ON THE ESTIMATION OF THE FUNDAMENTAL MODAL PROPERTIES OF ITALIAN HISTORICAL MASONRY TOWERS

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ABSTRACT – The preservation of historical structures is a very challenging problem that requires strong interdisciplinary efforts, in particular in those countries where seismic risk is relevant and the integration between conservation and protection can be very difficult. Recent Italian earthquakes hit many valuable masonry structures marking the urban and rural landscape. Among those, masonry towers are often impressive signs of past cultures and folks. Non-destructive testing of such structures has been widely investigated and discussed in the technical literature. In the present paper, some aspects related to output-only modal identification of tower-like masonry structures for their non-destructive characterization in view of seismic performance assessment are discussed. Specific attention is paid to the role of Operational Modal Analysis in the seismic analysis of masonry towers and the development of empirical predictive correlations.

Keywords: Operational Modal Analysis, Historical structures, Seismic protection, Masonry towers

1. **Introduction**

Preservation of structures and monuments belonging to the National cultural heritage is a challenge that involves several technical and design issues. This circumstance is confirmed by the fundamental principles reported in the International guidelines on conservation of historical structures [De Naeyer *et al.*, 2000], where maintenance and repairs are "fundamental parts of the process of heritage conservation". The related actions have to be based on systematic research, inspection, control, monitoring and testing, taking into consideration any possible decay; the latter has to be foreseen, reported on, and mitigated by means of appropriate preventive measures.

From an engineering perspective, materials and structures cannot represent the only option to maintain or even improve the level of safety. In fact, non-destructive tests and monitoring can mitigate the uncertainties about the behaviour of arts, materials and structures in operational conditions and under extreme events. The huge number of potentially vulnerable historical structures exposed to natural hazards, such as earthquakes, has fostered a very active discussion about the topics of seismic risk assessment and reduction of the cultural heritage in Italy. The effects of recent earthquakes (L'Aquila, 2009; Emilia, 2012) stressed the need for a modern approach to the evaluation and upgrading of the structural performance of historical structures. Substantial attention has been paid to the development of specific codes or recommendations, because historical structures share common characters with existing structures but they also show some peculiarities [NTC, 2008; EN1998-3, 2005]. Based on the most important results of the research in the field, the Italian Guidelines (Direttiva) for interventions on historical structures [MiBac, 2010; DPCM n. 66, 2011] recommend dedicated investigation and analysis procedures.

The Direttiva confirms the relevant role of Non Destructive Testing (NDT) to enhance the knowledge about the behaviour of historical structures. In particular, potentialities of ambient vibration modal identification techniques for the estimation of the dynamic properties of structures and components in view of seismic performance assessment are recognised. The extensive literature about Operational Modal Analysis (OMA) confirms the wide applicative perspectives of these techniques. Typical applications are the calibration of Finite Element (FE) models, and vibration based Structural Health Monitoring (SHM). In the first case, the experimentally identified modal properties of the investigated structure are used to validate and refine the numerical model; as a result, an indirect, non-destructive estimation of selected - geometric or mechanical – parameters can be achieved [Rainieri *et al.*, 2013]. OMA also plays a key role in damage detection and the development of effective SHM systems. The continuous monitoring of the modal parameters of the structure over time [Rainieri and Fabbrocino, 2010; Rainieri *et al.*, 2011] and the analysis of their variations can provide useful information about the presence and eventually the location of damage at an early stage [Farrar and Worden, 2012]. On the analogy, the analysis of the variations of the experimental estimates of the modal properties can give information about the effectiveness of certain types of strengthening interventions [Ramos *et al.*, 2013].

Attention is herein focused on the analysis of the dynamic behaviour of historical masonry towers by OMA techniques and the opportunities they provide for seismic assessment. Damaged masonry towers were the symbol of the destructive power of the earthquake during the Emilia seismic sequence in 2012. They represent landmarks in many Italian urbanized areas and historical districts. They are typically equipped with bells, when erected in the vicinity of churches, or lights if erected on the sea to serve as beacon. In principle, they are characterized by a simple structural scheme, but their slenderness, and the uncertainties about the mechanical properties of masonry and the characteristics of foundations and subsoil often make the accurate prediction of their structural response pretty complex. In what follows, the main outcomes of the experimental campaign on the historical bell tower of the Saint Bartolomeo's church in the Carthusian Monastery of Trisulti in Collepardo (Italy) are briefly reported. It represents an interesting and explanatory case study where OMA provided a quick and reliable insight in the dynamic behaviour of a structure characterized by complex constructional layout and shallow foundation on rock soil (Class A according to relevant National and International seismic codes). A short discussion about the use of the experimental estimates of the modal properties in the context of LV1 seismic vulnerability analysis of the bell tower is reported. After this explanatory case study, the development of predictive correlations for the estimation of the fundamental dynamic properties of historical masonry towers is discussed. Complementing the results of an extensive experimental campaign on masonry towers located in the Molise region [Rainieri *et al.*, 2009; Fabbrocino *et al.*, 2012] with data obtained from review and careful analysis of the technical literature on the subject, a database of experimental estimates and geometric information has been assembled and used to check the performance of available formulations for the prediction of the fundamental period. The same database has been used to define a new relation between the fundamental period and the height of masonry towers that outperforms the other correlations, including the one suggested in one of the annexes of the Direttiva and not validated yet.

2. **Output only dynamic identification of the historical bell tower of the Carthusian Monastery of Trisulti**

The experimental analysis of the dynamic response to ambient vibrations of the San Bartolomeo's church bell tower in the Carthusian Monastery of Trisulti in Collepardo is herein discussed. Data obtained from the analysis of documents collected in the historical archives provided a sound basis for the understanding of the construction features of the tower and guided the definition of the experimental layout. This was aimed at the clear identification of the fundamental bending modes of the structure and the related distribution of the modal displacements along the height in view of LV1 analyses, as discussed next in the paper. Due to the limited number of sensors and time constraints, this resulted in a less accurate definition of the shape of torsional modes (see also Section 2.1).

The Carthusian monastery of Trisulti (Figure 1) lies on the Ernici mountains in the Apennines of Lazio (Central Italy). The erection of the primitive monastery is attributed to Saint Dominic by Sora, but nowadays only ruins of that ancient complex have survived, including those of the church and the chapter house [Taglienti, 1985]. In 1204 the Pope, Innocent III, entrusted to the Carthusian order what remains of the Benedictine monastery of Saint Bartolomeo. The risk of landslides and the need for a rearrangement of the existing complex to fit the rules of the different monastic organization led to the erection of another Carthusian monastery in a more suitable place. The construction of the new monastery was completed in 1211; thereafter, the complex was further enlarged and remodelled in the Baroque Age and later in the Eighteenth Century.

Figure 1. *The Carthusian monastery of Trisulti in Collepardo (2014).*

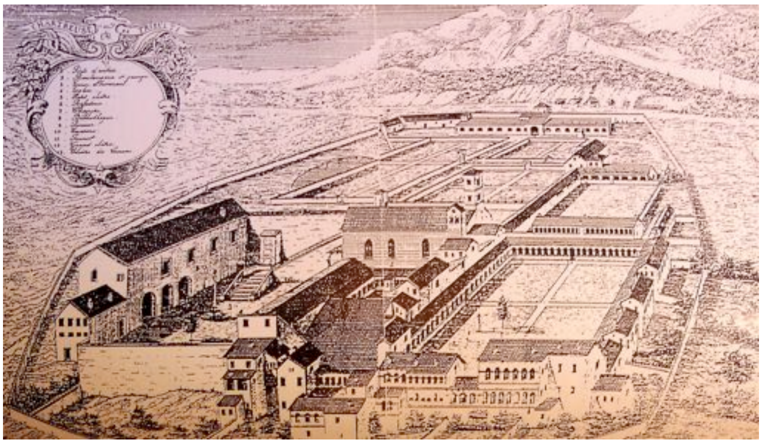


Figure 2. *Historical picture of Trisulti and of the old bell tower [Hogg et al., 1991].*

The heart of the Trisulti Charterhouse is the Church of San Bartolomeo, erected at the beginning of the Thirteenth Century. The interior is characterized by a single nave covered by a pointed vault. The old bell tower (Figure 2) stood on the vault of the presbytery of the church [Hoog *et al.*, 1991] and was accessible through a carved stone winding staircase, which still exists. The construction of a new bell tower started in 1746 by Giuseppe Pozzi. According to the original design the tower was totally made by squared limestone. However, after the 1746 earthquake, the structure showed several cracks. Thus, the initial project was modified, and the bell tower was made by limestone with a well-defined and marked cornice for the first half; the remaining half was made by brick masonry to reduce the total weight of the structure. The bell tower (Figure 3) was completed on January 10, 1749 [Taglienti, 1987].

In 2010, the bell tower underwent a major restoration, made mandatory by the presence of noticeable forms of alteration and degradation as well as of micro cracks and fractures (Figure 4). The limestone basement and the upper part of the tower made by brick masonry showed black-brownish deposits of varying thickness and layers of algae and lichens. Additionally, losses of several millimetres of material were evident in a number of points of the structure as a result of exfoliation, micro-cracks and erosion (caused by freezing/thawing phenomena and other environmental effects). Large cracks were present in the brick masonry near the metallic elements (bells anchoring, railings and reinforcements). Most of the cracks were stitched with the insertion of fiberglass rebars (10 mm and 12 mm diameters) embedded in epoxy resin. Another relevant crack pattern was found in the middle column made by bricks on the outside wall that overlooks the square of the church; this crack was attributed to the stress induced by the swinging bells. Consequently, the whole bell complex, consisting of a central large bell and four smaller bells, was anchored to a new steel frame aiming at reducing the stresses induced on the walls of the tower.

Figure 3. *San Bartolomeo's Church bell tower, North view (left) and elevation (right).*

Figure 4. *The bell tower before restoration in 2010 (Trisulti National Library, 2010).*

The irregularities of shape (Figure 3) and material along the height of the bell tower and the uncertainties in reconstructing the restoration interventions carried out over the centuries made the accurate prediction of the dynamic and seismic behaviour of the structure quite complex. The interaction with the surrounding structure of the church and the level of restraint that it offers to the lower part of the tower required specific attention in view of the vulnerability assessment of the tower. In order to reduce the uncertainties and get a reference set of accurate modal parameter estimates, experimental investigations have been carried out, as discussed in the next section. It is worth noting that irregularities in shape and materials, uncertainties in determining geometry, structural scheme and details, mechanical properties of materials, and often the presence of hidden construction defects or damage (for instance, due to past earthquakes) are common characters for most of the historical masonry towers in Italy. For this reason, the Direttiva encourages the experimental identification of the modal properties for seismic vulnerability assessment, as more extensively illustrated in Section 3.

2.1 Operational Modal Analysis of Saint Bartolomeo's Church bell tower

The accurate experimental estimation of the modal properties of the bell tower in view of LV1 analyses required the selection of an appropriate experimental setup and the analysis of the recorded time series by well-established OMA techniques. The dynamic response of the structure was measured at five different levels by seven couples of accelerometers, some of which were placed at opposite corners of the tower to observe bending as well as torsion modes (Figure 5). Force balance accelerometers characterized by 200 Hz bandwidth (starting from DC), 140 dB dynamic range, 0.5 g full-scale range and 20 V/g sensitivity were adopted. The data acquisition system was able to record data from up to 32 channels, simultaneously sampled by dedicated 16-bit ADCs. 3600 s long records of the structural response to ambient vibrations were acquired at a sampling frequency of 100 Hz. In spite of the very quiet conditions of the surrounding environment, the adopted measurement chain was able to resolve the structural response to ambient vibrations without the need for additional excitations. Before extracting the modal properties, the time series were accurately inspected and validated in agreement with well-established procedures for random data analysis [Bendat and Piersol, 2000]. Offsets and spurious trends were removed. Since the objective of the tests was the identification of the parameters of the fundamental bending modes to be used in LV1 analyses, the investigated frequency range was limited by low-pass filtering and decimation (by a factor of four) of the data. Thus, the final sampling frequency of the analyzed records was 25 Hz (Nyquist frequency f_N equal to 12.5 Hz) and the investigated frequency range was limited to 10 Hz taking into account that the finite roll-off rate of the filter makes the data in the transition band $[0.8f_N, f_N]$ distorted. It is worth pointing out that the fundamental modes of the structure were expected to be well within the range [0, 10] Hz.

The focus on the fundamental bending modes of the tower is also witnessed by the adopted sensor layout (Figure 5). In fact, it was set in order to have estimates of the modal displacements of the fundamental bending modes along the whole height of the tower. Due to the limited number of available sensors and time constraints, the discrimination of torsional modes from bending modes was ensured by only four couples of accelerometers installed at the upper levels of the structure. This obviously resulted in a less accurate definition of the shape of torsional modes, as discussed next in the paper. However, this was not critical for the objectives of the study. The modal parameters were extracted by applying both parametric and non-parametric OMA procedures, working in time domain as well as frequency domain. Data processing was carried out in the frequency domain by applying the Hanning window and considering a 66% overlap in spectrum computation. The use of different OMA techniques ensures the reliability of the obtained results thanks to the possibility of carrying out cross-validation checks. The accuracy of modal parameter estimates is also ensured by the application of appropriate data processing techniques [Rainieri, 2014; Rainieri and Fabbrocino, 2014a]. Powerful and robust OMA techniques, such as the Frequency Domain Decomposition (FDD) [Brincker *et al.*, 2001; Rainieri and Fabbrocino, 2014b], the Covariance Driven Stochastic Subspace Identification (Cov-SSI) [Van Overschee and De Moor, 1996; Peeters, 2000; Rainieri and Fabbrocino, 2014b] and the Second Order Blind Identification (SOBI) [Poncelet *et al.* 2007; Rainieri, 2014; Rainieri and Fabbrocino, 2014b], were adopted.

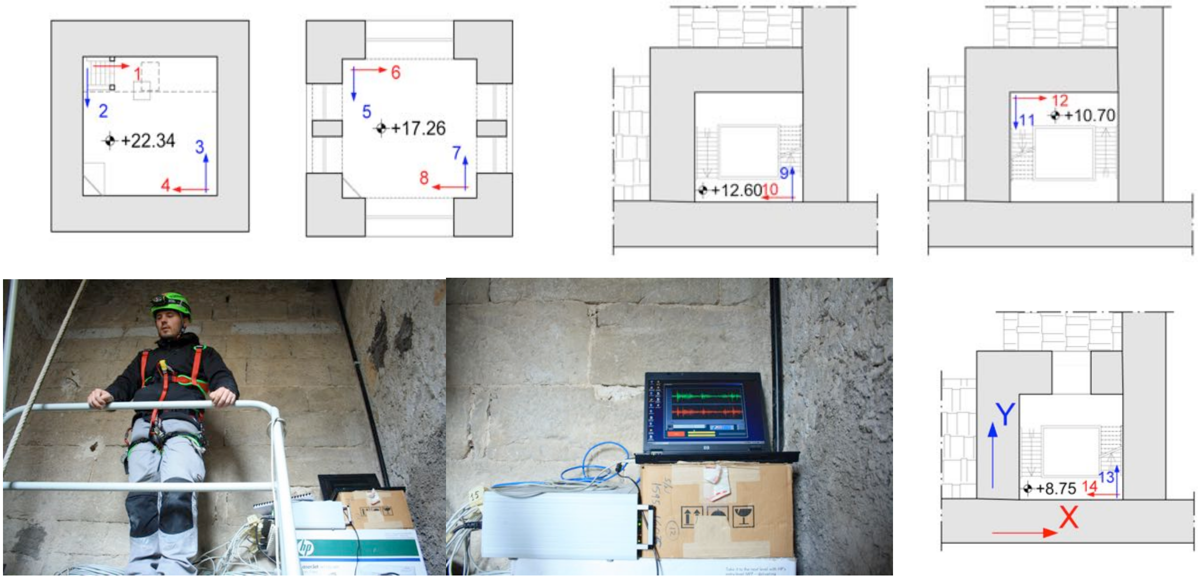


Figure 5. *Dynamic testing of the Saint Bartolomeo's church bell tower: sensor layout (from top to bottom instrumented sections, clockwise) and data acquisition system.*

The FDD is a frequency domain, non-parametric, output-only modal identification technique based on the Singular Value Decomposition (SVD) of the output Power Spectral Density (PSD) matrix. Structural resonances are identified from the singular value plots through peak picking; the corresponding singular vectors are good estimates of the corresponding mode shapes. In the enhanced version of FDD, damping is estimated by an Inverse Fourier Transform of the Power Spectral Density function of the Single Degree of Freedom (SDOF) system corresponding to a mode. The SDOF bell function is identified around the peak of the singular value plot by comparing the mode shape estimate at the resonance with the singular vectors associated with the frequency lines around the peak and retaining all lines above a pre-set threshold of vector correlation. Since partial identification of the SDOF bell functions usually affects the accuracy of damping estimates, the FDD method is herein used to estimate natural frequencies and mode shapes only.

The Cov-SSI method is a time domain, parametric, modal identification procedure based on a state-space description of the dynamic problem. The modal parameters are extracted from realizations of the state matrix and output matrix of the system obtained from measurements of its response to ambient vibrations through algebraic manipulations. In particular, the extraction of the modal properties by the Cov-SSI method is based on the decomposition of the Toeplitz matrix of covariances.

When applied to OMA, SOBI can be referred to as a non-parametric, time domain method, because an a-priori model is not fitted to the data. The method simply takes advantage of their temporal structure to extract the modal information. SOBI belongs to the wide class of Blind Source Separation (BSS) techniques [Ans *et al.*, 1985; Cardoso, 1998] that, given a series of observed signals, are aimed at recovering the underlying sources. It is shown that, under given assumptions, the modal coordinates act as virtual sources [Poncelet *et al.*, 2007], thus allowing the application of this method to output-only modal parameter identification. The definition of virtual sources [Kerschen *et al.*, 2007] establishes a one-to-one relationship between the mixing matrix and the mode shapes on one hand, and the sources and the modal coordinates on the other hand. Thus, the mode shapes are obtained from the columns of the mixing matrix, while natural frequencies and damping ratios are obtained through curve fitting of the sources. The results are summarized in Table 1, while the trace of the output PSD matrix is shown in Figure 6, where the modes are clearly denoted by major peaks in the plot. The identified mode shapes are shown in Figure 7. The graphical representation of the mode shapes is limited to the upper levels. In fact, as previously discussed, the adopted sensor layout was inadequate to observe torsional motion at the lower levels. However, the nature of the mode can be defined, and the modal displacements at the lower levels have been estimated in any case. In fact, the subsequent vulnerability analyses required the knowledge of the distribution of the modal displacements along the whole height of the structure for the fundamental bending modes.

Table 1. *Modal identification results.*

Mode	Type	FDD	Cov-SSI			SOBI		
		f (Hz)	f (Hz)	ξ (%)	$MAC_{FDD-SSI}$	f (Hz)	ξ (%)	$MAC_{FDD-SOBI}$
I	Transl. y	4.10	4.10	0.4	0.999	4.10	0.3	0.999
II	Transl. x	4.72	4.71	0.5	0.996	4.72	0.6	0.997
III	Transl. y & Torsion	8.16	8.18	1.0	0.985	8.15	1.2	0.998
IV	Torsional	9.14	9.15	0.5	0.979	9.13	0.5	0.998

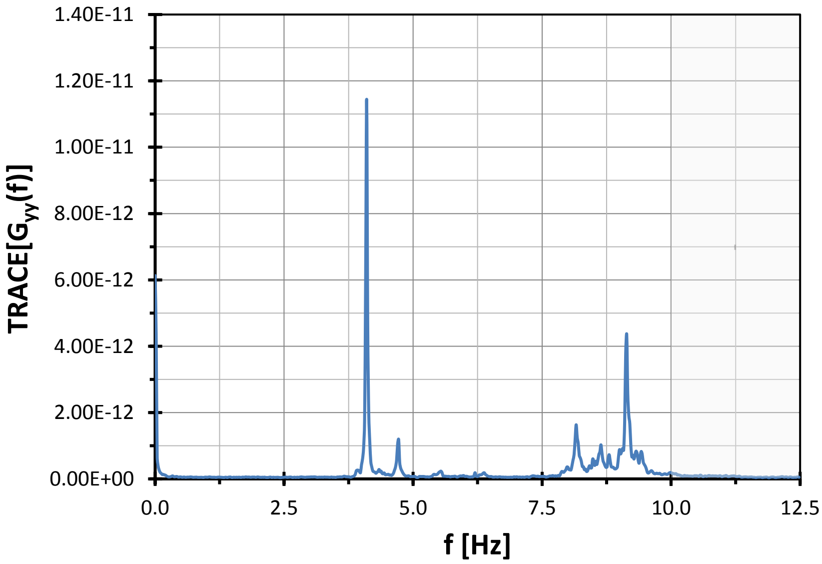


Figure 6. *Trace of the output PSD matrix.*

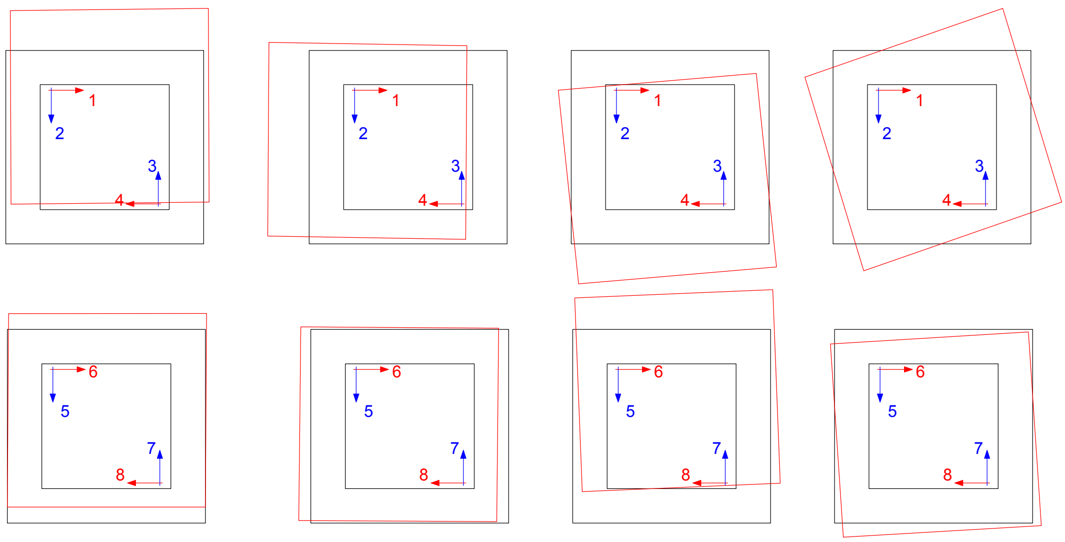
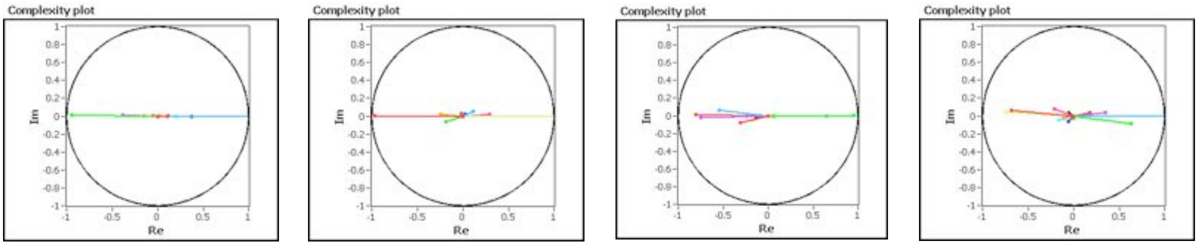


Figure 7. *Plots of the identified mode shapes: mode I to IV from left to right.*

Moreover, even if not reported, the estimated modal displacements at the lower levels of the tower showed negligible displacements in the X direction for bending as well as torsional modes, while larger displacements were observed in the Y direction, confirming that the nearby church offers different degrees of restraint in the two directions. The analysis of the complexity plots (Figure 8) shows that all the identified modes are normal.

The reliability of the obtained results was confirmed by cross-checks in terms of both natural frequencies and mode shape estimates. Consistent results, characterized by negligible scatter in terms of natural frequency estimates and good correlation in terms of mode shape estimates, as pointed out by the MAC values close to 1 in Table 1, were obtained from the different methods, thus confirming the success of the identification process.



(a) (b) (c) (d)

Figure 8. *Complexity plots: mode I (a), II (b), III (c), IV (d).*

3. **Prediction of the fundamental period of masonry towers**

OMA tests provide fundamental information to enhance the accuracy and reliability of numerical models and analysis procedures used for seismic vulnerability assessment. However, in the case of preliminary analyses, often characterized by limited budget availability, alternative options are essential to accurately estimate at least the natural period of the fundamental modes. In fact, the actual seismic response of masonry towers depends, among the rest, on the fundamental periods and their evolution along the transient base excitation. Their accurate evaluation is of interest even at low levels of vibration, because the structural behaviour in the linear regime and the initial seismic response depend on this set of dynamic properties. These play a relevant role also in the context of seismic vulnerability assessment at the large scale [Sepe *et al.*, 2008], when the knowledge of the dynamic parameters of structures and the underlying soil allows the identification of resonance phenomena (such an approach has been recently adopted for microzonation studies in urban and metropolitan areas; see, for instance, [Navarro *et al.*, 2004; Panou *et al.*, 2005; Mucciarelli *et al.*, 2008]).

In the case of historical towers and similar structures, the seismic behavior depends on factors such as slenderness, degree of connection between the walls, presence of lower nearby structures acting as horizontal restraints, soil-structure interaction mechanisms. Since masonry towers are usually characterized by a lower geometric and structural complexity with respect, for instance, to churches, they can be analyzed according to classical structural schemes through accurate and detailed linear models. Linear dynamic (modal) analyses play a primary role in the investigation of the amplification of motion along the structure. This is a critical aspect above all when there are bells in the upper part of the tower. The bell cell, in fact, causes a loss of regularity in elevation. The wide openings for the bells turn the walls into slender and poorly compressed columns. These are usually very vulnerable also in consideration of the amplification of motion from the base to the top of the structure.

A simplified model of masonry towers under seismic loading usually represents the structure as a cantilever beam subjected to the horizontal forces associated to the earthquake. When a linear static analysis is adopted, the total horizontal force depends on the value of the elastic response spectrum. This is evaluated at a period equal to the fundamental natural period of the structure in the considered direction. When the basic assessment LV1 [MiBac, 2010] of the seismic vulnerability of structures is considered, the Direttiva for intervention on historical structures recommends the use of fundamental natural periods obtained from ambient vibration tests, because the dynamic properties they provide can be addressed as representative of the linear elastic behaviour of the structure. The effect of high amplitude vibrations can be taken into account through a simplified approach based on the amplification of the period estimated in operational conditions by a factor in the range 1.4 ÷ 1.75. The amplification factor is introduced for analyses at the ultimate limit state. It accounts for the lengthening of the period induced by increasing levels of damage, cracking and other non-linear phenomena at high amplitude vibrations. Whenever OMA tests cannot be carried out for the structures of interest, reliable empirical correlations for the prediction of the natural period of the fundamental modes can support the execution of accurate, even if basic, LV1 seismic vulnerability analyses. Thus, an attractive use of modal data obtained from ambient vibration tests on a number of homogeneous structures (historical masonry towers) in similar conditions (operational conditions) concerns the development of empirical models and correlations for the prediction of the dynamic properties of the selected class of structures. An example of such models is the one proposed by Jeary to describe the dependence of damping on the amplitude of vibrations in buildings [Jeary, 1986]. This section illustrates how the modal parameter estimates obtained from ambient vibration testing of a number of Italian historic masonry towers have been processed to develop empirical correlations for the prediction of the natural periods of the fundamental modes of those structures as a function of relevant geometric parameters.

A large experimental campaign complemented by an extensive technical literature review represented the roadmap for the collection of a fairly large database of modal properties and relevant geometric data about the historical masonry towers in Italy. Attention has been limited to the Italian territory because the objective is to provide an effective design tool to the Italian professional engineers. The collection of data started from an extensive experimental campaign, based on ambient vibration tests, aimed at the identification of the fundamental dynamic properties of some historical bell towers (Figure 9) in the Molise region (Southern Italy). The results of this experimental campaign are summarized in Table 2.

Figure 9. *The investigated bell towers in the Molise region. IDs are in Table 2.*

Table 2. *Molise's bell towers: output-only modal identification results*.

ID	Bell Tower	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)	f_5 (Hz)	f_6 (Hz)
(a)	Santa Maria delle Rose (Bonefro)	2.96	3.52	6.04	6.96	8.44	-
(b)	San Pardo (Larino)	2.81	3.34	7.00	-	-	-
(c)	Santa Maria della Pietà (Larino)	3.61	3.86	4.34	4.75	7.77	8.43
(d)	Santa Maria Maggiore (Morrone del Sannio)	1.96	2.24	4.76	-	-	-
(e)	Sant'Alfonso dei Liguori (Colletorto)	4.44	5.16	6.96	7.68	8.80	-
(f)	San Giacomo (Santa Croce di Magliano)	3.06	3.44	4.08	-	-	-
(g)	Santa Maria delle Rose (Montorio nei Frentani)	2.74	3.43	3.80	6.02	7.39	7.70
(h)	Santa Maria Assunta (Provvidenti)	2.82	3.40	4.00	-	-	-
(i)	Santa Maria Assunta (Ripabottoni)	2.27	2.68	3.37	3.88	-	-

The dynamic response of the structures was measured at different levels by installing a variable number of accelerometers, ranging from 6 to 20, in couples at opposite corners of the towers, so that both translational and torsional modes could be observed. The accelerometers were of the force balance type, with bandwidth (-3 dB) of about 200 Hz (starting from DC) and high dynamic range (more than 140 dB). The full-scale range was 0.5 g and the sensitivity was 20 V/g. The records were 3600 s long and sampled at 100 Hz. After data validation, filtering and decimation, the ambient vibration response of the bell towers was analyzed by well-established techniques such as FDD, Cov-SSI and SOBI. Data processing in the frequency domain was carried out adopting Hanning window and 66% overlap in spectrum computation. More details about test execution and data processing can be found elsewhere [Rainieri *et al.*, 2009; Rainieri and Fabbrocino, 2012]. The identified modal properties were collected into a database, including also information about geometry, materials, type and number of sensors and other relevant structural detailing data. The database was complemented by data collected through an extensive literature review about masonry bell towers located in Italy, properly reviewed and validated. The validation consisted in a critical review of the reported results, excluding from the dataset inconsistent estimates, case studies characterized by very noisy measurements or poor analysis, and those reporting insufficient information or anomalous estimates (for instance, unrealistic damping).

As a final result, geometric data and experimental estimates of the modal properties in operational conditions for thirty Italian masonry bell towers have been collected in the database. These data have been used first of all to check the predictive performance of the empirical correlations reported in the Italian [NTC, 2008] and Spanish [NSCE-02, 2002] seismic codes and the one suggested in the Annex of the Direttiva related to masonry towers. The empirical correlation reported in [NTC, 2008] provides the fundamental period T_1 of masonry structures as a function of the height H (expressed in meters) as shown in Equation (1).

$$T_1 = 0.050 H^{3/4} \tag{1}$$

$$\omega_1 = \frac{\sqrt{L}}{0.06 H \sqrt{\frac{H}{2L + H}}} \tag{2}$$

$$T_1 = 0.0187 H \tag{3}$$

A linear relation between the fundamental natural period T_1 and the height H of the tower is, instead, reported in the Annex of the Direttiva, see Equation (3), without any reference to experimental or numerical results.

The Spanish Code [NSCE-02, 2002] recommends a formulation for the prediction of the fundamental frequency ω_1 of masonry structures as a function of the height H and the plan dimension L, Equation (2).

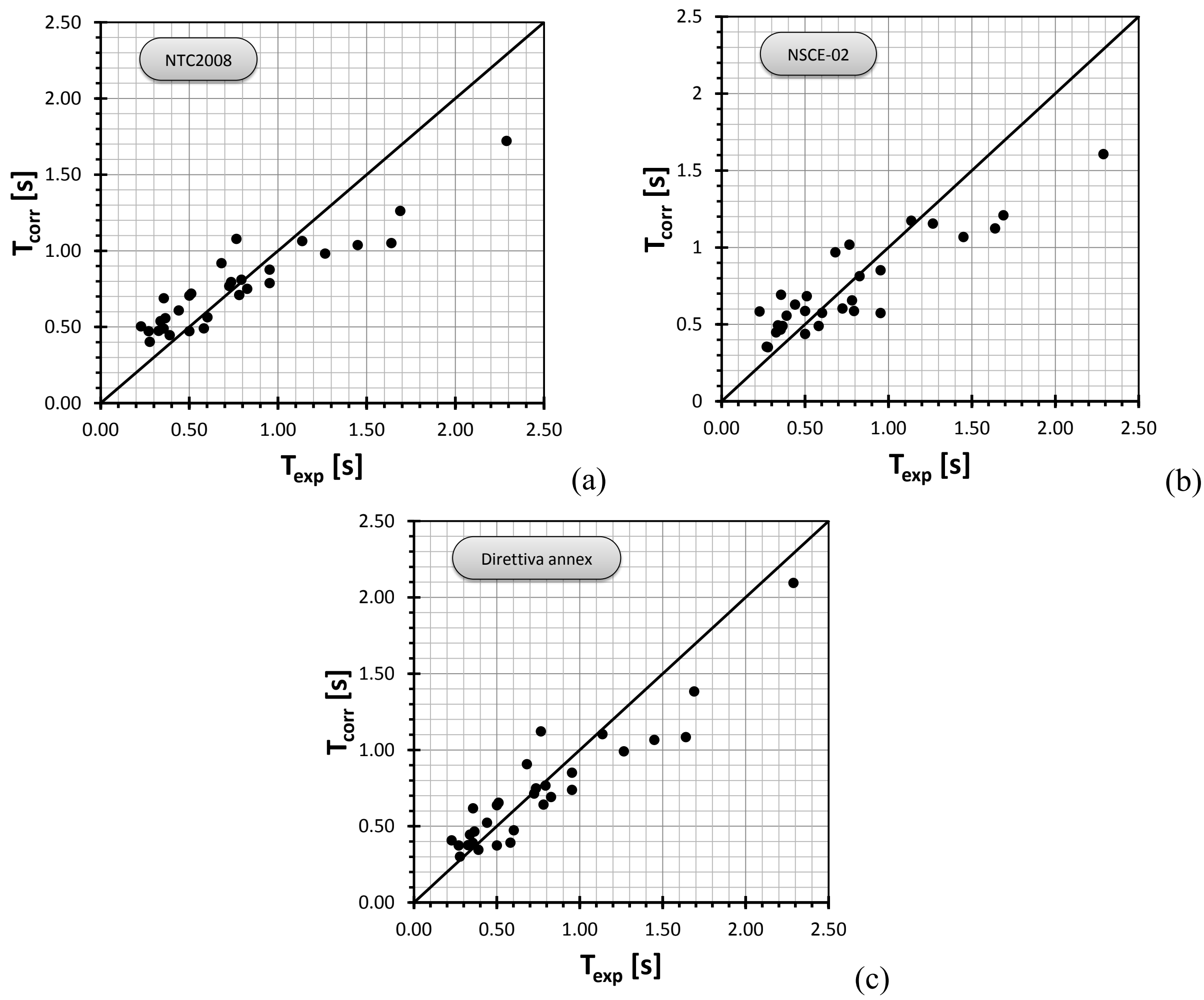


Figure 10. *Comparison between experimental and predicted values of the fundamental period of Italian masonry towers according to Equation (1) (a), Equation (2) (b) and Equation (3) (c) - the black line represents the bisector -*

The comparison of the results provided by the above mentioned correlations with the experimental values of the fundamental period collected into the database (Figure 10) show that Equation (1) and Equation (2) lead to an overestimation for low values of the natural period and to an underestimation at the higher values of the period. This confirms the need to calibrate specific correlations for the towers, as suggested also in the Direttiva. Equation (3) shows a good agreement with the experimental data, at least for periods lower than 1 s. Its performance at higher values of the period cannot be reliably assessed, due to the limited number of available experimental data and their dispersion; however, Equation (3) seems to fit the experimental data better than the previous formulations.

Based on the collected geometric and modal data, another empirical correlation for the prediction of the fundamental period as a function of geometric data has been calibrated by fitting the collected data with a simple mathematical law. A linear relationship between the fundamental period T_1 and the height H of the towers expressed in meters has been considered, on the analogy with former studies [Lagomarsino, 1993]. The coefficient has been estimated by a least squares approach, obtaining the following formulation:

$$T_1 = 0.02H \ [s] \tag{4}$$

In Figure 11 the values of the fundamental period predicted according to Equation (4) are reported as a function of the corresponding experimental estimates. The analysis of the distribution of the points with respect to the bisector (represented by the black line) remarks the good predictive performance of the correlation.

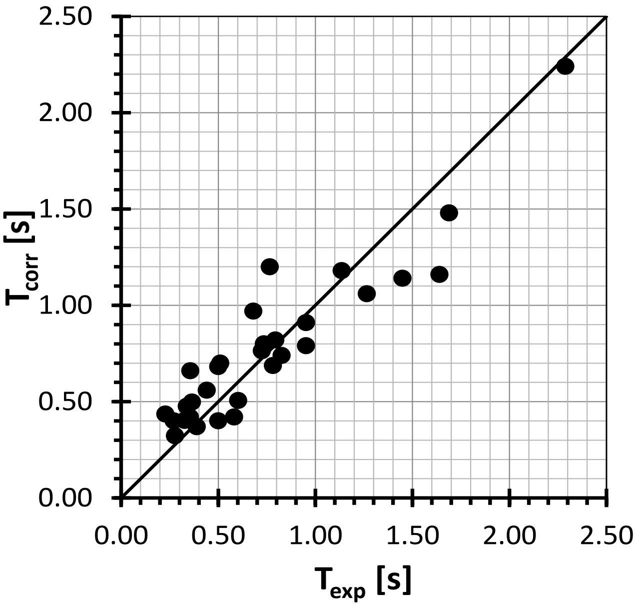


Figure 11. *Comparison between experimental and predicted values of the fundamental period of Italian masonry towers according to Equation (4).*

A quantitative comparison among the different formulations was also carried out by computing the L_2 norm of the proportional error between predicted and measured values:

$$L_2 = \sqrt{\sum_i |e_i|^2} \tag{5}$$

$$e_i = \frac{T_{\exp,i} - T_{corr,i}}{T_{corr,i}} \tag{6}$$

In Equation (6) $T_{exp,i}$ and $T_{corr,i}$ are the i-th experimental and predicted value of the fundamental period, respectively. The results are reported in Table 3. They basically confirm the superior predictive performance of Equation (3) and Equation (4) with respect to Equation (1) and Equation (2). Moreover, the herein reported results provide a rational validation of Equation (3) for the prediction of the fundamental period of the Italian historic masonry towers. In fact, even if that formulation is recommended in the Annex of the Direttiva, it is currently not supported by specific and well-documented background. Taking into account that the best fit of the experimental data is given by Equation (4), which shows good predictive performance in a fairly wide range of heights, it can be confidently adopted as an alternative to Equation (3) in the current engineering practice for the prediction of the fundamental period of the Italian historical masonry towers.

Table 3. *Quantitative comparisons among different predictive correlations.*

Equation	(1)	(2)	(3)	(4)
L_2 norm	1.60	1.68	1.42	1.35

The collected data have also made possible the development of correlations for the prediction of the natural period of higher modes as a function of the period of the fundamental mode. In fact, the natural period T_2 of the second mode of the considered masonry towers was found to be related to the natural period T_1 of the first mode as follows:

$$T_2 = 0.86 \cdot T_1 \tag{7}$$

In fact, as shown in Figure 12, the collected data show a linear correlation between the two values of the natural period.

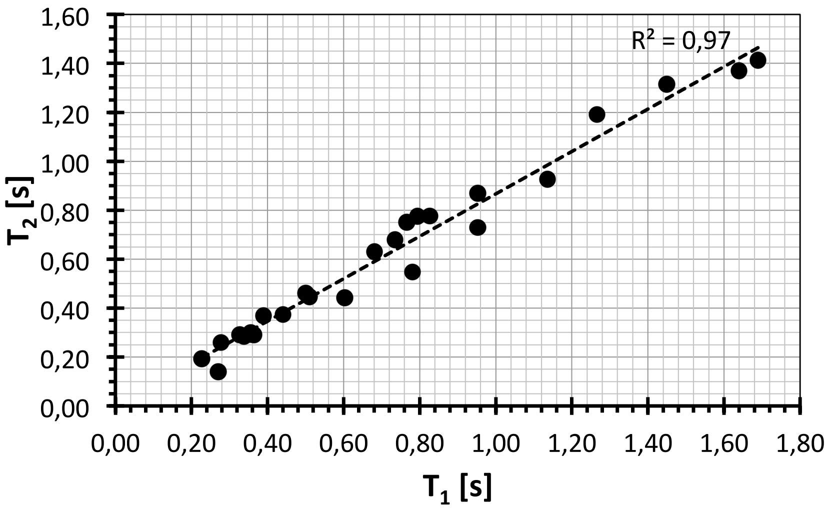


Figure 12. *Correlation between the natural periods of the first two modes for the Italian masonry towers*.

Finally, the period of the third (torsional) mode can be estimated from the fundamental period as follows:

$$T_3 = 0.4 \cdot T_1 \tag{8}$$

while the periods of the higher bending modes can be estimated from the periods of the corresponding fundamental modes as follows:

$$T_4 = 0.3 \cdot T_1 \tag{9}$$

$$T_5 = 0.3 \cdot T_2 \tag{10}$$

The correlations expressed by Equation (8), Equation (9) and Equation (10) are based on a more limited number of experimental values (about ten) and they have to be better assessed against a larger population of samples. Nevertheless, they are in good agreement with similar correlations available in the literature for masonry buildings [NSCE-02, 2002].

4. **Conclusions**

The applicative perspectives of OMA techniques in the context of basic (LV1) seismic vulnerability analyses of historical masonry towers have been discussed starting from an explanatory case aimed at the experimental analysis of the dynamic response of the bell tower of Saint Bartolomeo’s church at the Carthusian Monastery of Trisulti in Collepardo. Moreover, a database of geometric data and modal properties obtained from ambient vibration tests for a number of Italian historic masonry towers has been presented, and its role in the assessment of the performance of available correlations for the prediction of the fundamental period and the development of a new one has been illustrated. A simple, linear correlation between the fundamental period and the height of the towers has been validated. A number of quantitative and qualitative comparisons have shown that it can be confidently used in the engineering practice. Correlations for the prediction of the period of higher modes have also been reported. They basically agree with similar correlations available in the literature. In particular, the 3:1 ratio between the frequency of higher bending modes and those of the corresponding fundamental modes is in agreement with a cantilever scheme of the tower. Notwithstanding the encouraging results, these last correlations require further validation based on an extended dataset.

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