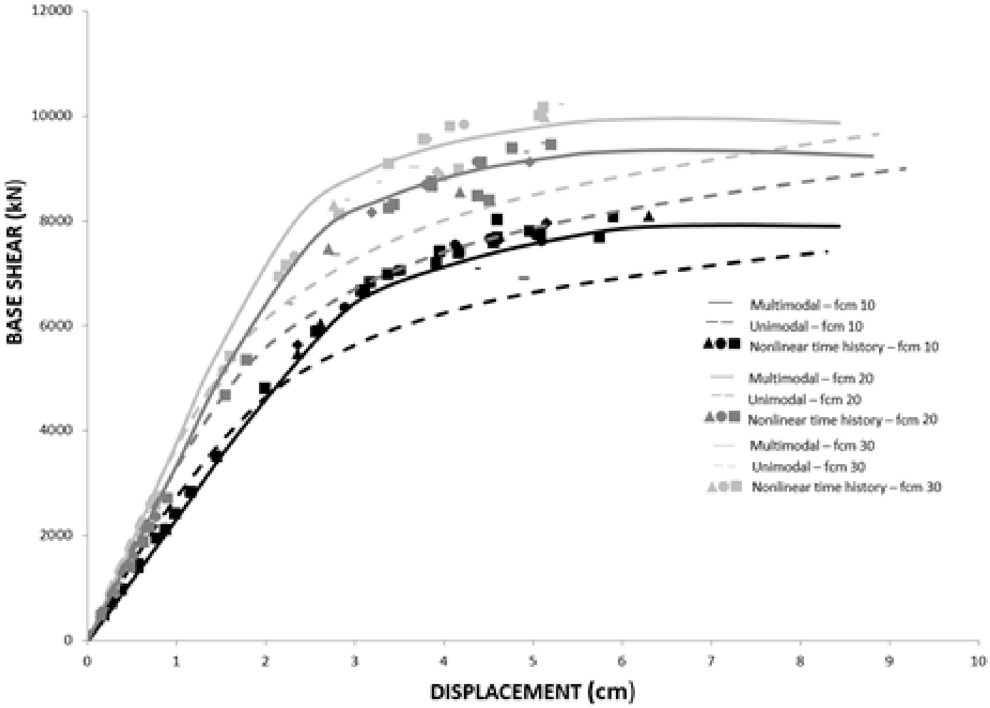

Figure 9 – *Comparison among NL analyses, varying concrete strength*

Additional analyses have been carried out, by considering the eventuality that concrete classes used in construction phase may be different from floor to floor, due to possible errors in execution (addition of water in the concrete mixture in order to improve the workability, inaccurate in-place concrete compaction, etc...). In particular, three scenarios are considered, by varying the concrete strength in a single floor at time, accordingly varying the numerical model. NLS analyses have been performed in the Y direction through MLP and comparing the results with IDA analyses (by using the same accelerograms in Figure 8). In particular, the capacity curve of the model with f_{cm}=30MPa has been compared with the results provided by multimodal NS analyses and IDA by changing the concrete strength of each floor, one at a time, from 30 MPa to 10 MPa. In Figure 10, the results of the analyses are shown. For the same comparison, Table 9 shows the percentage differences in terms of ΔV_b at two different values of roof drift (0.2% - elastic range; 0.4% - inelastic range). By varying the concrete strength at each floor, it was observed that the results have little variations, except for the case of first level. The results summarized in Table 9 and Figure 10 show that the failure behavior of the structure, neglecting brittle mechanisms and considering that in all analyses the system collapses for soft storey at the first level, is greatly accentuated decreasing the concrete strength of the structural elements at the first level. Finally, in Figure 11, are showed how plastic hinges are formed in one main frame, at 0.4% of roof drift.

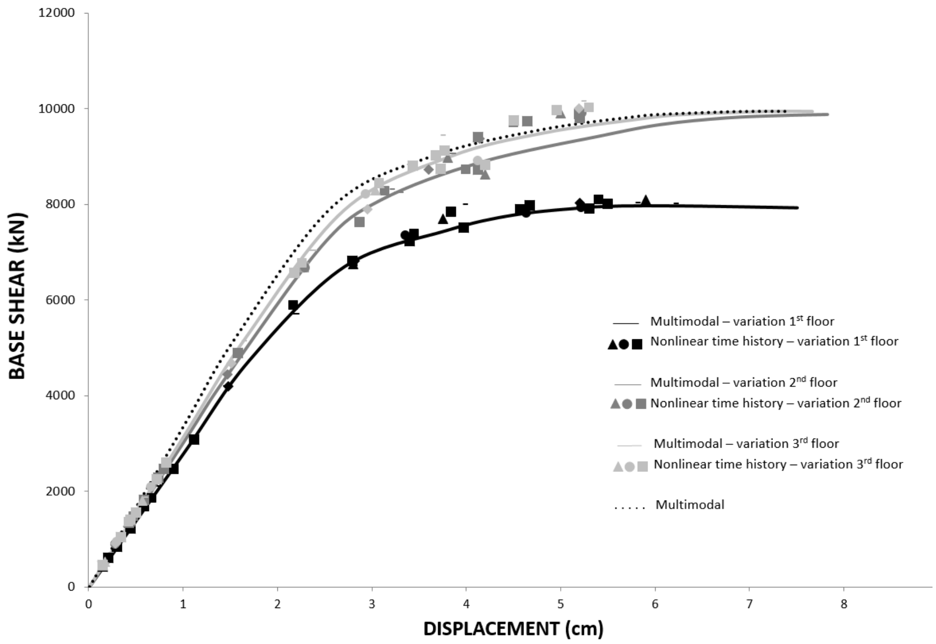

Figure 10 – *Comparison among NL analyses, varying concrete strength at each floor*

Table 9 – Percentage differences of base shear among different models, at defined values of Roof drift, due to the variation of f_{cm} in turn of each storey of the building

	variation 1st floor	variation 2nd floor	variation 3rd floor
Roof drift (%)	ΔV_b (%)	ΔV_b (%)	ΔV_b (%)
0.2	18.54	8.83	4.53
0.4	20.99	4.21	0.59

4. CONCLUDING REMARKS AND FUTURE DEVELOPMENTS

The research work presented is focused on the NL modeling and analysis of existing RC buildings characterized by an irregular configuration in plan. In particular, in a context in which the appropriateness and effectiveness of the conventional pushover method can be very limited, the objective is to appraise the influence of the dispersion of concrete strength on the vulnerability assessment by means of extensive comparisons among different structural models. A paradigmatic case study has been selected, constituted by a real school building that well represents structures built in Italy in the 60's. In fact it was designed only to vertical loads, according to the building codes of the time, which did not provide the presence of seismic actions. For the chosen building, extensive preliminary data were available, and it has been used as base for developing varied structural models on which performing sensitivity analyses. In particular, the available information about the building showed a large variation of the mechanical parameters of in situ concrete, which is a circumstance that may involve severe errors in the safety assessment of the school building, which is besides a strategic structure, for which particular attention is required.

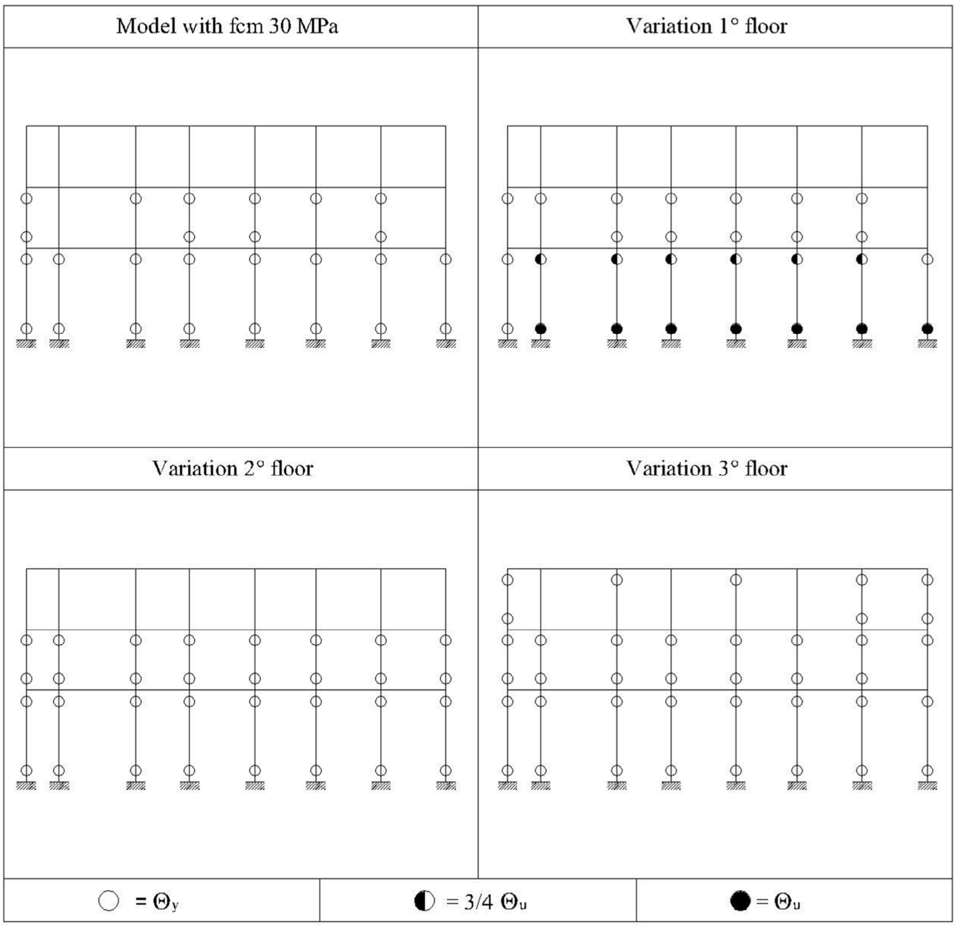

Figure 11 – *Plastic hinges formed in one main frame at 0.4% of Roof drift*

For the case study, different structural models have been implemented in order to appraise the influence of different factors. First of all, the importance of explicitly modeling the presence of secondary horizontal elements (double joists, concrete bands) has been investigated. Then, in order to consider the influence of mechanical parameters of in situ materials, the dispersion of concrete compressive strength has been parametrically considered, by assuming three different scenarios (10, 20 or 30 MPa), selected as plausible for the case study. For each of the three models, in the elastic range, modal parameters and stress states have been calculated, and comparisons with a reference model have shown a variation of periods and participating masses to motion up to 15% and a variation of main stresses states up to 17%. After this, NL analyses have been carried out, performing first conventional pushover analysis through unimodal, triangular inverse and uniform load profiles and then applying a more advanced approach by using a multimodal load profile calculated using a procedure based on weighted contribute of modal shapes of structure. The results of the different pushover analyses have been compared with those obtained by three incremental dynamic analyses performed through three spectrum compatible artificial accelerograms, with one component. The comparison among the results shows that an incorrect interpretation of the experimental (D e ND) tests causes uncertainty about response of structure which is significantly greater than that approximation connected to the analysis method. Furthermore, by varying the concrete strength at each floor, results show that overall, the differences obtained in structural response by varying the concrete strength at the 1st floor are much more relevant than in the other 2 cases. The further development of the research work, presently in progress, provides the modeling and analysis of additional case studies characterized by a more severe geometric irregularity, both in plan and in elevation. The program of analyses includes a wider variation of the mechanical parameters of in situ materials, and the influence of secondary structural elements, also by applying adaptive pushover methods, besides the approaches here adopted.

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